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强作用与手征规范场

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摘 要

本文引入了 $e^{i\alpha(x)\gamma_5}$ 定域规范变换及相应的规范场. 给出了手征规范不变的拉氏函数，并由此出发探讨了手征规范场与强相互作用的联系. 在特定条件下可给出唯象的 V-A 强相互作用形式.

一、引 言

1966年北京基本粒子理论组提出层子模型时，我们曾建议采用宇称守恒的 V-A 费米作用

$$\mathscr{L}_s = -f_s\{\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma_\mu\psi + \bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\gamma_\mu\gamma_5\psi\} \tag{1.1}$$

为强相互作用的等效拉氏函数[1]. 它有如下的特点:

(1) $SU(3)$ 或 $SU(4)$ 不变、菲尔兹换位不变，且在非相对论近似下给出自旋交换力.

(2) 赝矢为主，即 A 项是主要的.

(3) $\frac{f_s^2 m_\pi^2}{4\pi}|\Psi(0)|^2 = 3.6\times10^{-3}$，给出的 πNN 顶角为

$$\mathscr{H}_s = i\frac{20f_s\Psi(0)}{3\sqrt{2}}\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}N\cdot\frac{\partial\boldsymbol{\pi}}{\partial x_\mu}, \tag{1.2}$$

其中 $\Psi(0)$ 为介子波函数零点值.

用 $\mathscr{L}_s$ 计算低能强作用过程如自旋质量差、强衰变等等，发现理论与实验相当符合. 但由于费米型相互作用不能重正化，不能计算高次过程. 因此自然提出一个问题: 强作用的汤川形式是什么? 最小的内部对称性又是什么? 总之，强作用是否存在着规范场? 下面就这些问题加以探讨.

二、手征规范场

引入拉氏函数

$$\mathscr{L} = -\bar{\psi}\gamma_\mu\partial_\mu\psi, \tag{2.1}$$

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式中 ψ 是层子场量. 显然它在手征规范变换

$$\psi(x) \to \psi'(x) = e^{i\alpha\gamma_5}\psi(x) \tag{2.2}$$

下是不变的. 其中 α 是一常数.

定义手征流

$$j_\mu(x) = i\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x), \tag{2.3}$$

则有

$$\partial_\mu j_\mu(x) = 0. \tag{2.4}$$

令

$$H = -i\int d^3x j_4(x) = \int d^3x \psi^\dagger(x)\gamma_5\psi(x), \tag{2.5}$$

H 称为手征, 则 (2.4) 可化为:

$$\frac{dH}{dt} = 0. \tag{2.6}$$

由正则量子化条件 $\{\psi_\alpha(x), \psi_\beta^\dagger(x')\}_{t=t'} = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}')$ 可以导出

$$[\psi(x), H] = \gamma_5\psi(x). \tag{2.7}$$

现在如果认为手征规范变换在每个时空点都可独立选择即 $\alpha = \alpha(x)$ 是时空点的函数, 则微分符号变为:

$$\partial_\mu\psi \to \partial_\mu\psi' = e^{i\alpha\gamma_5}(\partial_\mu + i\gamma_5\partial_\mu\alpha)\psi.$$

为了保持拉氏函数的不变性, 引进手征规范场

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{f}\partial_\mu\alpha(x), \tag{2.8}$$

即 $A_\mu(x)$ 按 (2.8) 式变换, 则

$$(\partial_\mu - if\gamma_5A_\mu)\psi(x) \to (\partial_\mu - if\gamma_5A'_\mu)\psi' = e^{i\alpha(x)\gamma_5}(\partial_\mu - if\gamma_5A_\mu)\psi.$$

由此获得手征规范不变的拉氏函数:

$$\mathscr{L}(x) = -\bar{\psi}\gamma_\mu(\partial_\mu - if\gamma_5A_\mu)\psi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2. \tag{2.9}$$

这样我们得到了层子场与手征规范场的赝矢汤川相互作用.

