第4卷 第3期 高能物理与核物理 Vol. 4, No. 3
1980年5月 PHYSICA ENERGIAE FORTIS ET PHYSICA NUCLEARIS May, 1980

秩2紧致单纯李群的不可约表示(III)

孙洪洲
(北京大学物理系)

摘 要

本文应用前两篇文章(I),(II)所采用的方法讨论了 B_2, G_2 群的不可约表示,给出了求 B_2, G_2 群不可约表示的方法并给出了它们的一些低维的不可约表示.

一、B_2 群和 G_2 群

1. B_2 群和 G_2 群的无穷小算子

B_2 群的根图由图1给出,我们取 B_2 群的10个无穷小算子为[1-3]

$$A \equiv X_1 = \sqrt{6} H_1,$$

$$L_1 \equiv X_2 = -\sqrt{6} E_4, \quad L_0 \equiv X_3 = \sqrt{6} H_2, \quad L_{-1} \equiv X_4 = \sqrt{6} E_{-4};$$

$$T_1 \equiv X_5 = \sqrt{6} E_3, \quad T_0 \equiv X_6 = \sqrt{6} E_2, \quad T_{-1} \equiv X_7 = \sqrt{6} E_1;$$

$$V_1 \equiv X_8 = \sqrt{6} E_{-1}, \quad V_0 \equiv X_9 = -\sqrt{6} E_2, \quad V_{-1} \equiv X_{10} = \sqrt{6} E_{-3}. \qquad (1.1\text{-}1)$$

它们满足以下对易关系

$$[A, L_s] = 0; \quad [L_0, L_{\pm 1}] = \pm L_{\pm 1}, \quad [L_1, L_{-1}] = -L_0; \quad [A, T_s] = T_s;$$

$$[L_0, T_s] = sT_s; \quad [L_{\pm 1}, T_s] = \mp \left\{\frac{1}{2}(1 \mp s)(1 \pm s + 1)\right\}^{1/2} T_{s\pm 1};$$

$$[A, V_s] = -V_s; \quad [L_0, V_s] = sV_s; \quad [L_{\pm 1}, V_s] = \mp \left\{\frac{1}{2}(1 \mp s)(1 \pm s + 1)\right\}^{1/2} V_{s\pm 1};$$

$$[T_s, T_{s'}] = 0; \quad [V_s, V_{s'}] = 0. \qquad (1.1\text{-}2)$$

V_s, T_s 之间的对易关系可以写为

$$\{VT\}^0_0 - \{TV\}^0_0 = -\sqrt{3} A; \quad \{VT\}^1_s + \{TV\}^1_s = \sqrt{2} L_s;$$
$$\{VT\}^2_m - \{TV\}^2_m = 0. \qquad (1.1\text{-}3)$$

其中

$$\{VT\}^\xi_m = \sum_{rs} \langle 1r1s|\xi m\rangle V_r T_s, \quad \{TV\}^\xi_m = \sum_{rs} \langle 1r1s|\xi m\rangle T_r V_s. \qquad (1.1\text{-}3')$$

本文1978年12月8日收到.

由(1.1-1)可得[1-3]

$$T_s = (-)^{1+s}(V_{-s})^+. \tag{1.1-4}$$

B_2 群的 Casimir 算子可以写为

$$C = \frac{1}{6}\{L^2 + A(A+3) + 2\sqrt{3}(VT)^0_0\}. \tag{1.1-5}$$

实际上，B_2 群与 C_2 群同构，在 C_2 群中取

$$A = U_{\frac{1}{2}-\frac{1}{2}} - U_{-\frac{1}{2}\frac{1}{2}}, \quad L_0 = \nu_0 + \tau_0, \quad L_{\pm1} = \nu_{\pm1} + \tau_{\pm1},$$

$$T_1 = \sqrt{\frac{1}{2}}(\nu_1 - \tau_1) + U_{\frac{1}{2}\frac{1}{2}}, \; T_0 = \sqrt{\frac{1}{2}}(\nu_0 - \tau_0) + \sqrt{\frac{1}{2}}(U_{\frac{1}{2}-\frac{1}{2}} + U_{-\frac{1}{2}\frac{1}{2}}),$$

$$T_{-1} = \sqrt{\frac{1}{2}}(\nu_{-1} - \tau_{-1}) + U_{-\frac{1}{2}-\frac{1}{2}}; \quad V_1 = -\sqrt{\frac{1}{2}}(\nu_1 - \tau_1) + U_{\frac{1}{2}\frac{1}{2}},$$

$$V_0 = -\sqrt{\frac{1}{2}}(\nu_0 - \tau_0) + \sqrt{\frac{1}{2}}(U_{\frac{1}{2}-\frac{1}{2}} + U_{-\frac{1}{2}\frac{1}{2}}),$$

$$V_{-1} = -\sqrt{\frac{1}{2}}(\nu_{-1} - \tau_{-1}) + U_{-\frac{1}{2}-\frac{1}{2}}. \tag{1.1-6}$$

则 A, L_0, $L_{\pm1}$, T_s, V_s 满足的对易关系即是 (1.1-2), (1.1-3).

G_2 群的根图由图 2 给出[1-3]，我们选 G_2 群的无穷小算子为

$$\nu_1 \equiv X_1 = -2\sqrt{3}E_3, \quad \nu_0 \equiv X_2 = 2\sqrt{3}H_1, \quad \nu_{-1} \equiv X_3 = 2\sqrt{3}E_{-3};$$
$$\tau_1 \equiv X_4 = -2E_6, \quad \tau_0 \equiv X_5 = 2H_2, \quad \tau_{-1} \equiv X_6 = 2E_{-6};$$
$$U_{\frac{1}{2}\frac{3}{2}} \equiv X_7 = 2\sqrt{3}E_5, \quad U_{-\frac{1}{2}\frac{3}{2}} \equiv X_8 = -2\sqrt{3}E_{-1};$$
$$U_{\frac{1}{2}\frac{1}{2}} \equiv X_9 = 2\sqrt{3}E_4, \quad U_{-\frac{1}{2}\frac{1}{2}} \equiv X_{10} = 2\sqrt{3}E_{-2};$$
$$U_{\frac{1}{2}-\frac{1}{2}} \equiv X_{11} = 2\sqrt{3}E_2, \quad U_{-\frac{1}{2}-\frac{1}{2}} \equiv X_{12} = -2\sqrt{3}E_{-4};$$
$$U_{\frac{1}{2}-\frac{3}{2}} \equiv X_{13} = 2\sqrt{3}E_1, \quad U_{-\frac{1}{2}-\frac{3}{2}} \equiv X_{14} = 2\sqrt{3}E_{-5}. \tag{1.1-7}$$

