第16卷 第8期　　高能物理与核物理　　Vol. 16, No. 8
1992年8月　　HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS　　Aug., 1992

Chevalley 基下 WZNW 场论的 Hamilton 形式

杨 焕 雄
(西北大学物理研究所，西安 710069)

摘 要

本文在半单 Lie 代数的 Chevalley 正则基下给出了二维 WZNW 场论的 Hamilton 形式，并在此基础上计算了守恒流之间的 Poisson 括号，结果正是经典的 Kac-Moody 流代数。

作为 Kac-Moody 流代数和 Virasoro 代数的 Lagrange 实现，二维 WZNW 场论模型是最基本的共形场论之一[1,6]，其研究吸引着日益广泛的兴趣，并已经取得了多方面的进展。

一个重要的进展是，Witten 在他的著名论文[2]中建立起了二维 WZNW 场论的 Hamilton 正则形式，从而为规范 WZNW 场论[8]的正则量子化奠定了基础[3]。不过，Witten 的理论是在 Lie 代数的自然基下给出的。很难直接应用它实现 Balog、Fehéy、O'Raifeartaigh、Forgács 及 Wipf 等人提出的约束 WZNW 场论[4,5,9,10]的正则量子化。为了消除这一困难，本文试图在半单 Lie 代数的 Chevalley 基下将二维 WZNW 场论纳入 Hamilton 正则形式。

首先介绍本文的记号及半单 Lie 代数 $\mathscr{G}$ 的 Chevalley 基。用 Φ、Δ 分别表示根矢及素根的集合，则 $\mathscr{G}$ 的 Chevalley 基 $\{H_i \equiv H_{\alpha_i},\ E_\alpha\}$ 由下式定义：

$$[H_i, H_j] = 0,\ (i, j = 1, 2, \cdots, \mathrm{rank}\,\mathscr{G}\}, \tag{1a}$$

$$[H_i, E_\alpha] = K_{\alpha i} E_\alpha, \tag{1b}$$

$$[E_\alpha, E_\beta] = \sum_{ij} K_{ij}^{-1} K_{j\alpha} H_i \delta_{\alpha+\beta,0} + N_{\alpha,\beta} E_{\alpha+\beta}, \tag{1c}$$

式中 α、$\beta \in \Phi$，α_i、$\beta_i \in \Delta$，$K_{ab} = 2a \cdot b/b^2$，$\sum_b K_{ab}^{-1} K_{bc} = \delta_{ac}$，$N_{\alpha,\beta}$ 为常数(如果 $\alpha+\beta$ 不是 $\mathscr{G}$ 的相矢，则相应地有 $N_{\alpha,\beta} = 0$)。(1) 式已把 Serre 关系式考虑在内。在此 Chevalley 基下，$\mathscr{G}$ 的 Killing 双线性型为：

$$\mathrm{Tr}\,(H_i H_j) = \frac{2}{\alpha_i^2} K_{ij}, \quad \mathrm{Tr}(E_\alpha E_\beta) = \frac{2}{\alpha^2} \delta_{\alpha+\beta,0},$$

$$\mathrm{Tr}(H_i E_\alpha) = 0. \tag{2}$$

本文1991年10月6日收到。

二维 WZNW 模型的作用量为[4,7]，

$$S(g)=\frac{\kappa}{2}\int_{S_2}\mathrm{d}^2x\mathrm{Tr}(\partial_\mu gg^{-1}\partial^\mu gg^{-1})-\frac{\kappa}{3}\int_{B_3}\mathrm{d}^3\alpha\epsilon_{ijk}\mathrm{Tr}(\partial_i gg^{-1}\partial_j gg^{-1}\partial_k gg^{-1}),\tag{3}$$

式中 κ 为耦合常数，g 取值在某个连通的实 Lie 群 G 上（G 具有半单 Lie 代数 $\mathscr{G}$）。设群 G 流形上的群参数为 $\theta^a=\theta^a(x)$，$1\leqslant a\leqslant \dim G$，则

$$g=g(x)=g(\theta(x))\in G.$$

我们定义：

$$\partial_a gg^{-1}=\frac{\partial g}{\partial\theta^a}g^{-1}=\sum_{i=1}^{\mathrm{rank}\mathscr{G}}H_i\Omega_a^i(\theta)+\sum_{\alpha\in\Phi}E_\alpha\Omega_a^{-\alpha}(\theta),\tag{4a}$$

$$\mathrm{Tr}(\partial_a gg^{-1}[\partial_b gg^{-1},\ \partial_c gg^{-1}])=\partial_c\lambda_{ab}(\theta)+\partial_a\lambda_{bc}(\theta)+\partial_b\lambda_{ca}(\theta),\tag{4b}$$

