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单个电子在电磁场中加速问题

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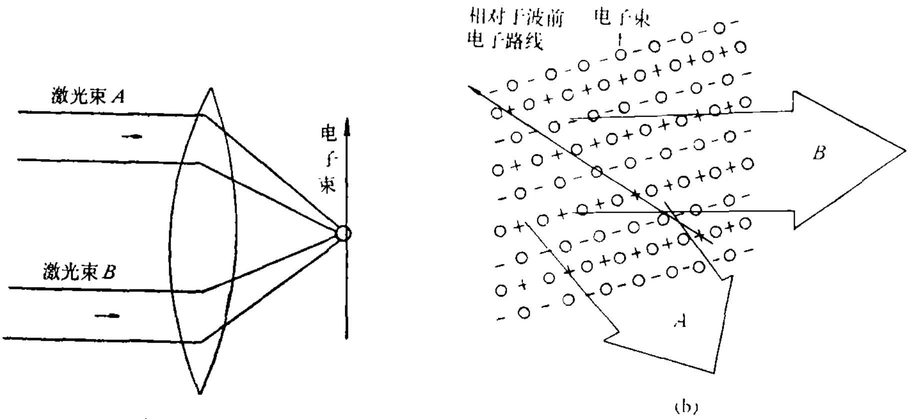
摘　　要

本文内容是应用作者变号场[2]性质，给出了电子在经典电磁场中加速问题的一个解决方法，对罗斯曼厄思所提出的设想，给出了一种新结果.

一、引　　言

英国《新科学家》1976年72卷1032期715页报道西德的电子同步加速实验室所提出的，用高功率激光束加速电子的新设想，高功率激光焦点处的电场可达100千兆电子伏/cm量级，而激光焦点的直径可能仅有几十微米，然而它至少具有使加速技术取得突破的潜力. 这个场所形成一种电磁波，在空间的任何一点，该场往往是正的即从一个方向驱动粒子，也往往是负的，即从相反方向驱动该粒子. 因此，就要求该场具有某种结构性质，使粒子能获得所预期的加速. 目前西德罗斯曼厄思提出一种设想，即利用所谓加速隧道. 如图:



(a)　　(b)

a图中所示两束激光在同一点上聚焦，并产生干涉图如(b)图，一电子束将在越过焦点时成像. 图内线代表两束光的波前;(＋)号代表能产生一电场使电子加速的干涉;(—)号代表使电子减速的电场;(○)号代表零场. 此种干涉图将连续不断波浪式穿过焦点. 罗斯

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曼厄思证明沿(b)图可得一(○),(+)的场而失去减速部分,得一加速隧道,但这样一来,得到的相速度比光速还快,而这是不能达到持续加速的目的. 为此罗斯曼厄思提出让光束聚焦于荧光晶体使相速度低于光速,而与被加速电子的运动速度同步.

上面所得到的场中有 +，○，−，是与作者所提出的变号场[1]一致的. 即具有非平直结构的场,在这样的场中可以找到一个场缝或势界,使得粒子从某一初速通过势界到达指定的预期的末速，而所经过时间是最小的. 在这里是说：可以找到电场分量与磁场分量积分(对时间积分)之间的符号函数型耦合性质,以保证达到电子加速的要求.

二、问题的叙述

设电荷在经典电磁场中运动方程为

$$m\frac{dV}{dt}=eE+\frac{e}{c}V\times H \tag{1}$$

其中

$$eE=-\frac{e}{c}\begin{pmatrix}\dfrac{\partial A_1}{\partial t}\\ \dfrac{\partial A_2}{\partial t}\\ \dfrac{\partial A_3}{\partial t}\end{pmatrix}-e\begin{pmatrix}\dfrac{\partial\varphi}{\partial x_1}\\ \dfrac{\partial\varphi}{\partial x_2}\\ \dfrac{\partial\varphi}{\partial x_3}\end{pmatrix},\quad H=\operatorname{rot}A=\begin{pmatrix}\dfrac{\partial A_3}{\partial x_2}-\dfrac{\partial A_2}{\partial x_3}\\ \dfrac{\partial A_1}{\partial x_3}-\dfrac{\partial A_3}{\partial x_1}\\ \dfrac{\partial A_2}{\partial x_1}-\dfrac{\partial A_1}{\partial x_3}\end{pmatrix},\quad V=\begin{pmatrix}V_1\\ V_2\\ V_3\end{pmatrix},$$