为了使层子场及手征规范场获得质量, 我们引进 Higgs 场:

$$\phi(x) = \frac{\sigma(x) + i\pi(x)}{\sqrt{2}}, \tag{2.10}$$

其中 $\sigma(x)$ 为实标量场, $\pi(x)$ 为实赝标量场. 手征规范变换对 $\bar{\psi}\psi$, $i\bar{\psi}\gamma_5\psi$ 而言可看成宇称空间的转动[3]:

$$\begin{aligned}\bar{\psi}\psi \to \bar{\psi}'\psi' &= \bar{\psi}e^{2i\alpha\gamma_5}\psi = \bar{\psi}\psi\cos 2\alpha + i\bar{\psi}\gamma_5\psi\sin 2\alpha,\\ i\bar{\psi}\gamma_5\psi \to i\bar{\psi}'\gamma_5\psi' &= i\bar{\psi}\gamma_5e^{2i\alpha\gamma_5}\psi = -\bar{\psi}\psi\sin 2\alpha + i\bar{\psi}\gamma_5\psi\cos 2\alpha.\end{aligned} \tag{2.11}$$

设 $\phi(x)$ 在手征规范变换下按 (2.11) 变即:

$$\begin{aligned}\sigma(x) \to \sigma'(x) &= \sigma(x)\cos 2\alpha + \pi(x)\sin 2\alpha,\\ \pi(x) \to \pi'(x) &= -\sigma(x)\sin 2\alpha + \pi(x)\cos 2\alpha,\end{aligned} \tag{2.12}$$

或

$$\phi(x) \to \phi'(x) = e^{-2i\alpha}\phi(x), \tag{2.13}$$

则 $\bar{\psi}(\sigma+i\gamma_5\pi)\psi$ 是宇称空间转动的不变量．再由 (2.13) 可见，$\phi(x)$ 的协变微分为：

$$(\partial_\mu+2ifA_\mu)\phi\to(\partial_\mu+2ifA'_\mu)\phi'=e^{-2i\alpha}(\partial_\mu+2ifA_\mu)\phi.$$

由此给出手征规范不变的拉氏函数：

$$\mathscr{L}=-\bar{\psi}\gamma_\mu(\partial_\mu-if\gamma_5A_\mu)\psi-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2-g\bar{\psi}(\sigma+i\gamma_5\pi)\psi$$
$$-(\partial_\mu-2ifA_\mu)\phi^*(\partial_\mu+2ifA_\mu)\phi-\mu^2\phi^*\phi-\lambda(\phi^*\phi)^2. \tag{2.14}$$

式中第三项使层子场 ψ 获得质量，第四、五项使手征规范场获得质量．$\mu^2<0$，$\lambda>0$.

三、 $\alpha A+\beta S$ 型强作用

假定 $\langle\sigma(x)\rangle_0\neq0$，$\langle\pi(x)\rangle_0=0$，

即

$$\langle\phi(x)\rangle_0=\frac{v}{\sqrt{2}},\ v\text{ 为常数}. \tag{3.1}$$

由 (2.14) 式可看出

$$v=\sqrt{\frac{-\mu^2}{\lambda}},\ \mu^2<0,\ \lambda>0. \tag{3.2}$$

按标准的办法把 $\phi(x)$ 参数化为

$$\phi(x)=e^{i\xi(x)/v}\frac{v+\eta(x)}{\sqrt{2}}=\frac{1}{\sqrt{2}}[v+\eta(x)+i\xi(x)+\cdots], \tag{3.3}$$

这里 $\eta(x)$ 为实标量场且 $\langle\eta(x)\rangle_0=0$，$\xi(x)$ 为实赝标量场 $\langle\xi(x)\rangle_0=0$．如果选取手征规范变换 $\alpha(x)=\xi(x)/2v$，则

$$\left.\begin{aligned}&\phi\to\phi'=e^{-2i\alpha(x)}\phi=\frac{v+\eta(x)}{\sqrt{2}},\\&A_\mu\to A'_\mu=A_\mu+\frac{1}{f}\partial_\mu\alpha(x)=A_\mu+\frac{1}{2fv}\partial_\mu\xi(x),\\&\psi\to\psi'=e^{i\alpha\gamma_5}\psi=e^{i\frac{\xi(x)}{2v}\gamma_5}\psi(x).\end{aligned}\right\} \tag{3.4}$$

$\mathscr{L}$ 在手征规范变换下不变．当用 ϕ', ψ', A'_μ 写出时为：

$$\mathscr{L}\equiv\mathscr{L}'=\mathscr{L}_0+\mathscr{L}_i+\text{常数项}, \tag{3.5}$$