这里，$\lambda_{ab}(\theta)$ 是群 G 流形上的反对称张量，而 $\Omega(\theta)$ 是非奇异矩阵。利用(4)式，作用量 $S(g)$ 可表为如下 2 维积分

$$S(g)=\int_{S_2}\mathrm{d}^2x\mathscr{L}(x),$$

$$\mathscr{L}(x)=\mathscr{L}(\theta^a,\dot\theta^a,\theta'^a)=\frac{\kappa}{2}\Big[\sum_{ij}\frac{2}{\alpha_i^2}K_{ij}\Omega_a^i\Omega_b^j+\sum_{\alpha\in\Phi}\frac{2}{\alpha^2}\Omega_a^\alpha\Omega_b^{-\alpha}\Big](\dot\theta^a\dot\theta^b-\theta'^a\theta'^b)-\kappa\lambda_{ab}\dot\theta^a\theta'^b,\tag{5}$$

$\left(式中约定\ \dot\theta^a=\dfrac{\partial\theta^a(x)}{\partial x^0},\quad \theta'^a=\dfrac{\partial\theta^a(x)}{\partial x^1}\right)$.

$\mathscr{L}(x)$ 正是二维 WZNW 场论中的 Lagrange 密度，含 $\lambda_{ab}(\theta)$ 的项代表着(3)式中拓扑项的贡献。

Euler-Lagrange 运动方程为：

$$\begin{aligned}0&=\frac{\partial\mathscr{L}}{\partial\theta^c}-\partial_0\left(\frac{\partial\mathscr{L}}{\partial\dot\theta^c}\right)-\partial_1\left(\frac{\partial\mathscr{L}}{\partial\theta'^c}\right)\\&=-\kappa\Big[\sum_{ij}\frac{2}{\alpha_i^2}K_{ij}(\partial_a\Omega_b^j-\partial_b\Omega_a^j)\Omega_c^i+\sum_{\alpha\in\Phi}\frac{2}{\alpha^2}(\partial_a\Omega_b^{-\alpha}-\partial_b\Omega_a^{-\alpha})\Omega_c^\alpha\Big]\cdot\dot\theta^a\theta'^b\\&\quad-\kappa\sum_{ij}\frac{2}{\alpha_i^2}K_{ij}(\partial_b\Omega_a^j)\Omega_c^i(\dot\theta^a\dot\theta^b-\theta'^a\theta'^b)\\&\quad-\kappa\sum_{\alpha\in\Phi}\frac{2}{\alpha^2}(\partial_b\Omega_a^{-\alpha})\Omega_c^\alpha(\dot\theta^a\dot\theta^b-\theta'^a\theta'^b)\\&\quad-\kappa\Big[\sum_{ij}\frac{2}{\alpha_i^2}K_{ij}\Omega_a^j\Omega_c^i+\sum_{\alpha\in\Phi}\frac{2}{\alpha^2}\Omega_a^\alpha\Omega_c^{-\alpha}\Big](\ddot\theta^a-\theta''^a),\\&\qquad(1\leqslant c\leqslant\dim G).\end{aligned}\tag{6}$$

在推导(6)式时，我们利用了附录中的（B.4）式。借助于（B.5）式，又可将(6)式写成左手流守恒律的形式，($\partial_-=\partial_0-\partial_1$)

$$\partial_-\mathscr{J}(H_i,x)=0,\quad \partial_-\mathscr{J}(E_\alpha,x)=0,\tag{7}$$

$$\begin{cases}\mathscr{J}(H_i,\,x)=\kappa(\dot\theta^a+\theta'^a)\sum_j \frac{2}{\alpha_j^2}K_{ji}\Omega_a^i,\ (i=1,2,\cdots,\mathrm{rank}\ \mathscr{G})\\ \mathscr{J}(E_\alpha,\,x)=\kappa(\dot\theta^a+\theta'^a)\frac{2}{\alpha^2}\Omega_a^\alpha.\ (\alpha\in\Phi)\end{cases}\tag{8}$$

