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三粒子系统的物理基

姚乃国 于祖荣
(南京大学)

摘要

本文利用 $SO(6)$ 和 $SU(4)$ 局部同构性质以及 $SU(3)$ 群的正则子群链与物理子群链之间的变换系数，构造出了三个全同粒子系统内部的物理基，并指出这个基与 Aguila 的超球函数基是一致的.

核物理中的三体问题涉及的面很广，它包括了三核子系统、三体集团、三体超核、三体衰变以及重子的夸克模型等等，所要研究的内容也很多，如核力问题，超子和核子的相互作用，轻核结构以及三体衰变机制等等，所以三体问题在核物理中始终占有一定地位.

处理三体系统的内部运动，常用超球函数基[1,2,3,4]. 在文献上有几种构造超球函数基的方法，它们各有长处. 本文则从 $SO(6)$ 群与 $SU(4)$ 群局部同构出发构造出了一组物理基，这样造出的基在使用时比较方便，并且很容易将它变换成文献[4]中的超球函数基.

为了描述内部运动，我们可引入 Jacobi 座标

$$\boldsymbol{X}^{(1)}=\sqrt{\frac{m\omega}{2\hbar}}(\boldsymbol{r}_1-\boldsymbol{r}_2),\qquad \boldsymbol{X}^{(2)}=\sqrt{\frac{m\omega}{6\hbar}}(\boldsymbol{r}_1+\boldsymbol{r}_2-2\boldsymbol{r}_3) \tag{1}$$

不失一般性，我们可采用谐振子模型，此时内部哈密顿量为

$$H_0=\frac{\hbar\omega}{2}\sum_{s=1}^{2}[(\boldsymbol{P}_s)^2+(\boldsymbol{X}_s)^2] \tag{2}$$

其中 $\boldsymbol{P}_s=\frac{1}{i}\boldsymbol{\nabla}_s$.

哈密顿(2)可以转到六维空间去处理. 为此我们将 $(\boldsymbol{X}^{(1)},\boldsymbol{X}^{(2)})$ 看作是六维空间的矢量 $\boldsymbol{X}$，其分量为

$$(X_1,X_2,\cdots X_6)=(X_1^{(1)},X_2^{(1)},X_3^{(1)},X_1^{(2)},X_2^{(2)},X_3^{(2)}) \tag{3}$$

每个分量对应的共轭动量是 $P_\alpha=\frac{1}{i}\frac{\partial}{\partial X_\alpha}$, $\alpha=1,2,\cdots 6$.

定义算符

$$K_{\alpha\beta}=X_\alpha P_\beta-X_\beta P_\alpha,\quad \alpha,\beta=1,2,\cdots 6 \tag{4}$$

则可将 H_0 写成

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$$H_0=\frac{\hbar\omega}{2}\left(\frac{K^2(\Omega)}{\rho^2}-\frac{\partial^2}{\partial\rho^2}-\frac{5}{\rho}\frac{\partial}{\partial\rho}+\rho^2\right) \tag{5}$$

其中

$$\rho^2=\sum_{\alpha=1}^{6}X_\alpha^2 \tag{6a}$$

$$K^2(\Omega)=\frac{1}{2}\sum_{\alpha,\beta}K_{\alpha\beta}^2 \tag{6b}$$

算符 $K^2(\Omega)$ 只与角变数有关．H_0 的本征函数方程可写成

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{h(h+4)}{\rho^2}+\varepsilon-\rho^2\right)F(\rho)=0 \tag{7a}$$

$$K^2(\Omega)Y_{[h]}(\Omega)=h(h+4)Y_{[h]}(\Omega) \tag{7b}$$

其中本征能量 $\varepsilon=2E_{nh}/\hbar\omega$，$E_{nh}=(2n+h+3)\hbar\omega$．$Y_{[h]}(\Omega)$ 只与角变数有关，称为超球函数．现在角变数共有 5 个，它的选择比较任意．随着 Ω 的不同选择，$Y_{[h]}(\Omega)$ 有不同的形式．符号 $[h]$ 代表包括 h 本身在内的 5 个量子数，显然量子数的选择也随角变数的选取而定．

从 (6a) 式可以看出，哈密顿量本征函数的径向部份在宇称算符和置换操作等作用下都是不变的，这些操作对波函数的影响都反映在 $Y_{[h]}(\Omega)$ 上．

(一)

从 $K_{\alpha\beta}$ 的定义式 (4)，很容易看出它们满足对易关系

$$[K_{\alpha\beta},K_{\gamma\delta}]=i(K_{\alpha\gamma}\delta_{\beta\delta}+K_{\beta\delta}\delta_{\alpha\gamma}-K_{\alpha\delta}\delta_{\beta\gamma}-K_{\beta\gamma}\delta_{\alpha\delta}) \tag{8a}$$

以及

$$K_{\alpha\beta}^+=K_{\alpha\beta}\qquad K_{\alpha\beta}=-K_{\beta\alpha}\qquad \alpha,\beta=1,2,\cdots 6 \tag{8b}$$