故有

$$V\times H=\begin{pmatrix}V_2H_3-V_3H_2\\ V_3H_1-V_1H_3\\ V_1H_2-V_2H_1\end{pmatrix}=\begin{pmatrix}0 & H_3 & -H_2\\ -H_3 & 0 & H_1\\ H_2 & -H_1 & 0\end{pmatrix}\begin{pmatrix}V_1\\ V_2\\ V_3\end{pmatrix}. \tag{2}$$

于是 (1) 可变为:

$$m\begin{pmatrix}\dfrac{dV_1}{dt}\\ \dfrac{dV_2}{dt}\\ \dfrac{dV_3}{dt}\end{pmatrix}=\begin{pmatrix}0 & H_3 & -H_2\\ -H_3 & 0 & H_1\\ H_2 & -H_1 & 0\end{pmatrix}\begin{pmatrix}V_1\\ V_2\\ V_3\end{pmatrix}+\begin{pmatrix}\dfrac{e}{c}\dfrac{\partial A_1}{\partial t}-e\dfrac{\partial\varphi}{\partial x_1}\\ \dfrac{e}{c}\dfrac{\partial A_2}{\partial t}-e\dfrac{\partial\varphi}{\partial x_2}\\ \dfrac{e}{c}\dfrac{\partial A_3}{\partial t}-e\dfrac{\partial\varphi}{\partial x_3}\end{pmatrix}. \tag{3}$$

初值和终值为:

$$\begin{pmatrix}V_1(t_0)\\ V_2(t_0)\\ V_3(t_0)\end{pmatrix}=\begin{pmatrix}V_{10}\\ V_{20}\\ V_{30}\end{pmatrix},\quad \begin{pmatrix}V_1(T)\\ V_2(T)\\ V_3(T)\end{pmatrix}=\begin{pmatrix}V_{11}\\ V_{12}\\ V_{13}\end{pmatrix}. \tag{4}$$

V_{11}, V_{12}, V_{13} 为指定的预期数值. 现在问题提为，电场强度分量与磁场强度怎样调整时，(3)—(4) 的解能使

$$\int_{t_0}^{T}dt=\min., \tag{5}$$

即

$$J[V(t_0),\ t_0] = \min. \int_{t_0}^{T} d\tau. \tag{6}$$

其中 T 是不固定的.

三、问 题 解 法

按动态规划[2]方法,我们由 (6) 可得哈米顿-耶可比方程

$$-\frac{\partial J}{\partial t} = \min_{B\in\mathscr{E}} \left[1 + \frac{\partial J}{\partial V_1}\frac{dV_1}{dt} + \frac{\partial J}{\partial V_2}\frac{dV_2}{dt} + \frac{\partial J}{\partial t}\frac{dV_3}{dt}\right]. \tag{7}$$

再由 (3), 知道 (3) 齐次共轭系的解为:

$$\boldsymbol{\phi}(t) = \exp\left[-\int_{t_0}^{T} \frac{e}{cm}\begin{pmatrix} 0 & H_3 & -H_2 \\ -H_3 & 0 & H_1 \\ H_2 & -H_1 & 0 \end{pmatrix} d\tau\right]\boldsymbol{\phi}(t_0). \tag{8}$$

我们将

$$\exp\left[-\int_{t_0}^{T} \frac{e}{cm}\begin{pmatrix} 0 & H_3 & -H_2 \\ -H_3 & 0 & H_1 \\ H_2 & -H_1 & 0 \end{pmatrix} d\tau\right]$$

展开,得:

$$\exp\left[-\int_{t_0}^{T} \frac{e}{cm}\begin{pmatrix} 0 & H_3 & -H_2 \\ -H_3 & 0 & H_1 \\ H_2 & -H_1 & 0 \end{pmatrix} d\tau\right]$$

$$= \frac{e}{cm}\left[I - \int_{t_0}^{T}\begin{pmatrix} 0 & H_3 & -H_2 \\ -H_3 & 0 & H_1 \\ H_2 & -H_2 & 0 \end{pmatrix} d\tau + \cdots\right].$$