引入矩阵 $L_{AB}=\mathrm{Tr}(AgBg^{-1})$ (这里 A、B 均为半单 Lie 代数 $\mathscr{G}$ 的 Chevalley 基)，利用 (B.9) 式把 Lagrange 密度改写为:

$$\begin{aligned}\mathscr{L}(x)=\frac{\kappa}{2}\Bigg\{&\sum_{ij}\Omega_a^i\Omega_b^j\left[\sum_{kl}\frac{\alpha_l^2}{2}K_{kl}^{-1}L_{il}L_{jk}+\sum_{\beta\in\Phi}\frac{\beta^2}{2}L_{i\beta}L_{j,-\beta}\right]\\&+2\sum_j\sum_{\alpha\in\Phi}\Omega_a^j\Omega_b^{-\alpha}\left[\sum_{kl}\frac{\alpha_l^2}{2}K_{kl}^{-1}L_{jk}L_{\alpha l}+\sum_{\beta\in\Phi}\frac{\beta^2}{2}L_{\alpha\beta}L_{j,-\beta}\right]\\&+\sum_{\alpha\in\Phi}\sum_{\gamma\in\Phi}\Omega_a^{-\alpha}\Omega_b^{-\gamma}\left[\sum_{kl}\frac{\alpha_l^2}{2}K_{kl}^{-1}L_{\gamma k}L_{\alpha l}+\sum_{\beta\in\Phi}\frac{\beta^2}{2}L_{\alpha\beta}L_{\gamma,-\beta}\right]\Bigg\}\\&\cdot(\dot\theta^a\dot\theta^b-\theta'^a\theta'^b)-\kappa\lambda_{ab}\dot\theta^a\theta'^b,\end{aligned}\tag{5$'$}$$

由此求 Euler-Lagrange 方程，便得到如下的右手流守恒律，($\partial_+=\partial_0+\partial_1$)

$$\partial_+\tilde{\mathscr{J}}(H_i,\,x)=0,\quad \partial_+\tilde{\mathscr{J}}(E_\alpha,\,x)=0,\tag{9}$$

$$\begin{cases}\tilde{\mathscr{J}}(H_i,\,x)=-\kappa(\dot\theta^a-\theta'^a)\left[\sum_j\Omega_a^jL_{ji}+\sum_{\beta\in\Phi}\Omega_a^{-\beta}L_{\beta i}\right],\\ \tilde{\mathscr{J}}(E_\alpha,\,x)=-\kappa(\dot\theta^a-\theta'^a)\left[\sum_j\Omega_a^jL_{j\alpha}+\sum_{\beta\in\Phi}\Omega_a^{-\beta}L_{\beta\alpha}\right],\end{cases}\tag{10}$$

$$(i=1,2,\cdots,\mathrm{rank}\mathscr{G},\ \alpha\in\Phi).$$

需要注意的是，流守恒方程(7)、(9)实际上是互相等价的。

下面通过标准程序把二维 WZNW 场论从已经得到的 Lagrange 形式过渡到 Hamilton 正则形式。具体做法是，现将 $\theta^a(1\leqslant a\leqslant \dim G)$ 作为群 G 流形上的正则坐标，并定义其共轭正则动量 π_a 和理论中的基本 Poisson 括号:

$$\pi_a=\frac{\partial\mathscr{L}}{\partial\dot\theta^a}=\kappa\left[\sum_{ij}\frac{2}{\alpha_i^2}K_{ij}\Omega_a^i\Omega_b^j+\sum_{\alpha\in\Phi}\frac{2}{\alpha^2}\Omega_a^\alpha\Omega_b^{-\alpha}\right]\dot\theta^b-\kappa\lambda_{ab}\theta'^b,\tag{11}$$

$$\{\theta^a(x),\,\pi_b(y)\}=\delta_b^a\delta(x_1-y_1),\tag{12}$$

$$(1\leqslant a,\,b\leqslant\dim G).$$

此外，定义场的正则 Hamilton 密度:

$$\begin{aligned}\mathscr{H}&\equiv\pi_a\dot\theta^a-\mathscr{L}\\&=\frac{\kappa}{2}\left[\sum_{ij}\frac{2}{\alpha_i^2}K_{ij}\Omega_a^i\Omega_b^j+\sum_{\alpha\in\Phi}\frac{2}{\alpha^2}\Omega_a^\alpha\Omega_b^{-\alpha}\right](\dot\theta^a\dot\theta^b+\theta'^a\theta'^b)\\&=\frac{1}{4\kappa}\sum_{ij}\frac{\alpha_j^2}{2}K_{ij}^{-1}[\mathscr{J}(H_i,\,x)\mathscr{J}(H_j,\,x)+\tilde{\mathscr{J}}(H_i,x)\tilde{\mathscr{J}}(H_j,x)]\\&\quad+\frac{1}{4\kappa}\sum_{\alpha\in\Phi}\frac{\alpha^2}{2}[\mathscr{J}(E_\alpha,\,x)\mathscr{J}(E_{-\alpha},\,x)+\tilde{\mathscr{J}}(E_\alpha,\,x)\tilde{\mathscr{J}}(E_{-\alpha},x)],\end{aligned}\tag{13}$$

显然，(13) 式右端的结果正是能量密度的 Sugawara 构造。$\mathscr{H}$ 又可表为如下与基无关的形式，

$$\mathscr{H}=\frac{1}{4\kappa}\mathrm{Tr}[\mathscr{J}^2(x)+\tilde{\mathscr{J}}^2(x)],\tag{14}$$

$$\mathscr{J}(x) = \sum_{ij} \frac{\alpha_i^2}{2} K_{ij}^{-1} H_i \mathscr{J}(H_j, x) + \sum_{\alpha \in \Phi} \frac{\alpha^2}{2} E_{-\alpha} \mathscr{J}(E_\alpha, x), \tag{15a}$$

$$\tilde{\mathscr{J}}(x) = \sum_{ij} \frac{\alpha_i^2}{2} K_{ij}^{-1} H_i \tilde{\mathscr{J}}(H_j, \alpha) + \sum_{\alpha \in \Phi} \frac{\alpha^2}{2} E_{-\alpha} \tilde{\mathscr{J}}(E_\alpha, x). \tag{15b}$$

由(13)式知,欲在 Hamilton 形式中写出场的正则运动方程,首先须求得守恒流之间的 Poisson 括号. 用正则力学变量表示的守恒流分量是:

$$\mathscr{J}(H_i, x) = \omega^{ai}\pi_a + \kappa\omega^{ai}\lambda_{ab}\theta'^b + \kappa \sum_j \frac{2}{\alpha_j^2} K_{ji}\Omega_a^j \theta'^a, \tag{16a}$$

$$\mathscr{J}(E_\alpha, x) = \omega^{a\alpha}\pi_a + \kappa\omega^{a\alpha}\lambda_{ab}\theta'^b + \kappa \frac{2}{\alpha^2} \Omega_a^\alpha \theta'^a, \tag{16b}$$

$$\begin{aligned}\tilde{\mathscr{J}}(H_i, x) = &- \sum_j L_{ji} \left[\sum_l \frac{\alpha_l^2}{2} K_{jl}^{-1}(\omega^{al}\pi_a + \kappa\omega^{al}\lambda_{ab}\theta'^b) - \kappa\Omega_a^j \theta'^a \right] \\ &- \sum_{\beta \in \Phi} L_{-\beta, i} \left[\frac{\beta^2}{2} (\omega^{a\beta}\pi_a + \kappa\omega^{a\beta}\lambda_{ab}\theta'^b) - \kappa\Omega_a^\beta \theta'^a \right],\end{aligned} \tag{16c}$$

$$\begin{aligned}\tilde{\mathscr{J}}(E_\alpha, x) = &- \sum_j L_{j\alpha} \left[\sum_l \frac{\alpha_l^2}{2} K_{jl}^{-1}(\omega^{al}\pi_a + \kappa\omega^{al}\lambda_{ab}\theta'^b) - \kappa\Omega_a^j \theta'^a \right] \\ &- \sum_{\beta \in \Phi} L_{-\beta, \alpha} \left[\frac{\beta^2}{2} (\omega^{a\beta}\pi_a + \kappa\omega^{a\beta}\lambda_{ab}\theta'^b) - \kappa\Omega_a^{+\beta} \theta'^a \right] \\ & i = 1, 2, \cdots, \mathrm{rank}\mathscr{G}, \quad \alpha \in \Phi.\end{aligned} \tag{16d}$$

(16)式中的 ω 是 Ω 的逆矩阵(参见附录 B).