其中

$$\mathscr{L}_0=-gv\bar{\psi}'\psi'-\bar{\psi}'\gamma_\mu\partial_\mu\psi'-\frac{1}{4}(\partial_\mu A'_\nu-\partial_\nu A'_\mu)^2$$
$$-2f^2v^2A'_\mu A'_\mu-\frac{1}{2}\partial_\mu\eta\partial_\mu\eta+\mu^2\eta^2-\lambda v\eta^3-\frac{\lambda}{4}\eta^4, \tag{3.6}$$

$$\mathscr{L}_i=if\bar{\psi}'\gamma_\mu\gamma_5A'_\mu\psi'-g\bar{\psi}'\psi'\eta-2f^2A'_\mu A'_\mu\eta^2-4vf^2A'_\mu A'_\mu\eta. \tag{3.7}$$

由 $\mathscr{L}_0$ 表式可给出粒子的质量：

$$m_\psi=gv\quad m_\eta^2=-2\mu^2\quad m_A^2=4f^2v^2. \tag{3.8}$$

当 m_A、m_η 很大时，在低能过程中 $\mathscr{L}_i$ 给出层子间的费米相互作用：

$$\mathscr{L}_s=\frac{1}{2}\left\{\frac{g^2}{m_\eta^2}\bar{\phi}\phi\bar{\phi}\phi-\frac{f^2}{m_A^2}\bar{\phi}\gamma_\mu\gamma_5\phi\,\bar{\phi}\gamma_\mu\gamma_5\phi\right\}. \tag{3.9}$$

$\mathscr{L}_s$ 给出的 πNN 顶角为：

$$\mathscr{H}_s=i\frac{5\Psi(0)}{6\sqrt{2}}\left[2\left(1+\frac{m_\pi}{m_N}\right)\left(\frac{f}{m_A}\right)^2-\left(1+\frac{m_\pi}{2m_N}\right)\left(\frac{g}{m_\eta}\right)^2\right]\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}N\cdot\frac{\partial\boldsymbol{\pi}}{\partial x_\mu}. \tag{3.10}$$

我们称 (3.9)、(3.10) 为 $\alpha A+\beta S$ 型强作用．与 (1.2) 比较有：

$$2\left(1+\frac{m_\pi}{m_N}\right)\left(\frac{f}{m_A}\right)^2-\left(1+\frac{m_\pi}{2m_N}\right)\left(\frac{g}{m_\eta}\right)^2=8f_s. \tag{3.11}$$

如果假定

$$\left(\frac{g}{m_\eta}\right)^2=2\left(\frac{f}{m_A}\right)^2, \tag{3.12}$$

则

$$\mathscr{L}_s=\frac{1}{2}\left(\frac{f}{m_A}\right)^2\{2\bar{\phi}\phi\bar{\phi}\phi-\bar{\phi}\gamma_\mu\gamma_5\phi\bar{\phi}\gamma_\mu\gamma_5\phi\}, \tag{3.13}$$

$$\mathscr{H}_s=i\frac{5\Psi(0)}{6\sqrt{2}}\left(\frac{f}{m_A}\right)^2\left(\frac{m_\pi}{m_N}\right)\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}N\cdot\frac{\partial\boldsymbol{\pi}}{\partial x_\mu}. \tag{3.14}$$

(3.13)、(3.14) 我们称之为 $A+2S$ 型强作用．

由 (1.2) 式及 (3.14) 式 (3.8) 式可得：

$$\left(\frac{f}{m_A}\right)^2\frac{\Psi(0)m_\pi}{4\pi}\sim 1,\quad v^2=\frac{\Psi(0)m_\pi}{16\pi}. \tag{3.15}$$

四、粒子数规范场

引入拉氏函数

$$\mathscr{L}=-\bar{\phi}\gamma_\mu\partial_\mu\phi \tag{4.1}$$

和粒子数规范变换

$$\phi\to\phi'=e^{i\beta}\phi(x), \tag{4.2}$$

β 为常数．则 (4.1) 中 $\mathscr{L}$ 在 (4.2) 变换下不变．因而导出守恒定律：几率(或粒子数)守恒：

$$\partial_\mu j_\mu=0,\quad j_\mu=i\bar{\phi}\gamma_\mu\phi. \tag{4.3}$$

令

$$N=-i\int d^3x j_4(x)=\int d^3x\phi^+(x)\phi(x), \tag{4.4}$$

则 (4.4) 可改写为

$$\frac{dN}{dt}=0. \tag{4.5}$$

由

$$\{\psi_\alpha(x),\ \psi_\beta^+(x')\}_{t=t'} = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}') \tag{4.6}$$