取一次近似

$$\boldsymbol{\phi}(t) \cong -\frac{e}{cm}\int_{t_0}^{T}\begin{bmatrix} 0 & -H_3 & H_2 \\ H_3 & 0 & -H_1 \\ -H_2 & H_1 & 0 \end{bmatrix} d\tau\boldsymbol{\phi}(t_0) + \boldsymbol{\phi}(t_0)\frac{e}{cm}$$

$$= -\frac{e}{cm}\int_{t_0}^{T}\begin{bmatrix} 0 & \left(\frac{\partial A_1}{\partial x_2} - \frac{\partial A_2}{\partial x_1}\right) & \left(\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}\right) \\ \left(\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}\right) & 0 & \left(\frac{\partial A_2}{\partial x_3} - \frac{\partial A_3}{\partial x_2}\right) \\ \left(\frac{\partial A_3}{\partial x_1} - \frac{\partial A_1}{\partial x_3}\right) & \left(\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}\right) & 0 \end{bmatrix}$$

$$\cdot\ d\tau\boldsymbol{\phi}(t_0) + \frac{e}{cm}\boldsymbol{\phi}(t_0), \tag{9}$$

在 [1, 2] 中已经证明了

$$\frac{\partial J}{\partial V_1} = \psi_1,\quad \frac{\partial J}{\partial V_2} = \psi_2,\quad \frac{\partial J}{\partial V_3} = \psi_3, \tag{10}$$

将(10)代入(7)得：

$$-\frac{\partial J}{\partial t}=\min_{E\in\mathscr{E}}.\left[1+\frac{1}{m}\psi_1\left(\frac{e}{c}E_1+H_3V_2-H_2V_3\right)+\frac{1}{m}\psi_2\left(\frac{e}{c}E_2+H_1V_2-H_3V_1\right)\right.$$
$$\left.+\frac{1}{m}\psi_3\left(\frac{e}{c}E_3+H_2V_1-H_1V_2\right)\right]. \tag{11}$$

其中

$$\begin{cases}\psi_1\cong\psi_1(t_0)\dfrac{e}{cm}-\dfrac{e}{cm}\left[\psi_2(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_1}{\partial x_2}-\frac{\partial A_2}{\partial x_1}\right)dt+\psi_3(t_0)\right.\\ \qquad\left.\cdot\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_1}{\partial x_3}-\frac{\partial A_3}{\partial x_1}\right)dt\right],\\ \psi_2\cong\psi_2(t_0)\dfrac{e}{cm}-\dfrac{e}{cm}\left[\psi_1(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_2}{\partial x_1}-\frac{\partial A_1}{\partial x_2}\right)dt+\psi_3(t_0)\right.\\ \qquad\left.\cdot\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_2}{A x_3}-\frac{\partial A_3}{A x_2}\right)dt\right],\\ \psi_3\cong\psi_3(t_0)\dfrac{e}{cm}-\dfrac{e}{cm}\left[\psi_1(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_3}{\partial x_1}-\frac{\partial A_1}{\partial x_3}\right)dt+\psi_2(t_0)\right.\\ \qquad\left.\cdot\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_3}{\partial x_2}-\frac{\partial A_2}{\partial x_3}\right)dt\right].\end{cases} \tag{12}$$

故

$$\begin{cases}\psi_1E_1=\dfrac{e}{cm}\left\{\psi_1(t_0)-\left[\psi_2(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_1}{\partial x_2}-\frac{\partial A_2}{\partial x_1}\right)dt+\psi_3(t_0)\right.\right.\\ \qquad\left.\left.\cdot\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_1}{\partial x_3}-\frac{\partial A_3}{\partial x_1}\right)dt\right]\right\}\left(\dfrac{e}{mc}\dfrac{\partial A_1}{\partial t}-\dfrac{e}{m}\dfrac{\partial\varphi}{\partial x_1}\right),\\ \psi_2E_2=\dfrac{e}{cm}\left\{\psi_2(t_0)-\left[\psi_1(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_2}{\partial x_1}-\frac{\partial A_1}{\partial x_2}\right)dt+\psi_3(t_0)\right.\right.\\ \qquad\left.\left.\cdot\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_2}{\partial x_3}-\frac{\partial A_3}{\partial x_2}\right)dt\right]\right\}\left(\dfrac{e}{mc}\dfrac{\partial A_2}{\partial t}-\dfrac{e}{m}\dfrac{\partial\varphi}{\partial x_2}\right),\\ \psi_3E_3=\dfrac{e}{cm}\left\{\psi_3(t_0)-\left[\psi_1(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_3}{\partial x_1}-\frac{\partial A_1}{\partial x_3}\right)dt+\psi_2(t_0)\right.\right.\\ \qquad\left.\left.\cdot\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_3}{\partial x_2}-\frac{\partial A_2}{\partial x_3}\right)dt\right]\right\}\left(\dfrac{e}{cm}\dfrac{\partial A_3}{\partial t}-\dfrac{e}{m}\dfrac{\partial\varphi}{\partial x_3}\right).\end{cases} \tag{13}$$