它们所满足的对易关系为[1-3]

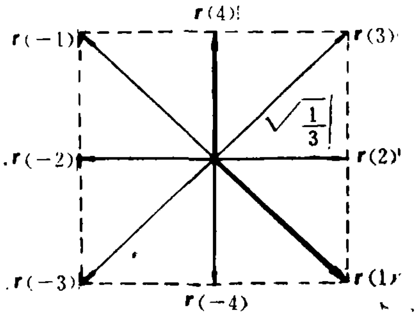
$$[\nu_0, \nu_{\pm1}] = \pm\nu_{\pm1}, \quad [\nu_1, \nu_{-1}] = -\nu_0, \quad [\tau_0, \tau_{\pm1}] = \pm\tau_{\pm1}, \quad [\tau_1, \tau_{-1}] = -\tau_0,$$
$$[\nu_s, \tau_r] = 0, \quad [\nu_0, U_{pq}] = pU_{pq}, \; [\tau_0, U_{pq}] = qU_{pq},$$
$$[\nu_{\pm1}, U_{pq}] = \mp\left\{\frac{1}{2}\left(\frac{1}{2}\mp p\right)\left(\frac{1}{2}\pm p + 1\right)\right\}^{\frac{1}{2}} U_{p\pm1,\,q},$$
$$[\tau_{\pm1}, U_{pq}] = \mp\left\{\frac{1}{2}\left(\frac{3}{2}\mp q\right)\left(\frac{3}{2}\pm q + 1\right)\right\}^{\frac{1}{2}} U_{p,\,q\pm1}. \tag{1.1-8}$$

U_{pq} 间的对易关系可以写为

$$\{UU\}^{0,1}_{0,s} = \sqrt{\frac{5}{2}}\tau_s, \quad \{UU\}^{0,3}_{0,m} = 0, \quad \{UU\}^{1,0}_{s,0} = \sqrt{\frac{9}{2}}\nu_s, \quad \{UU\}^{1,2}_{m,m'} = 0, \tag{1.1-9}$$

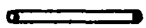
其中

$$\{UU\}^{\xi,\eta}_{m,m'} = \sum_{pp'qq'}\left\langle\frac{1}{2}p\frac{1}{2}p'\middle|\xi m\right\rangle\left\langle\frac{3}{2}q\frac{3}{2}q'\middle|\eta m'\right\rangle U_{pq}U_{p'q'}. \tag{1.1-9'}$$



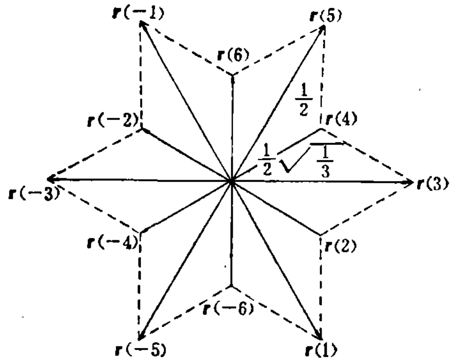
$N_{41}=N_{42}=\sqrt{\frac{1}{6}}$

(a)



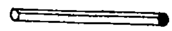
(b)

图 1　(a) B_2 群的根图(黑线为素根)
(b) B_2 群的邓金图



$N_{61}=N_{64}=N_{42}=N_{15}=\frac{1}{2}\sqrt{\frac{1}{2}}$
$N_{62}=\sqrt{\frac{1}{6}}$

(a)



(b)

图 2　(a) G_2 群的根图(黑线为素根)
(b) G_2 群的邓金图

由 (1.1-7) 可得

$$U_{pq}=(-)^{p+q}(U_{-p-q})^{+}. \tag{1.1-10}$$

G_2 群的 Casimir 算子可以写为

$$C=\frac{1}{12}\{3\nu^2+3\tau^2+2\sqrt{2}(UU)^{0;0}_{0;0}\}. \tag{1.1-11}$$

2. B_2 群的不可约表示

为了完全标记 B_2 群不可约表示 $(\lambda\mu)$ 的表示空间 $R^{(\lambda\mu)}$ 的基矢，除了算符 A, L_0 以外，还要引入 $f=\frac{1}{2}(10-6)=2$ 个外加算符. 从 1 节中容易看出算符 L^2, $(VT)^0_0$ 满足这一要求. 但是从 (1.1-5) 可以看出：$(VT)^0_0$ 与 $A(A+3)$; L^2 等构成 B_2 群的 Casimir 算子，故 $(VT)^0_0$. 不能取为外加算符. 不难看出

$$B=(VV)^0_0(TT)^0_0$$

满足作为外加算符的所有条件. 这样，我们就可以取 A, B, L^2, L_0 的共同本征函数标记 $R^{(\lambda\mu)}$ 的基矢.