把(12)式和(16)式结合起来,便可求得守恒流满足的 Poisson 括号. 例如:

$$\begin{aligned}\{\mathscr{J}(H_i, x), \mathscr{J}(H_j, y)\} = &\{\omega^{ai}\pi_a, \omega^{cj}\pi_c\} + \{\omega^{ai}\pi_a, \kappa\omega^{cj}\lambda_{cd}\theta'^d\} \\ &+ \left\{\omega^{ai}\pi_a, \kappa \sum_l \frac{2}{\alpha_l^2} K_{lj}\Omega_c^l \theta'^c\right\} \\ &+ \{\kappa\omega^{ai}\lambda_{ab}\theta'^b, \omega^{cj}\pi_c\} \\ &+ \left\{\kappa \sum_k \frac{2}{\alpha_k^2} K_{ki}\Omega_a^k \theta'^a, \omega^{cj}\pi_c\right\},\end{aligned}$$

利用附录中的 (B.2)、(B.5) 及 (B.6) 式知:

$$\begin{aligned}\{\omega^{ai}\pi_a, \omega^{cj}\pi_c\} &= [(\partial_c\omega^{ai})\omega^{cj}\pi_a - (\partial_a\omega^{cj})\omega^{ai}\pi_c]\delta(x_1 - y_1) \\ &= \omega^{ai}\omega^{bj}\pi_c \left[\sum_l \omega^{cl}(\partial_a\Omega_b^l - \partial_b\Omega_a^l) \right. \\ &\quad \left. + \sum_{\beta \in \Phi} \omega^{c\beta}(\partial_a\Omega_b^{-\beta} - \partial_b\Omega_a^{-\beta}) \right] \delta(x_1 - y_1) = 0,\end{aligned}$$

又注意到 $\{\theta'^a(x), \pi_b(y)\} = \delta_b^a \delta'(x_1 - y_1)$, 则:

$$\begin{aligned}&\{\omega^{ai}\pi_a, \kappa\omega^{cj}\lambda_{cd}\theta'^d\} + \{\kappa\omega^{ai}\lambda_{ab}\theta'^b, \omega^{cj}\pi_c\} \\ &= \kappa\omega^{ai}\left[-(\partial_a\omega^{cj})\lambda_{cd}\theta'^d - \omega^{cj}(\partial_a\lambda_{cd})\theta'^d + \frac{\partial}{\partial x'}(\omega^{cj}\lambda_{ca}) \right] \delta(x_1 - y_1) \\ &\quad + \kappa\omega^{cj}[(\partial_c\omega^{ai})\lambda_{ab}\theta'^b + \omega^{ai}(\partial_c\lambda_{ab})\theta'^b]\delta(x_1 - y_1) \\ &\quad + \kappa\omega^{ai}\lambda_{ab} \frac{\partial}{\partial x'} \omega^{bj}\delta(x_1 - y_1)\end{aligned}$$

$$= -\kappa\omega^{ai}\omega^{bj}\theta'^{c}[\partial_a\lambda_{bc} + \partial_b\lambda_{ca} + \partial_c\lambda_{ab}]\delta(x_1 - y_1)$$
$$+ \kappa\lambda_{bc}\theta'^{c}[\omega^{ai}(\partial_a\omega^{bi}) - \omega^{ai}(\partial_a\omega^{bj})]\delta(x_1 - y_1)$$
$$= \kappa\lambda_{bc}\theta'^{c}\omega^{ai}\omega^{dj}\left[\sum_l \omega^{bl}(\partial_a\Omega^l_d - \partial_d\Omega^l_a) + \sum_{\beta\in\Phi}\omega^{b\beta}(\partial_a\Omega^{-\beta}_d - \partial_d\Omega^{-\beta}_a)\right]$$
$$= 0,$$