将(13)代入(11)中，并考虑到"min."条件，则必有

$$\begin{cases}\dfrac{e}{c}\dfrac{\partial A_1}{\partial t}-e\dfrac{\partial\varphi}{\partial x}=-\operatorname{sign}\left[\psi_2(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_2}{\partial x_1}-\frac{\partial A_1}{\partial x_2}\right)dt\right],\\ \dfrac{e}{c}\dfrac{\partial A_1}{\partial t}-e\dfrac{\partial\varphi}{\partial x}=-\operatorname{sign}\left[\psi_3(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_3}{\partial x_1}-\frac{\partial A_1}{\partial x_3}\right)dt\right],\end{cases} \tag{14}$$

$$\begin{cases}\dfrac{e}{c}\dfrac{\partial A_2}{\partial t}-e\dfrac{\partial\varphi}{\partial x_2}=-\operatorname{sign}\left[\psi_1(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_1}{\partial x_2}-\frac{\partial A_2}{\partial x_1}\right)dt\right],\\ \dfrac{e}{c}\dfrac{\partial A_2}{\partial t}-e\dfrac{\partial\varphi}{\partial x_2}=-\operatorname{sign}\left[\psi_3(t_0)\displaystyle\int_{t_0}^{T}\left(\frac{\partial A_3}{\partial x_2}-\frac{\partial A_2}{\partial x_3}\right)dt\right],\end{cases} \tag{15}$$

$$\left\{\begin{aligned} &\frac{e}{c}\frac{\partial A_2}{\partial t} - e\frac{\partial \varphi}{\partial x_2} = -\text{sign}\left[\psi_1(t_0)\int_{t_0}^{T}\left(\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}\right)dt\right],\\ &\frac{e}{c}\frac{\partial A_3}{\partial t} - e\frac{\partial \varphi}{\partial x_3} = -\text{sign}\left[\psi_2(t_0)\int_{t_0}^{T}\left(\frac{\partial A_2}{\partial x_3} - \frac{\partial A_3}{\partial x_2}\right)dt\right],\end{aligned}\right. \tag{16}$$

即

$$\left\{\begin{aligned} &E_1 = -\text{sign}\int_{t_0}^{T} H_3 d\tau,\\ &E_1 = -\text{sign}\int_{t_0}^{T}(-H_2)d\tau,\\ &E_2 = -\text{sign}\int_{t_0}^{T}(-H_3)d\tau,\\ &E_2 = -\text{sign}\int_{t_0}^{T} H_1 d\tau,\\ &E_3 = -\text{sign}\int_{t_0}^{T} H_2 d\tau,\\ &E_3 = -\text{sign}\int_{t_0}^{T}(-H_1)d\tau.\end{aligned}\right. \tag{17}$$

这样，我们从(17)得知电场分量和磁场分量对时间积分后之间的符号函数关系，也就是说，电场和磁场在由(17)所规定之情形下电子快速加速在理论上便成为正确的因而是可行的. 罗斯曼厄思[3]所提出的调谐速度问题，在理论上便获得实现.

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ON THE ACCELERATION OF SINGLE ELECTRON IN THE ELECTROMAGNETIC FIELD

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ABSTRACT

We study the Rossmanith[3] problem for the acceleration of single electron in the electromagnetic field. By the method of a new field theory of [2], we give some new and more important results, such as formula (17) in section 3 of this paper.