但是，由于算符 B 比较复杂，在实际计算中取 A, B, L^2, L_0 的共同本征函数来标记 $R^{(\lambda\mu)}$ 的基矢并不方便，所以我们还是取 A, L^2, L_0 的共同本征函数来标记 $R^{(\lambda\mu)}$ 的基矢. 由于 A, L^2, L_0 三个算符的共同本征函数不能唯一地标记 $R^{(\lambda\mu)}$ 的基矢，这时需要引入流动指标 i. 即，我们用 $\left|\begin{matrix}(\lambda\mu)\\ e\Lambda Ki\end{matrix}\right\rangle$ 来标记 $R^{(\lambda\mu)}$ 的基矢，它们满足

$$A\left|\begin{matrix}(\lambda\mu)\\ e\Lambda Ki\end{matrix}\right\rangle=e\left|\begin{matrix}(\lambda\mu)\\ e\Lambda Ki\end{matrix}\right\rangle;$$

$$L^2\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=\Lambda(\Lambda+1)\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle,\quad L_0\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=K\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle; \tag{1.2-1}$$

其中 i 是流动指标．这时 B_2 群的无穷小算子 A, L_0, $L_{\pm1}$, T_s, V_s 所对应的矩阵为:

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon'\Lambda'K'i'\end{matrix}\right|A\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=\epsilon\delta(\epsilon',\epsilon)\delta(\Lambda',\Lambda)\delta(K',K)\delta(i',i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon'\Lambda'K'i'\end{matrix}\right|L_0\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=K\delta(\epsilon',\epsilon)\delta(\Lambda',\Lambda)\delta(K',K)\delta(i',i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon'\Lambda'K'i'\end{matrix}\right|L_{\pm1}\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=\mp\left\{\frac{1}{2}(\Lambda\mp K)(\Lambda\pm K+1)\right\}^{1/2}$$
$$\cdot\delta(\epsilon',\epsilon)\delta(\Lambda',\Lambda)\delta(K',K\pm1)\delta(i',i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon'\Lambda'K'i'\end{matrix}\right|T_s\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon+1\Lambda'i'\end{matrix}\right\|T\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle$$
$$\cdot\frac{\langle\Lambda K1s|\Lambda'K'\rangle}{\sqrt{(2\Lambda'+1)}}\delta(\epsilon',\epsilon+1),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon'\Lambda'K'i'\end{matrix}\right|V_s\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle=\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'i'\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle$$
$$\cdot\frac{\langle\Lambda K1s|\Lambda'K'\rangle}{\sqrt{(2\Lambda'+1)}}\delta(\epsilon',\epsilon-1). \tag{1.2-2}$$

由(1.1-4)可得

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon+1\lambda'i'\end{matrix}\right\|T\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle=(-)^{1+\Lambda'-\Lambda}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon+1\Lambda'i'\end{matrix}\right\rangle. \tag{1.2-3}$$

由(1.1-3)得

$$D_\epsilon(\Lambda',\Lambda i\tilde{\Lambda}\tilde{i})=F_0\delta(\Lambda,\tilde{\Lambda})\delta(i\tilde{i})+F_1D_{\epsilon+1}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda-1)$$
$$+F_2D_{\epsilon+1}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda)+F_3D_{\epsilon+1}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda+1). \tag{1.2-4}$$

其中

$$D_\epsilon(\Lambda',\Lambda i\Lambda\tilde{i})=\sum_{i'}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'i'\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'i'\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\tilde{\Lambda}\tilde{i}\end{matrix}\right\rangle,$$

$$D_{\epsilon+1}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda')=\sum_{i'}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon+1\Lambda'i'\end{matrix}\right\rangle\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon\tilde{\Lambda}\tilde{i}\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon+1\Lambda'i'\end{matrix}\right\rangle, \tag{1.2-4'}$$

系数 F_0, F_1, F_2, F_3 由表1给出．表示空间 $R^{(\lambda\mu)}$ 的基矢间有以下关系

$$\sqrt{(2\Lambda'+1)}\left\{V\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle\right\}_{\Lambda'K'}=\sum_{i'}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'i'\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle\left|\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'K'i'\end{matrix}\right\rangle. \tag{1.2-5}$$

由[3]知，B_2 群不可约表示 $(\lambda\mu)$ 的最高数 $\boldsymbol{W}$ 满足

$$2\boldsymbol{W}\cdot\boldsymbol{r}(1)/|\boldsymbol{r}(1)|^2=\mu,\quad 2\boldsymbol{W}\cdot\boldsymbol{r}(4)/|\boldsymbol{r}(4)|^2=\lambda \tag{1.2-6}$$

这对应于

$$\epsilon_{\max}=\frac{1}{2}(\lambda+2\mu),\quad \Lambda_0=\frac{\lambda}{2},\quad i=1 \tag{1.2-7}$$

表 1　系数 F_0, F_1, F_2, F_3

Λ'　Λ　$\tilde{\Lambda}$	F_0	F_1	F_2	F_3
$\Lambda-1$　Λ　Λ	$(2\Lambda-1)(\epsilon+\Lambda+1)$	$\frac{1}{\Lambda(2\Lambda+1)}$	$\frac{-(2\Lambda-1)}{\Lambda(2\Lambda+1)}$	$\frac{2\Lambda-1}{2\Lambda+1}$
Λ　Λ　Λ	$(2\Lambda+1)(\epsilon+1)$	$\frac{-1}{\Lambda}$	$\frac{\Lambda(\Lambda+1)-1}{\Lambda(\Lambda+1)}$	$\frac{1}{\Lambda+1}$
$\Lambda+1$　Λ　Λ	$(2\Lambda+3)(\epsilon-\Lambda)$	$\frac{2\Lambda+3}{2\Lambda+1}$	$\frac{2\Lambda+3}{(\Lambda+1)(2\Lambda+1)}$	$\frac{1}{(\Lambda+1)(2\Lambda+1)}$
Λ　Λ　$\Lambda+1$			$-\frac{1}{\Lambda+1}$	$\sqrt{\frac{(2\Lambda)(2\Lambda+1)\cdot(2\Lambda+1)(2\Lambda+4)}{(2\Lambda+2)(2\Lambda+3)}}$
$\Lambda+1$　Λ　$\Lambda+1$			$\sqrt{\frac{(2\Lambda)(2\Lambda+1)\cdot(2\Lambda+3)(2\Lambda+4)}{(2\Lambda+1)(2\Lambda+2)}}$	$\frac{1}{\Lambda+1}$
$\Lambda+1$　Λ　$\Lambda+2$				1

(1.2-7) 式指出，对于 $\epsilon=\epsilon_{\max}$ 时，流动指标只可能取一个数值 $i=1$. 以后凡是流动指标只取一个值时，我们就略去它.

将 (1.2-7) 代入 (1.2-4)，并按以下方法规定流动指标 i，我们就可以完全地求得 B_2 群的不可约表示 $(\lambda\mu)$.