所以，$K_{\alpha\beta}$，$\alpha,\beta=1,2,\cdots 6$ 是 $SO(6)$ 群的生成元．而 $K^2=\frac{1}{2}\sum_{\alpha\beta}K_{\alpha\beta}^2$ 是它的两次 Casimir 算符．因为 $SO(6)$ 群与 $US(4)$ 群是局部同构的，所以它们的生成元之间有下列关系[5]：

$$H_a=E_{aa}+E_{44}=K_{aa+3}\quad a=1,2,3 \tag{9a}$$

$$E_{ab}=\frac{1}{2}(iK_{ab}+K_{ab+3}+K_{ba+3}+iK_{a+3b+3})a,\ b=1,2,3\quad a\neq b \tag{9b}$$

$$E_{a4}=\frac{1}{2}(-iK_{bc}+iK_{b+3c+3}-K_{bc+3}+K_{cb+3})\quad a,\ b,\ c\ \text{轮换取}\ 1,2,3 \tag{9c}$$

$$E_{4a}=E_{a4}^+ \tag{9d}$$

其中 $E_{\mu\nu}$，$\mu,\nu=1,2,3,4$，是 $SU(4)$ 的生成元．它们有下列性质

$$[E_{\mu\nu}E_{\sigma\tau}]=E_{\mu\tau}\delta_{\nu\sigma}-E_{\sigma\nu}\delta_{\mu\tau}\quad \mu,\nu,\sigma,\tau=1,2,3,4 \tag{10a}$$

$$E_{\mu\nu}^+=E_{\nu\mu} \tag{10b}$$

$$\sum_{\mu=1}^{4}E_{\mu\mu}=0 \tag{10c}$$

从(4)式和(9)式以及下面定义的 $\boldsymbol{Z}, \boldsymbol{Z}^*$,

$$\boldsymbol{Z} = \frac{1}{\sqrt{2}}(\boldsymbol{X}^{(1)} + i\boldsymbol{X}^{(2)}), \quad \boldsymbol{Z}^* = \frac{1}{\sqrt{2}}(\boldsymbol{X}^{(1)} - i\boldsymbol{X}^{(2)}) \tag{11}$$

可以将(9)式改写成

$$H_a = Z_a\frac{\partial}{\partial Z_a} - Z_a^*\frac{\partial}{\partial Z_a^*} \qquad a = 1, 2, 3 \tag{12 a}$$

$$E_{ab} = Z_a\frac{\partial}{\partial Z_b} - Z_b^*\frac{\partial}{\partial Z_a^*} \qquad a \neq b \quad a, b = 1, 2, 3 \tag{12 b}$$

$$E_{a4} = -Z_b^*\frac{\partial}{\partial Z_c} + Z_c^*\frac{\partial}{\partial Z_b} \qquad a, b, c \text{ 轮换取 } 1, 2, 3 \tag{12 c}$$

$$E_{a4} = (E_{4a})^+ \tag{12 d}$$

由(12)式,我们可以很容易找出 $SU(4)$ 的 GZ(Gel'fand-Zetlin) 基.

(二)

我们知道 $SO(6)$ 的不可约表示 (m_1, m_2, m_3) 与 $SU(4)$ 的不可约表示 $[m_1 + m_2, m_1 - m_3, m_2 - m_3, 0]$ 相对应[5]. 因为现在 $SO(6)$ 的不可约表示是 $(h, 0, 0)$ 所以 $SU(4)$ 的不可约表示为 $[hh00]$, 对应于这个表示的最高权态为

$$|G\rangle_{HW} = \left|\begin{matrix} h & & h & & 0 & & 0 \\ & h & & h & & 0 & \\ & & h & & h & & \\ & & & h & & & \end{matrix}\right\rangle \tag{13}$$

因为 $|G\rangle_{HW}$ 是 $H_a(a = 1, 2, 3)$ 的本征态,同时又满足 $E_{ab}|G\rangle_{HW} = 0$ 当 $a < b$ 时,所以我们得到

$$|G\rangle_{HW} = \frac{(Z_3^*)^h}{\sqrt{h!}} \tag{14}$$

从(14)式并用 Moshinsky 的下降算符[6] 就可以求出与群链 $SU(4) \supset SU(3) \supset SU(2)$ 对应的所有 GZ 态. 例如可以得到群 $SU(3)$的最高权态

$$|G\rangle_{HW(SU_3)} = \left|\begin{matrix} h & & h & & 0 & & 0 \\ & h & & p & & 0 & \\ & & h & & p & & \\ & & & h & & & \end{matrix}\right\rangle = \frac{1}{\sqrt{(h-p)!p!}}(-Z_1)^{h-p}(Z_3^*)^p \tag{15}$$

至于更一般的 GZ 态,当然可以继续用 Moshinsky 算符作用在(15)式上求得,但计算比较麻烦,所以改用我们比较熟悉的张量基方法[7,8]. 为此我们将前述算符重新组成

$$Q_0 = 2H_3 - H_1 - H_2 \tag{16 a}$$

$$\nu_0 = \frac{i}{2}(E_{21} - E_{12}) \tag{16 b}$$

$$\nu_\pm = \mp\frac{1}{\sqrt{8}}(E_{11} - E_{22} \pm iE_{12} \pm iE_{21}) \tag{16 c}$$

$$T_{\pm\frac{1}{2}} = \mp \frac{1}{2}(E_{31} \pm iE_{32}) \tag{16 d}$$

$$V_q = (-)^{\frac{1}{2}-q}(T_{-q})^+ q = \pm \frac{1}{2} \tag{16 e}$$

这些算符的对易式很容易求出，或请参阅原文[7,8]．在此，基底函数取为算符 Q_0，ν^2 和 $2\nu_0$ 的本征函数 $\chi((\lambda\mu)\epsilon\,\Lambda K)$