$$\left\{\omega^{ai}\pi_a, \kappa\sum_l \frac{l}{\alpha_l^2}K_{lj}\Omega^l_c\theta'^{c}\right\}$$
$$= \omega^{ai}\kappa\sum_l \frac{2}{\alpha_l^2}K_{lj}\{\pi_a, \Omega^l_c\theta'^{c}\}$$
$$= \kappa\omega^{ai}\sum_l \frac{2}{\alpha_l^2}K_{lj}[-\partial_a\Omega^l_c\theta'^{c}\delta(x_1-y_1) + \partial_1\Omega^l_a\delta(x_1-y_1) + \Omega^l_a\delta'(x_1-y_1)]$$
$$= \kappa\omega^{ai}\sum_l \frac{2}{\alpha_l^2}K_{lj}[(\partial_c\Omega^l_a - \partial_a\Omega^l_c)\theta'^{c}\delta(x_1 - y_1) + \Omega^l_a\delta'(x_1 - y_1)]$$
$$= \frac{2\kappa}{\alpha_i^2}K_{ij}\delta'(x_1 - y_1),$$
$$\therefore \quad \left\{\kappa\sum_k \frac{2}{\alpha_k^2}K_{ki}\Omega^k_a\theta'^{a}, \omega^{cj}\pi_c\right\} = \frac{2\kappa}{\alpha_i^2}K_{ij}\delta'(x_1 - y_1),$$

在以上各 Poisson 括号的计算中，我们不加说明地应用了附录中得到的数学公式.将以上结果综合起来得:

$$\{\mathscr{J}(H_i, x), \mathscr{J}(H_j, y)\} = \frac{4\kappa}{\alpha_i^2}K_{ij}\delta'(x_1 - y_1), \tag{17a}$$

同理有:

$$\{\mathscr{J}(H_i, x), \mathscr{J}(E_\beta, y)\} = K_{\beta i}\mathscr{J}(E_\beta, x)\delta(x_1 - y_1), \tag{17b}$$
$$\{\mathscr{J}(E_\alpha, x), \mathscr{J}(E_\beta, y)\} = \delta_{\alpha+\beta,0}\left[\sum_{ij}K^{-1}_{ij}K_{j\alpha}\mathscr{J}(H_i, x)\delta(x_1 - y_1) + \frac{4\kappa}{\alpha^2}\delta'(x_1 - y_1)\right] + N_{\alpha,\beta}\mathscr{J}(E_{\alpha+\beta}, x)\delta(x_1 - y_1), \tag{17c}$$

$$\{\tilde{\mathscr{J}}(H_i, x), \tilde{\mathscr{J}}(H_j, y)\} = -\frac{4\kappa}{\alpha_i^2}K_{ij}\delta'(x_1 - y_1), \tag{18a}$$
$$\{\tilde{\mathscr{J}}(H_i, x), \tilde{\mathscr{J}}(E_\beta, y)\} = K_{\beta i}\tilde{\mathscr{J}}(E_\beta, x)\delta(x_1 - y_1), \tag{18b}$$
$$\{\tilde{\mathscr{J}}(E_\alpha, x), \tilde{\mathscr{J}}(E_\beta, y)\} = \delta_{\alpha+\beta,0}\left[\sum_{ij}K^{-1}_{ij}K_{j\alpha}\tilde{\mathscr{J}}(H_i, x)\delta(x_1 - y_1) - \frac{4\kappa}{\alpha^2}\delta'(x_1 - y_1)\right] + N_{\alpha,\beta}\tilde{\mathscr{J}}(E_{\alpha+\beta}, x)\delta(x_1 - y_1), \tag{18c}$$

$$\{\mathscr{J}(H_i, x), \tilde{\mathscr{J}}(H_j, y)\} = 0, \tag{19a}$$
$$\{\mathscr{J}(H_i, x), \tilde{\mathscr{J}}(E_\beta, y)\} = 0, \tag{19b}$$
$$\{\mathscr{J}(E_\alpha, x), \tilde{\mathscr{J}}(H_j, y)\} = 0, \tag{19c}$$
$$\{\mathscr{J}(E_\alpha, x), \tilde{\mathscr{J}}(E_\beta, y)\} = 0, \tag{19d}$$

$$(i, j = 1, 2, \cdots, \mathrm{rank}\mathscr{G}, \ \alpha, \beta \in \Phi),$$

(17)、(18) 正是 Chevalley 基下写出的 Kac-Moody 流代数，(19) 式则是 WZNW 场论具有共形不变性的体现。作为(17)—(19)的推论，我们有：

$$\{\mathscr{H}(x), \mathscr{J}(H_i, y)\} = \mathscr{J}(H_i, x)\delta'(x_1 - y_1) \tag{20a}$$

$$\{\mathscr{H}(x), \mathscr{J}(E_\alpha, y)\} = \mathscr{J}(E_\alpha, x)\delta'(x_1 - y_1), \tag{20b}$$