设 ϵ 确定时，所有的基矢 $\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda Ki\end{matrix}\right\rangle$ 都已确定，于是可以计算出所有的量 $D_\epsilon(\Lambda',\Lambda i\tilde{\Lambda}\tilde{i})$. 为了方便，我们规定 Λi 的大小，设 Λi, $\tilde{\Lambda}\tilde{i}$ 满足 $\Lambda<\tilde{\Lambda}$ 或 $\Lambda=\tilde{\Lambda}$, $i<\tilde{i}$, 则称 Λi 小于 $\tilde{\Lambda}\tilde{i}$, 记为 $\Lambda i<\tilde{\Lambda}\tilde{i}$. 设 $\Lambda_1 i_1$ 是满足 $D_\epsilon(\Lambda',\Lambda i\tilde{\Lambda}\tilde{i})>0$ 的最小的 Λi 值，我们定义

$$\sqrt{(2\Lambda'+1)}\left\{V\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_1 i_1\end{matrix}\right\rangle\right\}_{\Lambda'K'}=\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'_1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_1 i_1\end{matrix}\right\rangle\left|\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'K'1\end{matrix}\right\rangle,\quad(1.2\text{-}8)$$

于是有

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'i'\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_1 i_1\end{matrix}\right\rangle=\{D\epsilon(\Lambda',\Lambda_1 i,\Lambda_1 i_1)\}^{\frac{1}{2}}=0,\qquad i'=1,\quad i'\geqslant 2$$

利用 (1.2-4′) 可得

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle=\frac{D_\epsilon(\Lambda',\Lambda_1 i_1,\Lambda i)}{\{D_\epsilon(\Lambda',\Lambda_1 i_1,\Lambda_1 i_1)\}^{\frac{1}{2}}}.\qquad(1.2\text{-}9)$$

这样，我们就求得了所有的

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle.$$

再设 $\Lambda_2 i_2$ 是满足下式的最小的 Λi,

$$D_\epsilon(\Lambda',\Lambda i,\Lambda i)-\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle^2>0,$$

于是有

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'2\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle=0,\qquad 当\ \Lambda i<\Lambda_2 i_2\ 时$$

定义 $\left|\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'2\end{matrix}\right\rangle$, 使得

$$\sqrt{(2\Lambda'+1)}\left\{V\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_2 i_2\end{matrix}\right\rangle\right\}_{\Lambda'K'} = \left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_2 i_2\end{matrix}\right\rangle\left|\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'K'1\end{matrix}\right\rangle + \left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'2\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_2 i_2\end{matrix}\right\rangle\left|\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'K'2\end{matrix}\right\rangle, \tag{1.2-10}$$

于是有

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'2\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle \begin{cases} =0, & \text{当 } \Lambda i < \Lambda_2 i_2 \text{ 时} \\ =D_\epsilon(\Lambda', \Lambda_2 i_2, \Lambda i) - \left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_2 i_2\end{matrix}\right\rangle & \\ \quad\cdot\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle\Big/\left\{D_\epsilon(\Lambda', \Lambda_2 i_2, \Lambda_2 i_2)\right. & \\ \quad\left.-\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'1\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_2 i_2\end{matrix}\right\rangle^2\right\}^{\frac{1}{2}}, & \\ & \text{当 } \Lambda i \geqslant \Lambda_2 i_2 \text{ 时} \end{cases} \tag{1.2-11}$$

设 $\Lambda_l i_l$ 是满足

$$D_\epsilon(\Lambda', \Lambda_l i_l, \Lambda_l i_l) - \sum_{k=1}^{l-1}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'k\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_l i_l\end{matrix}\right\rangle^2 > 0$$

的最小的 Λi 值，定义

$$\sqrt{(2\Lambda'+1)}\left\{V\left|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_l i_l\end{matrix}\right\rangle\right\}_{\Lambda'K'} = \sum_{k=1}^{l}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'k\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_l i_l\end{matrix}\right\rangle\left|\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'K'k\end{matrix}\right\rangle, \tag{1.2-12}$$

这样，可以得到

$$\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'l\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle \begin{cases} =0, & \text{当 } \Lambda i < \Lambda_l i_l \text{ 时} \\ =D_\epsilon(\Lambda', \Lambda_l i_l, \Lambda i) - \sum_{k=1}^{l-1}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'k\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_l i_l\end{matrix}\right\rangle & \\ \quad\cdot\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'k\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda i\end{matrix}\right\rangle\Big/\left\{D_\epsilon(\Lambda', \Lambda_l i_l, \Lambda_l i_l)\right. & \\ \quad\left.-\sum_{k=1}^{l-1}\left\langle\begin{matrix}(\lambda\mu)\\ \epsilon-1\Lambda'k\end{matrix}\right\|V\left\|\begin{matrix}(\lambda\mu)\\ \epsilon\Lambda_l i_l\end{matrix}\right\rangle^2\right\}^{\frac{1}{2}}. & \end{cases} \tag{1.2-13}$$

继续应用以上方法即可求出 B_2 群的所有不可约表示 $(\lambda\mu)$.

应用以上方法，我们求出了 B_2 群的不可约表示 $(\lambda 0)$, (0μ), (1μ), (22) 等，结果在表 2 中给出

3. G_2 群的不可约表示

由于 G_2 群在目前还没有具体应用，在这里不准备进行详细的讨论，我们仅给出 G_2 群的一些低维不可约表示

为了完全标记 G_2 群的不可约表示 $(\lambda\mu)$ 的表示空间 $R^{(\lambda\mu)}$ 的基矢，除了 ν_0, τ_0 以外还需要 $f=\frac{1}{2}(14-6)=4$ 个外加算符[1-3]. 我们可以找到两个这样的算符 ν^2, τ^2. 其他

表 2　(a) B_2 群的不可约表示 $(\lambda\,0)$, $(0\,\mu)$, $(1\,\mu)$ 中的 $\left\langle {(\lambda\mu) \atop \varepsilon'\Lambda'} \right\| V \left\| {(\lambda\mu) \atop \varepsilon\Lambda} \right\rangle$ (其中 $\varepsilon = \frac{\lambda}{2} + \mu - a$, $\Lambda = \frac{\lambda}{2} + a - b$)