$$Q_0\chi((\lambda\mu)\epsilon\,\Lambda K) = \epsilon\,\chi((\lambda\mu)\epsilon\,\Lambda K) \tag{17 a}$$

$$\nu^2\chi((\lambda\mu)\epsilon\,\Lambda K) = \Lambda(\Lambda+1)\chi((\lambda\mu)\epsilon\,\Lambda K) \tag{17 b}$$

$$2\nu_0\chi((\lambda\mu)\epsilon\,\Lambda K) = K\chi((\lambda\mu)\epsilon\,\Lambda K) \tag{17 c}$$

其中算符 $\nu^2 = -\nu_1\nu_{-1} - \nu_{-1}\nu_1 + \nu_0^2$．这组基与 GZ 基的关系可通过下述手续得到，设

$$\chi((\lambda\mu)\epsilon_{\min}\Lambda_0 K) = \sum_q C_q \left| \begin{matrix} h & & p & & 0 \\ & h & & p & \\ & & q & & \end{matrix} \right\rangle \tag{18}$$

其中我们已将 GZ 态简写成 $\left| \begin{matrix} h & & p & & 0 \\ & h & & p & \\ & & q & & \end{matrix} \right\rangle$ 的形式．在(18)式中出现的符号有下列关系：

$h = \lambda + \mu$，$p = \mu$，$\epsilon_{\min} = -\lambda - 2\mu$，$\Lambda_0 = \frac{\lambda}{2}$，展开系数 C_q 则可解下列方程组求出：

$$\sum_q \left\{ A_q A_{q'} \left\langle \begin{matrix} h & & p & & 0 \\ & h & & p & \\ & & h & & \end{matrix} \right| E_{12}^{h-q'+1}E_{21}^{h-q} - E_{12}^{h-q'}E_{21}^{h-q+1} \left| \begin{matrix} h & & p & & 0 \\ & h & & p & \\ & & h & & \end{matrix} \right\rangle - W\delta_{qq'} \right\} C_q = 0 \tag{19 a}$$

$$\sum_q C_q^* C_q = 1 \tag{19 b}$$

其中

$$A_q = \sqrt{\frac{(q-p)!}{(h-q)!(h-p)!}} \tag{20}$$

(19 *a*) 式中的矩阵元是很容易算出的，因而很容易得到 $\chi((\lambda\mu)\epsilon_{\min}\Lambda_0 K)$．至于一般的 $\chi((\lambda\mu)\epsilon\,\Lambda K)$ 则可从 $\chi((\lambda\mu)\epsilon_{\min}\Lambda_0 K)$ 以及反复使用

$$\chi((\lambda\mu)\epsilon + 3\tilde{\Lambda}\tilde{K}) = \frac{\sqrt{2\tilde{\Lambda}+1}}{\langle \epsilon + 3\tilde{\Lambda}\|T\|\epsilon\Lambda\rangle} \sum_{Kq} C^{\tilde{\Lambda}\frac{\tilde{K}}{2}}_{\Lambda\frac{K}{2}\frac{1}{2}q} T_q \chi((\lambda\mu)\epsilon\,\Lambda K) \tag{21}$$

就可求出．其中 $C^{\tilde{\Lambda}\frac{\tilde{K}}{2}}_{\Lambda\frac{K}{2}\frac{1}{2}q}$ 是通常的 C-G 系数，约化矩阵元 $\langle \epsilon + 3\tilde{\Lambda}\|T\|\epsilon\,\Lambda\rangle$ 的数值可照文献 [7,8] 所述的方法求出．

但是，$\chi((\lambda\mu)\epsilon\,\Lambda K)$ 是非物理基，而物理上要求的是群链 $SU(3) \supset SO(3) \supset SO(2)$ 的基．在文献[9]中我们已给出了两者的联系．

附表 1 给出了我们的计算结果．为了与文献[4]的超球函数 $Y^{(\lambda\mu)}_{\alpha LM}(\Omega)$ 基比较，可将我们的结果乘以 $\rho^{-h}(\rho = \boldsymbol{Z}\cdot\boldsymbol{Z}^*)$，并换为球座标，再转至体座标系和引入 Dalitz 变数．即可得到文献(4)的 $Y^{(\lambda\mu)}_{\alpha LM}(\Omega)$．