$$\{\mathscr{H}(x), \tilde{\mathscr{J}}(H_i, y)\} = -\tilde{\mathscr{J}}(H_i, x)\delta'(x_1 - y_1), \tag{20c}$$

$$\{\mathscr{H}(x), \tilde{\mathscr{J}}(E_\alpha, y)\} = -\tilde{\mathscr{J}}(E_\alpha, x)\delta'(x_1 - y_1), \tag{20d}$$

$$(i = 1, 2, \cdots, \text{rank}\mathscr{G};\ \alpha \in \Phi).$$

WZNW 场的 Hamilton 量为：

$$H = \int dx_1 \mathscr{H}(x), \tag{21}$$

于是，用守恒流分量表出的场的 Hamilton 正则运动方程是：

$$\partial_{x_0}\mathscr{J}(H_i, x) = \{\mathscr{J}(H_i, x), H\} = \partial_{x_1}\mathscr{J}(H_i, x), \tag{22a}$$

$$\partial_{x_0}\mathscr{J}(E_\alpha, x) = \{\mathscr{J}(E_\alpha, x), H\} = \partial_{x_1}\mathscr{J}(E_\alpha, x), \tag{22b}$$

$$\partial_{x_0}\tilde{\mathscr{J}}(H_i, x) = \{\tilde{\mathscr{J}}(H_i, x), H\} = -\partial_{x_1}\tilde{\mathscr{J}}(H_i, x), \tag{22c}$$

$$\partial_{x_0}\tilde{\mathscr{J}}(E_\alpha, x) = \{\tilde{\mathscr{J}}(E_\alpha, x), H\} = -\partial_{x_1}\tilde{\mathscr{J}}(E_\alpha, x), \tag{22d}$$

这与前面得到的 Lagrange 方程(7)、(9)一致。

至此完成了 Chevalley 基下二维 WZNW 场论的 Hamilton 形式的讨论。

作者感谢侯伯宇教授的热情指导及赵柳同学的许多有益的讨论。

附　　录

(A)

本附录给出 $N_{\alpha,\beta}$ 的计算法则. $N_{\alpha,\beta}$ 由 (1c) 式定义：

$$[E_\alpha, E_\beta] = \sum_{ij} K_{ij}^{-1} K_{i\alpha} H_i \delta_{\alpha+\beta,0} + N_{\alpha,\beta} E_{\alpha+\beta}, \tag{1c}$$

通常取 $H_i^t = H_i$、$E_\alpha^t = E_{-\alpha}$[1) ($i = 1, 2, \cdots$, rank$\mathscr{G}$; $\alpha \in \Phi$)。在如此选择的 Chevalley 基下容易证明[4]：

$$N_{\alpha,\beta} = -N_{\beta,\alpha} (N_{\alpha,-\alpha} = 0), \tag{A.1}$$

$$N_{\alpha,\beta} = -N_{-\alpha,-\beta}, \tag{A.2}$$

$$N_{\alpha,\beta} = \frac{(\alpha+\beta)^2}{\beta^2} N_{-\alpha,\alpha+\beta}, \tag{A.3}$$

(B)

下面给出矩阵元 $\Omega_a^i(\theta)$、$L_{AB}(\theta)$ 及张量 $\lambda_{ab}(\theta)$ 满足的数学等式.

1. $\lambda_{ab}(\theta)$ 是反对称张量：

$$\lambda_{ab} = -\lambda_{ba}, \tag{B.1}$$

2. $\Omega_a^i(\theta)$ 及 $\lambda_{ab}(\theta)$ 的运算规则是：

$$\partial_a \Omega_b^i - \partial_b \Omega_a^i = \sum_j \sum_{\beta\in\Phi} K_{ij}^{-1} K_{j\beta} \Omega_a^{-\beta} \Omega_b^\beta, \tag{B2a}$$

$$\partial_a \Omega_b^{-\alpha} - \partial_b \Omega_a^{-\alpha} = \sum_j K_{\alpha j} (\Omega_a^j \Omega_b^{-\alpha} - \Omega_a^{-\alpha} \Omega_b^j) + \sum_{\beta\in\Phi} \frac{\alpha^2}{(\alpha+\beta)^2} N_{\alpha,\beta} \Omega_a^{-\alpha-\beta} \Omega_b^\beta, \tag{B2b}$$

---

1) t 表示“转置”.