$\varepsilon'\Lambda'$ \ $(\lambda\mu)$	$(\lambda\ 0)$	$(0\ \mu)$	$(1\ \mu)$
$\varepsilon-1\ \ \Lambda-1$	$\sqrt{\frac{(b+1-a)(\lambda-b)(\lambda-b-1)\cdot(\lambda+1+a-b)}{(\lambda+2a-2b)}}$	$\sqrt{\frac{(b+2)(a-b)(2\mu+1-b)}{2}}$	$\sqrt{\frac{(b+2)(a+1-b)(a-b)(2\mu+3-b)}{(2a+1-2b)}}$　b 偶 $\sqrt{\frac{(b+1)(a+1-b)(a-b)(2\mu+2-b)}{(2a+1-2b)}}$　b 奇
$\varepsilon-1\ \ \Lambda$	$\sqrt{\frac{(2a+1-b)(\lambda+2)^2(\lambda-b)\cdot(\lambda+1+2a-2b)}{2(\lambda+2a-2b)(\lambda+2+2a-2b)}}$	0	$\sqrt{\frac{(a+1-b)(2a+3-b)(2\mu+3-b)}{(2a+1-2b)(2a+3-2b)}}$　b 偶 $-\sqrt{\frac{(b+1)(a+1-b)(2\mu+1-2a+b)}{(2a+1-2b)(2a+3-2b)}}$　b 奇
$\varepsilon-1\ \ \Lambda+1$	$\sqrt{\frac{(2a+1-b)(2a+2-b)\cdot(b-a)(\lambda+2+a-b)}{(\lambda+2+2a-2b)}}$	$\sqrt{\frac{(a+1-b)(2a+3-b)\cdot(2\mu-2a+b)}{2}}$	$\sqrt{\frac{(a+1-b)(a+2-b)(2a+3-b)(2\mu-2a+b)}{(2a+3-2b)}}$　b 偶 $\sqrt{\frac{(a+1-b)(a+2-b)(2a+4-b)(2\mu+1-2a+b)}{(2a+3-2b)}}$　b 奇
	$a=0,1,2,\cdots,\lambda$ $b=a,a+1,\cdots,b_0$ $b_0=\{2a,\lambda\}_{\min}$	$a=0,1,2,\cdots,2\mu$ $b=b_0,b_0+2,\cdots,<a$ $b_0=\{0,2(a-\mu)\}_{\max}$	$a=0,1,2,\cdots,2\mu+1$ $b=b_0,b_0+1,\cdots,a$ $b_0=\{0,2(a-\mu)-1\}_{\max}$

表2 (b) B_2 群不可约表示(21)中的 $\left\langle {(\lambda\mu) \atop \epsilon'\Lambda' i'} \right\| V \left\| {(\lambda\mu) \atop \epsilon\Lambda i} \right\rangle$

$\epsilon'\Lambda'i'$ \ $\epsilon\Lambda i$	2 1	1 0	1 1	1 2	0 1_1	0 1_2	0 2	−1 0	−1 1	−1 2
2 1										
1 0	$\sqrt{4}$									
1 1	$\sqrt{9}$									
1 2	$\sqrt{5}$									
0 1_1		$\sqrt{7}$	$\sqrt{72/14}$	$\sqrt{40/14}$						
0 1_2		0	$\sqrt{75/14}$	$-\sqrt{135/14}$						
0 2			$\sqrt{15/2}$	$\sqrt{15/2}$						
−1 0					$\sqrt{25/7}$	$-\sqrt{24/7}$				
−1 1					0	$\sqrt{21/2}$	$-\sqrt{15/2}$			
−1 2					$\sqrt{160/14}$	$\sqrt{15/14}$	$\sqrt{15/2}$			
−2 1								$\sqrt{4}$	$-\sqrt{9}$	$\sqrt{5}$

的外加算符不去找了，而一般地用流动指标来标记；即我们用 ν^2, ν_0, τ^2, τ_0 的共同本征函数

$$\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle$$

作为 $R^{(\lambda\mu)}$ 的基矢，它们满足

$$\nu^2\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\alpha(\alpha+1)\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle;$$

$$\nu_0\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\epsilon\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle;$$

$$\tau^2\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\Lambda(\Lambda+1)\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle;$$

$$\tau_0\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=K\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle. \tag{1.3-1}$$

其中 i 为流动指标，这时 G_2 群的无穷小算子所对应的矩阵为：

$$\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\epsilon'\Lambda'K'i'\end{matrix}\right|\nu_0\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\epsilon\delta(\alpha',\alpha)\delta(\epsilon',\epsilon)\delta(\Lambda',\Lambda)\delta(K',K)\delta(i',i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\epsilon'\Lambda'K'i'\end{matrix}\right|\nu_{\pm1}\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\mp\left\{\frac{1}{2}(\alpha\mp\epsilon)(\alpha\pm\epsilon+1)\right\}^{\frac{1}{2}}$$
$$\cdot\delta(\alpha',\alpha)\delta(\epsilon',\epsilon\pm1)\delta(\Lambda',\Lambda)\delta(K'K)\delta(i',i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\epsilon'\Lambda'K'i'\end{matrix}\right|\tau_0\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=K\delta(\alpha',\alpha)\delta(\epsilon',\epsilon)\delta(\Lambda',\Lambda)\delta(K',K)\delta(i'i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\epsilon'\Lambda'K'i'\end{matrix}\right|\tau_{\pm1}\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\mp\left\{\frac{1}{2}(\Lambda\mp K)(\Lambda\pm K+1)\right\}^{\frac{1}{2}}$$
$$\cdot\delta(\alpha',\alpha)\delta(\epsilon',\epsilon)\delta(\Lambda',\Lambda)\delta(K'K\pm1)\delta(i',i),$$

$$\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\epsilon'\Lambda'K'i'\end{matrix}\right|U_{pq}\left|\begin{matrix}(\lambda\mu)\\ \alpha\epsilon\Lambda Ki\end{matrix}\right\rangle=\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\Lambda'i'\end{matrix}\right\|U\left\|\begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix}\right\rangle$$
$$\cdot\frac{\langle\alpha\epsilon\,1/2\,p|\alpha'\epsilon'\rangle\langle\Lambda K\,3/2\,q|\Lambda'K'\rangle}{\sqrt{(2\alpha'+1)(2\Lambda'+1)}} \tag{1.3-2}$$