表 1 三粒子系统的物理基

$(\lambda\mu)$	L	M	物理基
(10)	1	1	$-\sqrt{\frac{1}{2}}(Z_1+iZ_2)$
	1	0	$-Z_3$
	1	−1	$\sqrt{\frac{1}{2}}(Z_1-iZ_2)$
(20)	2	2	$\sqrt{\frac{1}{8}}(Z_1+iZ_2)^2$
	2	1	$\sqrt{\frac{1}{2}}(Z_1+iZ_2)\cdot Z_3$
	2	0	$\sqrt{\frac{1}{12}}(2Z_3^2-Z_1^2-Z_2^2)$
	2	−1	$-\sqrt{\frac{1}{2}}(Z_1-iZ_2)\cdot Z_3$
	2	−2	$\sqrt{\frac{1}{8}}(Z_1-iZ_2)^2$
	0	0	$\sqrt{\frac{1}{6}}(Z_1^2+Z_2^2+Z_3^2)$
(01)	1	1	$\sqrt{\frac{1}{2}}(Z_1^*+iZ_2^*)$
	1	0	Z_3^*
	1	−1	$-\sqrt{\frac{1}{2}}(Z_1^*-iZ_2^*)$
(30)	3	3	$-\sqrt{\frac{1}{48}}(Z_1+iZ_2)^3$
	3	2	$-\sqrt{\frac{1}{8}}(Z_1+iZ_2)^2\cdot Z_3$
	3	1	$-\sqrt{\frac{1}{80}}(Z_1+iZ_2)(4Z_3^2-Z_1^2-Z_2^2)$
	3	0	$-\sqrt{\frac{1}{60}}(2Z_3^2-3Z_1^2-3Z_2^2)\cdot Z_3$
	3	−1	$\sqrt{\frac{1}{80}}(Z_1-iZ_2)(4Z_3^2-Z_1^2-Z_2^2)$
	3	−2	$-\sqrt{\frac{1}{8}}(Z_1-iZ_2)^2\cdot Z_3$
	3	−3	$\sqrt{\frac{1}{48}}(Z_1-iZ_2)^3$
	1	1	$-\sqrt{\frac{1}{20}}(Z_1+iZ_2)(Z_3^2+Z_1^2+Z_2^2)$
	1	0	$-\sqrt{\frac{1}{10}}(Z_3^2+Z_1^2+Z_2^2)\cdot Z_3$
	1	−1	$\sqrt{\frac{1}{20}}(Z_1-iZ_2)(Z_3^2+Z_1^2+Z_2^2)$

（续表 1）

$(\lambda\mu)$	L	M	物　理　基
(11)	2	2	$-\frac{1}{2}(Z_1+iZ_2)(Z_1^*+iZ_2^*)$
	2	1	$-\frac{1}{2}\{Z_3\cdot(Z_1^*+iZ_2^*)+Z_3^*\cdot(Z_1+iZ_2)\}$
	2	0	$-\sqrt{\frac{1}{6}}(2Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)$
	2	−1	$\frac{1}{2}\{Z_3\cdot(Z_1^*-iZ_2^*)+Z_3^*\cdot(Z_1-iZ_2)\}$
	2	−2	$\frac{1}{2}(Z_1-iZ_2)(Z_1^*-iZ_2^*)$
	1	1	$\frac{1}{2}\{Z_3(Z_1^*+iZ_2^*)-Z_3^*(Z_1+iZ_2)\}$
	1	0	$\frac{i}{\sqrt{2}}(Z_1^*Z_2-Z_1Z_2^*)$
	1	−1	$-\frac{1}{2}\{Z_3(Z_1^*-iZ_2^*)+Z_3^*(Z_1-iZ_2)\}$
(40)	4	4	$\sqrt{\frac{1}{384}}(Z_1+iZ_2)^4$
	4	3	$\sqrt{\frac{1}{48}}(Z_1+iZ_2)^3\cdot Z_3$
	4	2	$\sqrt{\frac{1}{672}}(Z_1+iZ_2)^2(6Z_3^2-Z_1^2-Z_2^2)$
	4	1	$\sqrt{\frac{1}{336}}(Z_1+iZ_2)\cdot Z_3\cdot(4Z_3^2-3Z_1^2-3Z_2^2)$
	4	0	$\sqrt{\frac{1}{6720}}(8Z_3^4-24Z_3^2(Z_1^2+Z_2^2)+3(Z_1^2+Z_2^2)^2)$
	4	−1	$-\sqrt{\frac{1}{336}}(Z_1-iZ_2)\cdot Z_3\cdot(4Z_3^2-3Z_1^2-3Z_2^2)$
	4	−2	$\sqrt{\frac{1}{672}}(Z_1-iZ_2)^2(6Z_3^2-Z_1^2-Z_2^2)$
	4	−3	$-\sqrt{\frac{1}{48}}(Z_1-iZ_2)^3\cdot Z_3$
	4	−4	$\sqrt{\frac{1}{384}}(Z_1-iZ_2)^4$
	2	2	$\sqrt{\frac{1}{112}}(Z_1+iZ_2)^2(Z_3^2+Z_1^2+Z_2^2)$
	2	1	$\sqrt{\frac{1}{28}}(Z_1+iZ_2)\cdot Z_3\cdot(Z_3^2+Z_1^2+Z_2^2)$
	2	0	$\sqrt{\frac{1}{168}}\{2Z_3^4+Z_3^2(Z_1^2+Z_2^2)-(Z_1^2+Z_2^2)^2\}$
	2	−1	$-\sqrt{\frac{1}{28}}(Z_1-iZ_2)\cdot Z_3\cdot(Z_3^2+Z_1^2+Z_2^2)$
	2	−2	$\sqrt{\frac{1}{112}}(Z_1-iZ_2)^2(Z_3^2+Z_1^2+Z_2^2)$
	0	0	$\sqrt{\frac{1}{120}}(Z_3^2+Z_1^2+Z_2^2)^2$