易导出

$$[\psi(x),\ N] = \psi(x), \tag{4.7}$$

N称粒子数或几率．在希尔伯空间，(4.2) 可表为幺正变换:

$$\psi'(x) = e^{-i\beta N}\psi(x)e^{i\beta N} = e^{i\beta}\psi(x). \tag{4.8}$$

如果认为粒子数规范变换在每个时空点可独立选择．即

$$\beta = \beta(x), \tag{4.9}$$

就导出了粒子数规范场的观念．

$$B_\mu(x) \to B'_\mu(x) = B_\mu(x) + \frac{1}{g_B}\partial_\mu\beta(x), \tag{4.10}$$

协变微分为:

$$(\partial_\mu - ig_BB_\mu)\psi \to (\partial_\mu - ig_BB'_\mu)\psi' = e^{i\beta(x)}(\partial_\mu - ig_BB_\mu)\psi. \tag{4.11}$$

由此，粒子数规范不变的拉氏函数为:

$$\mathscr{L} = -\bar{\psi}\gamma_\mu(\partial_\mu - ig_BB_\mu)\psi - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2. \tag{4.12}$$

为使粒子数规范场 B_μ 获得质量，引入复数场:

$$\chi = \frac{\chi_1(x) + i\chi_2(x)}{\sqrt{2}}, \tag{4.13}$$

其中 χ_1, χ_2 为实标量场．则粒子数规范不变的拉氏函数为:

$$\begin{aligned}\mathscr{L} = &-\bar{\psi}\gamma_\mu(\partial_\mu - ig_BB_\mu)\psi - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2\\ &-(\partial_\mu - 2ig_BB_\mu)\chi^*(\partial_\mu + 2ig_BB_\mu)\chi - \nu^2(\chi^*\chi) - \delta(\chi^*\chi)^2.\end{aligned} \tag{4.14}$$

由 (4.14) 可见，χ 的引入不能使 ψ 获得质量．

五、V-A 型强相互作用

由 (2.14)，(4.14) 可见，手征规范与粒子数规范不变的拉氏函数可写成:

$$\begin{aligned}\mathscr{L} = &-\bar{\psi}\gamma_\mu(\partial_\mu - if\gamma_5A_\mu - ig_BB_\mu)\psi - g\bar{\psi}(\sigma + i\pi\gamma_5)\psi\\ &-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2\\ &-(\partial_\mu - 2ifA_\mu)\phi^*(\partial_\mu + 2ifA_\mu)\phi - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2\\ &-(\partial_\mu - 2ig_BB_\mu)\chi^*(\partial_\mu + 2ig_BB_\mu)\chi - \nu^2(\chi^*\chi) - \delta(\chi^*\chi)^2,\end{aligned} \tag{5.1}$$

ϕ, χ 由 (2.10) 及 (4.13) 式表出．假定

$$\left.\begin{aligned}\langle\sigma\rangle_0 &= v = \sqrt{\frac{-\mu^2}{\lambda}},\\ \langle\pi\rangle_0 &= \langle\chi_2\rangle_0 = 0,\\ \langle\chi_1\rangle_0 &= w = \sqrt{\frac{-\nu^2}{\delta}},\end{aligned}\right\} \tag{5.2}$$

将 $\phi(x)$, $\chi(x)$ 参数化:

$$\left.\begin{aligned}\phi(x) &= e^{i\xi(x)/v}\frac{v+\eta(x)}{\sqrt{2}},\\ \chi(x) &= e^{i\zeta(x)/w}\frac{w+\tau(x)}{\sqrt{2}}.\end{aligned}\right\}\tag{5.3}$$

同时作手征规范变换和粒子数规范变换并选

$$\alpha(x)=\frac{\xi(x)}{2v},\quad \beta(x)=\frac{\zeta(x)}{2w},\tag{5.4}$$

则有:

$$\left.\begin{aligned}&\phi(x)\to\phi'(x)=e^{-2i\alpha(x)}\phi=\frac{v+\eta(x)}{\sqrt{2}},\\ &A_\mu(x)\to A'_\mu(x)=A_\mu(x)+\frac{1}{2fv}\partial_\mu\xi(x),\\ &\psi(x)\to\psi'(x)=e^{i\gamma_5\alpha(x)}\psi=e^{\frac{i\xi(x)}{2v}\gamma_5+\frac{i\zeta(x)}{2w}}\psi(x),\\ &\chi(x)\to\chi'(x)=e^{-2i\beta(x)}\chi=\frac{w+\tau(x)}{\sqrt{2}},\\ &B_\mu(x)\to B'_\mu(x)=B_\mu(x)+\frac{1}{2gw}\partial_\mu\zeta(x).\end{aligned}\right\}\tag{5.5}$$