由 (1.1-10) 可得

$$\left\langle\begin{matrix}(\lambda\mu)\\ \alpha'\Lambda'i'\end{matrix}\right\|U\left\|\begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix}\right\rangle=(-)^{\alpha'+\Lambda'-\alpha-\Lambda}\left\langle\begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix}\right\|U\left\|\begin{matrix}(\lambda\mu)\\ \alpha'\lambda'i'\end{matrix}\right\rangle; \tag{1.3-3}$$

由 (1.1-9) 可得

$$D_\alpha(\Lambda',\Lambda i\tilde{\Lambda}\tilde{i})=(2\alpha+1)\{F_0\delta(\Lambda\tilde{\Lambda})\delta(i\tilde{i})+F_1D_{\alpha+\frac{1}{2}}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda-3/2)$$
$$+F_2D_{\alpha+\frac{1}{2}}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda-1/2)+F_3D_{\alpha+\frac{1}{2}}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda+1/2)$$
$$+F_4D_{\alpha+\frac{1}{2}}(\Lambda i\tilde{\Lambda}\tilde{i},\Lambda+3/2). \tag{1.3-4}$$

其中

表 3 系数 F_0, F_1, F_2, F_3, F_4

Λ'	Λ	$\tilde{\Lambda}$	F_0	F_1	F_2
$\Lambda-3/2$	Λ	Λ	$\frac{3(2\Lambda-2)(2\alpha+2\Lambda+2)}{4}$	$\frac{(2\Lambda-2)[(2\Lambda+3)(2\Lambda+2)-3(2\Lambda+1)]}{(2\alpha+2)(2\Lambda+1)(2\Lambda)(2\Lambda-1)}-\frac{1}{2\alpha+1}$	$\frac{6(2\Lambda-2)}{(2\alpha+2)(2\Lambda+1)(2\Lambda)(2\Lambda-1)}$
$\Lambda-1/2$	Λ	Λ	$\frac{(2\Lambda)(6\alpha+2\Lambda+8)}{4}$	$\frac{6}{(2\alpha+2)(2\Lambda+1)(2\Lambda-1)}$	$\frac{(2\Lambda+2)^2(2\Lambda-3)-3(2\Lambda+3)(2\Lambda-4)}{(2\alpha+2)(2\Lambda+2)(2\Lambda+1)(2\Lambda-1)}-\frac{1}{2\alpha+1}$
$\Lambda+1/2$	Λ	Λ	$\frac{(2\Lambda+2)(6\alpha-2\Lambda+6)}{4}$	$\frac{-3(2\Lambda+2)}{(2\alpha+2)(2\Lambda+1)(2\Lambda)}$	$\frac{2\Lambda(2\Lambda-1)+3(2\Lambda-4)}{(2\alpha+2)(2\Lambda+1)(2\Lambda)}$
$\Lambda+3/2$	Λ	Λ	$\frac{3(2\Lambda+4)(2\alpha-2\Lambda)}{4}$	$\frac{(2\Lambda+4)}{(2\alpha+2)(2\Lambda+1)}$	$\frac{3(2\Lambda+4)}{(2\alpha+2)(2\Lambda+2)(2\Lambda+1)}$
$\Lambda-1/2$	Λ	$\Lambda+1$			$\frac{-(2\Lambda+4)(2\Lambda-1)}{(2\alpha+2)(2\Lambda+2)(2\Lambda+1)}+\frac{1}{2\alpha+1}$
$\Lambda+1/2$	Λ	$\Lambda+1$			$\frac{-2}{(2\alpha+2)(2\Lambda+1)}\sqrt{\frac{3(2\Lambda+4)(2\Lambda-1)}{(2\Lambda+3)(2\Lambda)}}$
$\Lambda+3/2$	Λ	$\Lambda+1$			$\frac{(2\Lambda+4)}{(2\alpha+2)(2\Lambda+2)}\sqrt{\frac{(2\Lambda+5)(2\Lambda-1)}{(2\Lambda+3)(2\Lambda+1)}}$
$\Lambda+1/2$	Λ	$\Lambda+2$			
$\Lambda+3/2$	Λ	$\Lambda+2$			
$\Lambda+3/2$	Λ	$\Lambda+3$			