（续表 1）

$(\lambda\mu)$	L	M	物　　理　　基
(21)	3	3	$\frac{1}{4}(Z_1+iZ_2)^2(Z_1^*+iZ_2^*)$
	3	2	$\sqrt{\frac{1}{24}}(Z_1+iZ_2)\{2Z_3(Z_1^*+iZ_2^*)+Z_3^*(Z_1+iZ_2)\}$
	3	1	$\sqrt{\frac{1}{240}}\{(Z_1^*+iZ_2^*)(4Z_3^2-Z_1^2-Z_2^2)+2(Z_1+iZ_2)(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	3	0	$\sqrt{\frac{1}{5}}\{Z_3\cdot(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}-\sqrt{\frac{1}{20}}Z_3^*\cdot(Z_1+iZ_2)(Z_1-iZ_2)$
	3	−1	$-\sqrt{\frac{1}{240}}\{(Z_1^*-iZ_2^*)(4Z_3^2-Z_1^2-Z_2^2)+2(Z_1-iZ_2)(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	3	−2	$\sqrt{\frac{1}{24}}(Z_1-iZ_2)\{2Z_3(Z_1^*-iZ_2^*)+Z_3^*(Z_1-iZ_2)\}$
	3	−3	$-\sqrt{\frac{1}{4}}(Z_1-iZ_2)^2(Z_1^*-iZ_2^*)$
	2	2	$\sqrt{\frac{1}{12}}(Z_1+iZ_2)\{(Z_1+iZ_2)Z_3^*-(Z_1^*+iZ_2^*)Z_3\}$
	2	1	$\sqrt{\frac{1}{12}}\{(Z_1^*+iZ_2^*)(-Z_3^2+Z_1^2+Z_2^2)+(Z_1+iZ_2)(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	2	0	$\frac{i}{\sqrt{2}}Z_3(Z_1Z_2^*-Z_2Z_1^*)$
	2	−1	$\sqrt{\frac{1}{12}}\{(Z_1^*-iZ_2^*)(-Z^2+Z_1^2+Z_2^2)+(Z_1-iZ_2)(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	2	−2	$\sqrt{\frac{1}{12}}(Z_1-iZ_2)\{(Z_1^*-iZ_2^*)\cdot Z_3-Z_3^*\cdot(Z_1-iZ_2)\}$
	1	1	$\sqrt{\frac{1}{40}}\{(Z_1^*+iZ_2^*)(2Z_3^2+2Z_1^2+2Z_2^2)-(Z_1+iZ_2)(Z_3Z_3^*+Z_1Z_1^*+Z_2Z_2^*)\}$
	1	0	$\sqrt{\frac{1}{20}}\{Z_3\cdot(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}+\sqrt{\frac{1}{5}}Z_3^*\cdot(Z_1+iZ_2)(Z_1-iZ_2)$
	1	−1	$-\sqrt{\frac{1}{40}}\{(Z_1^*-iZ_2^*)(2Z_3^2+2Z_1^2+2Z_2^2)-(Z_1-iZ_2)(Z_3Z_3^*+Z_1Z_1^*+Z_2Z_2^*)\}$
(02)	2	2	$\sqrt{\frac{1}{8}}(Z_1^*+iZ_2^*)^2$
	2	1	$\sqrt{\frac{1}{2}}(Z_1^*+iZ_2^*)\cdot Z_3^*$
	2	0	$\sqrt{\frac{1}{12}}(2Z_3^{*2}-Z_1^{*2}-Z_2^{*2})$
	2	−1	$-\sqrt{\frac{1}{2}}(Z_1^*-iZ_2^*)\cdot Z_3^*$
	2	−2	$\sqrt{\frac{1}{8}}(Z_1^*-iZ_2^*)^2$
	0	0	$\sqrt{\frac{1}{6}}(Z_3^{*2}+Z_1^{*2}+Z_2^{*2})$