在 (5.5) 变换下

$$\mathscr{L}=\mathscr{L}'\equiv\mathscr{L}_0+\mathscr{L}_i+\text{常数项},\tag{5.6}$$

其中

$$\begin{aligned}\mathscr{L}_0=&-\bar{\psi}'\gamma_\mu\partial_\mu\psi'-gv\bar{\psi}'\psi'-\frac{1}{4}(\partial_\mu A'_\nu-\partial_\nu A'_\mu)^2-2f^2v^2A'_\mu A'_\mu\\ &-\frac{1}{2}\partial_\mu\eta\partial_\mu\eta+\mu^2\eta^2-\lambda v\eta^3-\frac{\lambda}{4}\eta^4\\ &-\frac{1}{4}(\partial_\mu B'_\nu-\partial_\nu B'_\mu)^2-2g_B^2w^2B'_\mu B'_\mu-\frac{1}{2}\partial_\mu\tau\partial_\mu\tau+\nu^2\tau^2\\ &-\delta w\tau^3-\frac{\delta}{4}\tau^4,\end{aligned}\tag{5.7}$$

$$\begin{aligned}\mathscr{L}_i=&-g\bar{\psi}'\psi'\eta+if\bar{\psi}'\gamma_\mu\gamma_5A'_\mu\psi'-2f^2A'_\mu A'_\mu\eta^2-4vf^2A'_\mu A'_\mu\eta\\ &+ig_B\bar{\psi}'\gamma_\mu B'_\mu\psi'-2g_B^2B'_\mu B'_\mu\tau^2-4wg_B^2B'_\mu B'_\mu\tau.\end{aligned}\tag{5.8}$$

由 $\mathscr{L}_0$ 可读出粒子的质量:

$$m_\psi=gv,\quad m_A=2fv,\quad m_\eta^2=-2\mu^2,\quad m_B=2g_Bw,\quad m_\chi^2=-2\nu^2.\tag{5.9}$$

因 $\mathscr{L}_i$ 中有规范场与层子场相互作用的项

$$-g\bar{\psi}'\psi'\eta(x)+if\bar{\psi}'\gamma_\mu\gamma_5\psi'A'_\mu+ig_B\bar{\psi}'\gamma_\mu\psi'B'_\mu,$$

当 m_η、m_A、m_B 很大时,$\mathscr{L}_i$ 在低能强作用过程中给出 $\alpha S+\beta V+\gamma A$ 型强作用,

$$\mathscr{L}_s=\frac{1}{2}\left\{\left(\frac{g}{m_\eta}\right)^2\bar{\psi}'\psi'\bar{\psi}'\psi'-\left(\frac{g_B}{m_B}\right)^2\bar{\psi}'\gamma_\mu\psi'\bar{\psi}'\gamma_\mu\psi'-\left(\frac{f}{m_A}\right)^2\bar{\psi}'\gamma_\mu\gamma_5\psi'\bar{\psi}'\gamma_\mu\gamma_5\psi'\right\}.\tag{5.10}$$

当 $\mu^2=\nu^2$, $\lambda=\delta$ 时，即 $w=v$ 时，

有

$$\left(\frac{f}{m_A}\right)^2=\left(\frac{g_B}{m_B}\right)^2=\frac{1}{4v^2}.$$

如果再加上条件 $(g/m_\eta)^2 \ll 1$. 即 m_η 很大，则 (5.10) 中 S 型项可略去且最后给出 V-A 型的等效拉氏量:

$$\mathscr{L}_{s,\mathrm{eff}}=-\frac{1}{8v^2}\{\bar{\Phi}'\gamma_\mu\psi'\bar{\Phi}'\gamma_\mu\psi'+\bar{\Phi}'\gamma_\mu\gamma_5\psi'\bar{\Phi}'\gamma_\mu\gamma_5\psi'\}. \tag{5.11}$$

由 (1.1)，

$$f_s=\frac{1}{8v^2}\quad 或\quad v^2=0.59\Psi(0)m_\pi, \tag{5.12}$$

我们得到如下结论:

低能强作用可能与手征规范场和粒子数规范场相联系.

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STRONG INTERACTION AND CHIRAL GAUGE FIELD

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ABSTRACT

The concept of $e^{i\alpha(x)\gamma_5}$ local gauge transformation is introduced in this article. The chiral gauge invariant Lagrangian is given. The relationship between chiral gauge field and strong interaction is also discussed. In a special condition, we obtain the effective Lagrangian for V-A type strong interaction.