（续表 3）

Λ'	Λ	$\tilde{\Lambda}$	F_3	F_4
$\Lambda-3/2$	Λ	Λ	$\frac{-3(2\Lambda-2)}{(2\alpha+2)(2\Lambda+1)(2\Lambda)}$	$\frac{(2\Lambda-2)}{(2\alpha+2)(2\Lambda+1)}$
$\Lambda-1/2$	Λ	Λ	$\frac{(2\Lambda+3)(2\Lambda+2)-3(2\Lambda+6)}{(2\alpha+2)(2\Lambda+2)(2\Lambda+1)}$	$\frac{3(2\Lambda)}{(2\alpha+2)(2\Lambda+3)(2\Lambda+1)}$
$\Lambda+1/2$	Λ	Λ	$\frac{(2\Lambda)^2(2\Lambda+5)+3(2\Lambda+6)(2\Lambda-1)}{(2\alpha+2)(2\Lambda+3)(2\Lambda+1)(2\Lambda)}-\frac{1}{2\alpha+1}$	$\frac{6}{(2\alpha+2)(2\Lambda+3)(2\Lambda+1)}$
$\Lambda+3/2$	Λ	Λ	$\frac{6(2\Lambda+4)}{(2\alpha+2)(2\Lambda+3)(2\Lambda+2)(2\Lambda+1)}$	$\frac{(2\Lambda+4)[(2\Lambda)(2\Lambda-1)+3(2\Lambda+1)]}{(2\alpha+2)(2\Lambda+3)(2\Lambda+2)(2\Lambda+1)}-\frac{1}{2\alpha+1}$
$\Lambda-1/2$	Λ	$\Lambda+1$	$\frac{-2\sqrt{3(2\Lambda+4)(2\Lambda+3)(2\Lambda)(2\Lambda-1)}}{(2\alpha+2)(2\Lambda+3)(2\Lambda+2)(2\Lambda+1)}$	$\frac{2\Lambda}{(2\alpha+2)(2\Lambda+2)}\sqrt{\frac{(2\Lambda+5)(2\Lambda-1)}{(2\Lambda+3)(2\Lambda+1)}}$
$\Lambda+1/2$	Λ	$\Lambda+1$	$\frac{-12}{(2\alpha+2)(2\Lambda+3)(2\Lambda+1)}+\frac{1}{2\alpha+1}$	$\frac{2}{(2\alpha+2)(2\Lambda+3)}\sqrt{\frac{3(2\Lambda+5)(2\Lambda)}{(2\Lambda+4)(2\Lambda+1)}}$
$\Lambda+3/2$	Λ	$\Lambda+1$	$\frac{2\sqrt{3(2\Lambda+5)(2\Lambda+4)(2\Lambda+1)(2\Lambda)}}{(2\alpha+2)(2\Lambda+3)(2\Lambda+2)(2\Lambda+1)}$	$\frac{-(2\Lambda+5)(2\Lambda)}{(2\alpha+2)(2\Lambda+3)(2\Lambda+2)}+\frac{1}{2\alpha+1}$
$\Lambda+1/2$	Λ	$\Lambda+2$	$\frac{2\Lambda}{(2\alpha+2)(2\Lambda+3)}-\frac{1}{2\alpha+1}$	$\frac{\sqrt{(2\Lambda+6)(2\Lambda+4)(2\Lambda+2)(2\Lambda)}}{(2\alpha+2)(2\Lambda+4)(2\Lambda+3)}$
$\Lambda+3/2$	Λ	$\Lambda+2$	$\frac{\sqrt{(2\Lambda+6)(2\Lambda+4)(2\Lambda+2)(2\Lambda)}}{(2\alpha+2)(2\Lambda+3)(2\Lambda+2)}$	$\frac{2\Lambda+6}{(2\alpha+2)(2\Lambda+3)}-\frac{1}{2\alpha+1}$
$\Lambda+3/2$	Λ	$\Lambda+3$		$\frac{1}{2\alpha+1}$

$$D_\alpha(\Lambda', \Lambda i\tilde{\Lambda}\tilde{i}) = \sum_{i'} \left\langle \begin{matrix}(\lambda\mu)\\ \alpha-1/2\ \Lambda' i'\end{matrix} \middle\| U \middle\| \begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix} \right\rangle \left\langle \begin{matrix}(\lambda\mu)\\ \alpha-1/2\ \Lambda' i'\end{matrix} \middle\| U \middle\| \begin{matrix}(\lambda\mu)\\ \alpha\tilde{\Lambda}\tilde{i}\end{matrix} \right\rangle,$$

$$D_{\alpha+\frac{1}{2}}(\Lambda i\tilde{\Lambda}\tilde{i}, \Lambda') = \sum_{i'} \left\langle \begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix} \middle\| U \middle\| \begin{matrix}(\lambda\mu)\\ \alpha+1/2\ \Lambda' i'\end{matrix} \right\rangle \left\langle \begin{matrix}(\lambda\mu)\\ \alpha\tilde{\Lambda}\tilde{i}\end{matrix} \middle\| U \middle\| \begin{matrix}(\lambda\mu)\\ \alpha+1/2\ \Lambda' i'\end{matrix} \right\rangle. \quad (1.3\text{-}4')$$

系数 F_0, F_1, F_2, F_3, F_4 由表 3 给出，

表示空间 $R^{(\lambda\mu)}$ 的基矢间有以下关系

$$\sqrt{(2\alpha'+1)(2\Lambda'+1)}\left\{U \left| \begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix} \right\rangle \right\}_{\alpha'\epsilon'\Lambda'K'}$$

$$= \sum_{i} \left\langle \begin{matrix}(\lambda\mu)\\ \alpha'\Lambda' i'\end{matrix} \middle\| U \middle\| \begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix} \right\rangle \left| \begin{matrix}(\lambda\mu)\\ \alpha'\epsilon'\Lambda' K' i'\end{matrix} \right\rangle. \quad (1.3\text{-}5)$$

由[3]知，G_2 群的不可约表示 $(\lambda\mu)$ 的最高数 $\boldsymbol{W}$ 满足

$$\begin{aligned} 2\boldsymbol{W}\cdot\boldsymbol{r}(1)/|\boldsymbol{r}(1)|^2 &= \mu, \\ 2\boldsymbol{W}\cdot\boldsymbol{r}(6)/|\boldsymbol{r}(6)|^2 &= \lambda, \end{aligned} \quad (1.3\text{-}6)$$

这对应于

$$\epsilon_{\max} = \frac{1}{2}(\lambda+2\mu), \quad \Lambda_0 = \frac{\lambda}{2}, \qquad i=1. \quad (1.3\text{-}6')$$

(1.3-6′)指出，对于 $\epsilon=\epsilon_{\max}$ 时，流动指标 i 只能取一个数值 $i=1$. 以后凡是流动指标只取一个值时，我们就略去它.

把(1.3-6′)代入(1.3-4)，并按照 1.2 节中所使用的同样方法规定流动指标 i，我们就可以完全地求出 G_2 群的所有不可约表示 $(\lambda\mu)$.

应用这个方法，我们求出了 G_2 群的 (10)，(01)，(20)，(11)，(30)，(02)，(40) 等不可约表示，结果在表 4 中给出.

应用与 I, II[4] 中完全相同的方法也可以计算出 B_2 群，G_2 群的约化系数. 关于这个问题就不在这里讨论了.

表 4 G_2 群的一些不可约表示 $(\lambda\mu)$ 中的 $\left\langle \begin{matrix}(\lambda\mu)\\ \alpha'\Lambda' i'\end{matrix} \middle\| U \middle\| \begin{matrix}(\lambda\mu)\\ \alpha\Lambda i\end{matrix} \right\rangle$.