（续表 1）

$(\lambda\mu)$	L	M	物　　理　　基
	5	5	$-\sqrt{\frac{1}{3840}}(Z_1+iZ_2)^5$
	5	4	$-\sqrt{\frac{1}{384}}(Z_1+iZ_2)^4\cdot Z_3$
	5	3	$-\sqrt{\frac{1}{6912}}(Z_1+iZ_2)^3(4Z_3^2-Z_1^2-Z_2^2)$
	5	2	$-\sqrt{\frac{1}{288}}(Z_1+iZ_2)^2\cdot Z_3\cdot(2Z_3^2-Z_1^2-Z_2^2)$
	5	1	$-\sqrt{\frac{1}{8064}}(Z_1+iZ_2)\{8Z_3^4-12Z_3^2(Z_1^2+Z_2^2)+(Z_1^2+Z_2^2)^2\}$
	5	0	$-\sqrt{\frac{1}{60480}}Z_3\cdot\{8Z_3^4-40Z_3^2(Z_1^2+Z_2^2)+15(Z_1^2+Z_2^2)^2\}$
	5	−1	$\sqrt{\frac{1}{8064}}(Z_1-iZ_2)\{8Z_3^4-12Z_3^2(Z_1^2+Z_2^2)+(Z_1^2+Z_2^2)^2\}$
	5	−2	$-\sqrt{\frac{1}{288}}(Z_1-iZ_2)^2\cdot Z_3\cdot(2Z_3^2-Z_1^2-Z_2^2)$
	5	−3	$\sqrt{\frac{1}{6912}}(Z_1-iZ_2)^3(4Z_3^2-Z_1^2-Z_2^2)$
	5	−4	$-\sqrt{\frac{1}{384}}(Z_1-iZ_2)^4\cdot Z_3$
(50)	5	−5	$\sqrt{\frac{1}{3840}}(Z_1-iZ_2)^5$
	3	3	$-\sqrt{\frac{1}{864}}(Z_1+iZ_2)^3\cdot(Z_3^2+Z_1^2+Z_2^2)$
	3	2	$-\frac{1}{12}(Z_1+iZ_2)^2\cdot Z_3\cdot(Z_3^2+Z_1^2+{}_2^2)$
	3	1	$-\sqrt{\frac{1}{1440}}(Z_1+iZ_2)\{4Z_3^4+3Z_3^2(Z_1^2+Z_2^2)-(Z_1^2+Z_2^2)^2\}$
	3	0	$-\sqrt{\frac{1}{1080}}Z_3\cdot\{2Z_3^4-Z_3^2(Z_1^2+Z_2^2)-3(Z_1^2+Z_2^2)^2\}$
	3	−1	$\sqrt{\frac{1}{1440}}(Z_1-iZ_2)\{4Z_3^4+3Z_3^2(Z_1^2+Z_2^2)-(Z_1^2+Z_2^2)^2\}$
	3	−2	$-\frac{1}{12}(Z_1-iZ_2)^2\cdot Z_3\cdot(Z_3^2+Z_1^2+Z_2^2)$
	3	−3	$\sqrt{\frac{1}{864}}(Z_1-iZ_2)^3(Z_3^2+Z_1^2+Z_2^2)$
	1	1	$-\sqrt{\frac{1}{560}}(Z_1+iZ_2)(Z_3^2+Z_1^2+Z_2^2)^2$
	1	0	$-\sqrt{\frac{1}{280}}Z_3(Z_3^2+Z_1^2+Z_2^2)^2$
	1	−1	$\sqrt{\frac{1}{560}}(Z_1-iZ_2)(Z_3^2+Z_1^2+Z_2^2)^2$

（续表 1）

$(\lambda\mu)$	L	M	物理基
(31)	4	4	$-\sqrt{\frac{1}{96}}(Z_1+iZ_2)^3\cdot(Z_1^*+iZ_2^*)$
	4	3	$-\sqrt{\frac{1}{192}}(Z_1+iZ_2)^2\{3Z_3(Z_1^*+iZ_2^*)+Z_3^*(Z_1+iZ_2)\}$
	4	2	$-\sqrt{\frac{3}{3584}}(Z_1+iZ_2)(Z_1^*+iZ_2^*)(8Z_3^2-Z_1^2-Z_2^2)$
			$-\sqrt{\frac{3}{896}}(Z_1+iZ_2)^2(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)$
			$-\sqrt{\frac{1}{10752}}(Z_1+iZ_2)^3(Z_1^*-iZ_2^*)$
	4	1	$-\sqrt{\frac{1}{336}}Z_3(Z_1^*+iZ_2^*)(2Z_3^2-Z_1^2-Z_2^2)$
			$-\sqrt{\frac{1}{1344}}(Z_1+iZ_2)^2\{Z_3(Z_1^*-iZ_2^*)-Z_3^*(Z_1-iZ_2)\}$
			$-\sqrt{\frac{1}{84}}Z_3(Z_1+iZ_2)(3Z_3Z_3^*-2Z_1Z_1^*-2Z_2Z_2^*)$
	4	0	$-\sqrt{\frac{1}{1680}}\{4Z_3^2(2Z_3Z_3^*-3Z_1Z_1^*-3Z_2Z_2^*)-3(Z_1^2+Z_2^2)(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	4	-1	$\sqrt{\frac{1}{336}}Z_3(Z_1^*-iZ_2^*)(2Z_3^2-Z_1^2-Z_2^2)+\sqrt{\frac{1}{1344}}(Z_1-iZ_2)^2\{Z_3(Z_1^*+iZ_2^*)-Z_3^*(Z_1+iZ_2)\}$
			$+\sqrt{\frac{1}{84}}Z_3(Z_1-iZ_2)(3Z_3Z_3^*-2Z_1Z_1^*-2Z_2Z_2^*)$
	4	-2	$-\sqrt{\frac{3}{3584}}(Z_1-iZ_2)(Z_1^*-iZ_2^*)(8Z_3^2-Z_1^2-Z_2^2)$
			$-\sqrt{\frac{3}{896}}(Z_1-iZ_2)^2(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)$
			$-\sqrt{\frac{1}{10752}}(Z_1-iZ_2)^3(Z_1^*+iZ_2^*)$
	4	-3	$\sqrt{\frac{1}{192}}(Z_1-iZ_2)^2\{3Z_3(Z_1^*-iZ_2^*)+Z_3^*(Z_1-iZ_2)\}$
	4	-4	$-\sqrt{\frac{1}{96}}(Z_1-iZ_2)^3(Z_1^*-iZ_2^*)$
	3	3	$\frac{1}{8}(Z_1+iZ_2)^2\{Z_3(Z_1^*+iZ_2^*)-Z_3^*(Z_1+iZ_2)\}$
	3	2	$\sqrt{\frac{1}{1536}}(Z_1+iZ_2)(Z_1^*+iZ_2^*)(8Z_3^2-3Z_1^2-3Z_2^2)$
			$-\sqrt{\frac{1}{384}}(Z_1+iZ_2)^2(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)$
			$+\sqrt{\frac{1}{1536}}(Z_1+iZ_2)^3(Z_1^*-iZ_2^*)$
	3	1	$\sqrt{\frac{1}{2160}}Z_3(Z_1^*+iZ_2^*)(6Z_3^2-11Z_1^2-11Z_2^2)$
			$-\sqrt{\frac{1}{540}}Z_3(Z_1+iZ_2)(3Z_3Z_3^*-2Z_1Z_1^*-2Z_2Z_2^*)$
			$+\sqrt{\frac{1}{8640}}(Z_1+iZ_2)^2\{11Z_3(Z_1^*-iZ_2^*)+3Z_3^*(Z_1-iZ_2)\}$