$(\lambda\mu)=(10)$

$\alpha'\Lambda'$ \ $\alpha\Lambda$	1/2 1/2	0 1
1/2 1/2		
0 1	$2\sqrt{3}$	

$(\lambda\mu)=(01)$

$\alpha'\Lambda'$ \ $\alpha\Lambda$	1 0	1/2 3/2	0 1
1 0			
1/2 3/2	$3\sqrt{2}$		
0 1		$\sqrt{30}$	

$(\lambda\mu)=(20)$

$\alpha'\Lambda'$ \ $\alpha\Lambda$	1 1	1/2 1/2	1/2 3/2
1 1			
1/2 1/2	$\sqrt{6}$		
1/2 3/2	$\sqrt{30}$		
0 0		0	$\sqrt{14}$
0 2		$2\sqrt{5}$	$2\sqrt{5}$

$(\lambda\mu)=(11)$ （续表 4）

$\alpha'\Lambda'$ \ $\alpha\Lambda$	3/2 1/2	1 1	1 2	1/2 1/2	1/2 3/2	1/2 5/2
3/2 1/2						
1 1	$\sqrt{42}$					
1 2	$\sqrt{30}$					
1/2 1/2		$\frac{1}{2}\sqrt{75}$	$\frac{1}{2}\sqrt{105}$			
1/2 3/2		$\sqrt{9}$	$-\sqrt{63}$			
1/2 5/2		$\frac{1}{2}\sqrt{189}$	$\frac{1}{2}\sqrt{63}$			
0 1				$\frac{1}{2}\sqrt{63}$	$-\sqrt{21}$	$-\frac{1}{2}\sqrt{81}$
0 2				$\frac{1}{2}\sqrt{45}$	$-\sqrt{27}$	$\frac{1}{2}\sqrt{147}$

$(\lambda\mu)=(02)$

$\alpha'\Lambda'$ \ $\alpha\Lambda$	2 0	3/2 3/2	1 1	1 3	1/2 3/2	1/2 5/2
2 0						
3/2 3/2	$\sqrt{60}$					
1 1		$\sqrt{96}$				
1 3		$\sqrt{84}$				
1/2 3/2			$\sqrt{294/5}$	$-\sqrt{336/5}$		
1/2 5/2			$\sqrt{216/5}$	$\sqrt{504/5}$		
0 0					$\sqrt{30}$	
0 2					$\sqrt{36}$	$-\sqrt{84}$

$(\lambda\mu)=(30)$

$\alpha'\Lambda'$ \ $\alpha\Lambda$	3/2 3/2	1 0	1 1	1 2	1/2 1/2	1/2 3/2	1/2 5/2
3/2 3/2							
1 0	$\sqrt{24}$						
1 1	$\sqrt{60}$						
1 2	$\sqrt{60}$						
1/2 1/2		0	$\sqrt{9}$	$-\sqrt{45}$			
1/2 3/2		$\sqrt{30}$	$\sqrt{48}$	$\sqrt{12}$			
1/2 5/2			$\sqrt{27}$	$\sqrt{63}$			
0 1					$\sqrt{30}$	$-\sqrt{18/5}$	$-\sqrt{162/5}$
0 3						$\sqrt{252/5}$	$\sqrt{168/5}$

$(\lambda\mu)=(40)$ (续表4)

$\alpha'\Lambda'i'$ \ $\alpha\Lambda i$	2 2	3/2 1/2	3/2 3/2	3/2 5/2	1 1_1	1 1_2	1 2	1 3	1/2 1/2	1/2 3/2	1/2 5/2	1/2 7/2
2 2												
3/2 1/2	$\sqrt{75}$											
3/2 3/2	$\sqrt{120}$											
3/2 5/2	$\sqrt{105}$											
1 1_1		$\sqrt{81}$	$12\sqrt{6/29}$	$21/\sqrt{29}$								
1 1_2		0	$14\sqrt{33/145}$	$-24\sqrt{22/145}$								
1 2		$\sqrt{30}$	$\sqrt{108}$	$\sqrt{42}$								
1 3			$\sqrt{168/5}$	$\sqrt{672/5}$								
1/2 1/2					$\sqrt{330/29}$	$-21\sqrt{3/29}$	$\sqrt{33}$					
1/2 3/2					0	$2\sqrt{87/5}$	0	$-\sqrt{462/5}$				
1/2 5/2					$15\sqrt{14/29}$	$3\sqrt{77/145}$	$5\sqrt{3}$	$4\sqrt{3/5}$				
1/2 7/2							$6\sqrt{2}$	$6\sqrt{3}$				
0 0										6		
0 2									$\sqrt{42}$	$-\sqrt{42}$	$\sqrt{66/7}$	$6\sqrt{11/7}$
0 4											$18\sqrt{2/7}$	$6\sqrt{10/7}$

二、结 束 语

以上我们讨论了秩 2 紧致单纯李群的不可约表示，这个方法可以推广到更高秩的紧致单纯李群．从而也可以得到其不可约表示．这方面的工作正在进行中．

此外这个方法也可以用来讨论阶化李代数的不可约表示．

参 考 文 献

[1] G. Racah, Group Theory and Spectroscopy, Princeton, 1951.

[2] R. E. Behrends et al., *Rev. Mod. Phys.*, **34**(1962), 1.

[3] A. Salam, Theoretical Physics, p. 173, Viena, 1963.

[4] 孙洪洲，高能物理与核物理，**4** (1980), 73; 137.

ON THE IRREDUCIBLE REPRESENTATIONS OF THE COMPACT SIMPLE LIE GROUPS OF RANK 2 (III)

SUN HONG-ZHOU

(*Peking University*)

ABSTRACT

In this paper, using the method employed in two earlier papers: "On the Irreducible Representations of the Compact Simple Lie Groups of Rank 2(I) and (II)", the irreducible representations of the groups B_2 and G_2 are discussed. A method for calculating the irreducible representations of these groups is given. Moreover, some of the low dimensional representations of these groups are calculated.