（续表 1）

$(\lambda\mu)$	L	M	物　理　基
(31)	3	0	$-\frac{i}{\sqrt{80}}(Z_1Z_2^* - Z_1^*Z_2)(4Z_3^2 - Z_1^2 - Z_2^2)$
	3	−1	$\sqrt{\frac{1}{2160}}Z_3(Z_1^* - iZ_2^*)(6Z_3^2 - 11Z_1^2 - 11Z_2^2)$
			$-\sqrt{\frac{1}{540}}Z_3(Z_1 - iZ_2)(3Z_3Z_3^* - 2Z_1Z_1^* - 2Z_2Z_2^*)$
			$+\sqrt{\frac{1}{8640}}(Z_1 - iZ_2)^2\{11Z_3(Z_1^* + iZ_2^*) + 3Z_3^*(Z_1 + iZ_2)\}$
	3	−2	$-\sqrt{\frac{1}{1536}}(Z_1 - iZ_2)(Z_1^* - iZ_2^*)(8Z_3^2 - 3Z_1^2 - 3Z_2^2)$
			$+\sqrt{\frac{1}{384}}(Z_1 - iZ_2)^2(4Z_3Z_3^* - Z_1Z_1^* - Z_2Z_2^*)$
			$-\sqrt{\frac{1}{1536}}(Z_1 - iZ_2)^3(Z_1^* + iZ_2^*)$
	3	−3	$\frac{1}{8}(Z_1 - iZ_2)^2\{Z_3(Z_1^* - iZ_2^*) - Z_3^*(Z_1 - iZ_2)\}$
	2	2	$-\sqrt{\frac{5}{2688}}(Z_1 + iZ_2)(Z_1^* + iZ_2^*)(4Z_3^2 + 3Z_1^2 + 3Z_2^2)$
			$+\sqrt{\frac{1}{3360}}(Z_1 + iZ_2)^2(4Z_3Z_3^* - Z_1Z_1^* - Z_2Z_2^*)$
			$+\sqrt{\frac{5}{2688}}(Z_1 + iZ_2)^3(Z_1^* - iZ_2^*)$
	2	1	$-\sqrt{\frac{5}{1512}}Z_3(Z_1^* + iZ_2^*)(3Z_3^2 + 2Z_1^2 + 2Z_2^2)$
			$-\sqrt{\frac{1}{7560}}Z_3(Z_1 + iZ_2)(3Z_3Z_3^* - 2Z_1Z_1^* - 2Z_2Z_2^*)$
			$-\sqrt{\frac{5}{1512}}(Z_1 + iZ_2)^2\{3Z_3^*(Z_1 - iZ_2) - Z_3(Z_1^* - iZ_2^*)\}$
	2	0	$-\sqrt{\frac{1}{140}}\{Z_3^2(2Z_3Z_3^* - 3Z_1Z_1^* - 3Z_2Z_2^*) + (Z_1^2 + Z_2^2)(4Z_3Z_3^* - Z_1Z_1^* - Z_2Z_2^*)\}$
	2	−1	$\sqrt{\frac{5}{1512}}Z_3(Z_1^* - iZ_2^*)(3Z_3^2 + 2Z_1^2 + 2Z_2^2)$
			$+\sqrt{\frac{1}{7560}}Z_3(Z_1 - iZ_2)(3Z_3Z_3^* - 2Z_1Z_1^* - 2Z_2Z_2^*)$
			$+\sqrt{\frac{5}{1512}}(Z_1 - iZ_2)^2\{3Z_3^*(Z_1 + iZ_2) - Z_3(Z_1^* + iZ_2^*)\}$
	2	−2	$-\sqrt{\frac{5}{2688}}(Z_1 - iZ_2)(Z_1^* - iZ_2^*)(4Z_3^2 + 3Z_1^2 + 3Z_2^2)$
			$+\sqrt{\frac{1}{3360}}(Z_1 - iZ_2)^2(4Z_3Z_3^* - Z_1Z_1^* - Z_2Z_2^*)$
			$+\sqrt{\frac{5}{2688}}(Z_1 - iZ_2)^3(Z_1^* + iZ_2^*)$
	1	1	$-\sqrt{\frac{1}{360}}Z_3(Z_1^* + iZ_2^*)(3Z_3^2 + 2Z_1^2 + 2Z_2^2)$

（续表 1）

$\lambda\mu$	L	M	物　理　基
			$+\sqrt{\frac{1}{360}}Z_3(Z_1+iZ_2)(3Z_3Z_3^*-2Z_1Z_1^*-2Z_2Z_2^*)$
			$+\sqrt{\frac{1}{360}}(Z_1+iZ_2)^2\{Z_3(Z_1^*-iZ_2^*)+3Z_3^*(Z_1-iZ_2)\}$
	1	0	$\frac{i}{\sqrt{20}}(Z_1Z_2^*-Z_1^*Z_2)(Z_3^2+Z_1^2+Z_2^2)$
	1	-1	$-\sqrt{\frac{1}{360}}Z_3(Z_1^*-iZ_2^*)(3Z_3^2+2Z_1^2+2Z_2^2)$
			$+\sqrt{\frac{1}{360}}Z_3(Z_1-iZ_2)(3Z_3Z_3^*-2Z_1Z_1^*-2Z_2Z_2^*)$
			$+\sqrt{\frac{1}{360}}(Z_1-iZ_2)^2\{Z_3(Z_1^*+iZ_2^*)+3Z_3^*(Z_1+iZ_2)\}$
	3	3	$-\frac{1}{4}(Z_1+iZ_2)(Z_1^*+iZ_2^*)^2$
	3	2	$-\sqrt{\frac{1}{24}}(Z_1^*+iZ_2^*)\{Z_3(Z_1^*+iZ_2^*)+2Z_3^*(Z_1+iZ_2)\}$
	3	1	$-\sqrt{\frac{1}{240}}\{(Z_1+iZ_2)(4Z_3^{*2}-Z_1^{*2}-Z_2^{*2})+2(Z_1^*+iZ_2^*)(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	3	0	$\sqrt{\frac{1}{20}}Z_3(Z_1^{*2}+Z_2^{*2})-\sqrt{\frac{1}{5}}Z_3^*(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)$
	3	-1	$\sqrt{\frac{1}{240}}\{(Z_1-iZ_2)(4Z_3^{*2}-Z_1^{*2}-Z_2^{*2})+2(Z_1^*-iZ_2^*)(4Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	3	-2	$-\sqrt{\frac{1}{24}}(Z_1^*-iZ_2^*)\{Z_3(Z_1^*-iZ_2^*)+2Z_3^*(Z_1-iZ_2)\}$
	3	-3	$\frac{1}{4}(Z_1-iZ_2)(Z_1^*-iZ_2^*)^2$
(12)	2	2	$\sqrt{\frac{1}{12}}(Z_1^*+iZ_2^*)\{Z_3(Z_1^*+iZ_2^*)-Z_3^*(Z_1+iZ_2)\}$
	2	1	$\sqrt{\frac{1}{12}}\{(Z_1+iZ_2)(-Z_3^{*2}+Z_1^{*2}+Z_2^{*2})+(Z_1^*+iZ_2^*)(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	2	0	$-\frac{i}{\sqrt{2}}Z_3^*(Z_1Z_2^*-Z_1^*Z_2)$
	2	-1	$\sqrt{\frac{1}{12}}\{(Z_1-iZ_2)(-Z_3^{*2}+Z_1^{*2}+Z_2^{*2})+(Z_1^*-iZ_2^*)(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)\}$
	2	-2	$\sqrt{\frac{1}{2}}(Z_1^*-iZ_2^*)\{Z_3^*(Z_1-iZ_2)-Z_3(Z_1^*-iZ_2^*)\}$
	1	1	$\sqrt{\frac{1}{40}}\{(Z_1^*+iZ_2^*)(Z_3Z_3^*+Z_1Z_1^*+Z_2Z_2^*)-(Z_1+iZ_2)(2Z_3^{*2}+2Z_1^{*2}+2Z_2^{*2})\}$
	1	0	$-\sqrt{\frac{1}{20}}Z_3^*(Z_3Z_3^*-Z_1Z_1^*-Z_2Z_2^*)-\sqrt{\frac{1}{5}}Z_3(Z_1^*+iZ_2^*)(Z_1^*-iZ_2^*)$
	1	-1	$-\sqrt{\frac{1}{40}}\{(Z_1^*-iZ_2^*)(Z_3Z_3^*+Z_1Z_1^*+Z_2Z_2^*)-(Z_1-iZ_2)(2Z_3^{*2}+2Z_1^{*2}+2Z_2^{*2})\}$

如果再利用置换群 S_3 的投影算符以及下列等式

$$(12)\boldsymbol{Z} = -\boldsymbol{Z}^* \qquad (12)\boldsymbol{Z}^* = -\boldsymbol{Z}$$

$$(123)\boldsymbol{Z} = \exp\left(-\frac{2}{3}\pi i\right)\boldsymbol{Z} \qquad (123)\boldsymbol{Z}^* = \exp\left(\frac{2}{3}\pi i\right)\boldsymbol{Z}^* \tag{22}$$

就可容易地求得具有确定对称性的超球函数基．详细从略．

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THE PHYSICAL BASES OF THE THREE PARTICLE SYSTEM

Yao Nai-guo　Yu Zu-rong
(Nanjing University)

Abstract

In this paper, using the property of the local isomophism between the group $SO(6)$ and $SU(4)$, a set of physical bases of the three particles is derived. The bases are equivalent to the hyperspherical function bases.