$$\partial_c\lambda_{ab} + \partial_a\lambda_{bc} + \partial_b\lambda_{ca} = \sum_j \sum_{\beta\in\Phi} \frac{2}{\beta^2} K_{\beta j}(\Omega_a^j \Omega_b^{-\beta}\Omega_c^\beta + \Omega_a^\beta\Omega_b^j\Omega_c^{-\beta} + \Omega_a^{-\beta}\Omega_b^\beta\Omega_c^j)$$

$$+ \sum_{\gamma,\beta\in\Phi} \frac{l}{(\beta+\gamma)^2} N_{\beta,\gamma}\Omega_a^{-\beta-\gamma}\Omega_b^\gamma\Omega_c^\beta, \tag{B.3}$$

以上三式均可由定义式(4)直接得到. 将(B.2)及(B. 3)相结合,还可以得:

$$\partial_c\lambda_{ab} + \partial_a\lambda_{bc} + \partial_b\lambda_{ca} = \sum_{ij} \frac{2}{\alpha_i^2} K_{ij}(\partial_a\Omega_b^j - \partial_b\Omega_a^j)\Omega_c^i + \sum_{\beta\in\Phi} \frac{2}{\beta^2}(\partial_a\Omega_b^{-\beta} - \partial_b\Omega_a^{-\beta})\Omega_c^\beta, \tag{B.4}$$

3. $\Omega(\theta)$ 是非奇异矩阵，设其逆为 $\omega(\theta)$:

$$\begin{cases} \Omega_a^i\omega^{aj} = \delta^{ij}, \\ \Omega_a^i\omega^{a\beta} = 0, \; i, \; j = 1,2,\cdots, \; \mathrm{rank}\mathscr{G}. \\ \Omega_a^\alpha\omega^{aj} = 0, \; \alpha,\beta\in\Phi. \\ \Omega_a^\alpha\omega^{a\beta} = \delta_{\alpha+\beta,0}, \end{cases} \tag{B.5a}$$

$$\sum_i \omega^{ai}\Omega_b^i + \sum_{\alpha\in\Phi}\omega^{a\alpha}\Omega_b^{-\alpha} = \delta_b^a, \tag{B.5b}$$

则有:

$$\partial_b\omega^{ci} = -\omega^{ai}\left[\sum_j \omega^{cj}\partial_b\Omega_a^j + \sum_{\beta\in\Phi}\omega^{c\beta}\partial_b\Omega_a^{-\beta}\right], \tag{B.6a}$$

$$\partial_b\omega^{c\alpha} = -\omega^{a\alpha}\left[\sum_j \omega^{cj}\partial_b\Omega_a^j + \sum_{\beta\in\Phi}\omega^{c\beta}\partial_b\Omega_a^{-\beta}\right], \tag{B.6b}$$

4. 构造矩阵 $L_{AB} = \mathrm{Tr}(AgBg^{-1})$，其典型矩阵元为:

$$L_{ij} = \mathrm{Tr}(H_igH_jg^{-1}), \; L_{i\beta} = \mathrm{Tr}(H_igE_\beta g^{-1}),$$
$$L_{\alpha j} = \mathrm{Tr}(E_\alpha gH_jg^{-1}), \; L_{\alpha\beta} = \mathrm{Tr}(E_\alpha gE_\beta g^{-1}), \tag{B.7}$$

于是有:

$$g^{-1}H_ig = \sum_{jl}\frac{\alpha_l^2}{2}K_{jl}^{-1}L_{il}H_j + \sum_{\gamma\in\Phi}\frac{\gamma^2}{2}L_{i\gamma}E_{-\gamma}, \tag{B.8a}$$

$$g^{-1}E_\alpha g = \sum_{jl}\frac{\alpha_l^2}{2}K_{jl}^{-1}L_{\alpha l}H_j + \sum_{\gamma\in\Phi}\frac{\gamma^2}{2}L_{\alpha\gamma}E_{-\gamma}, \tag{B.8b}$$

因此,由 $\mathrm{Tr}(AB) = \mathrm{Tr}(g^{-1}Agg^{-1}Bg)$ 可得:

$$\frac{2}{\alpha_i^2}K_{ij} = \sum_{kl}\frac{\alpha_l^2}{2}K_{kl}^{-1}L_{il}L_{jk} + \sum_{\alpha\in\Phi}\frac{\alpha^2}{2}L_{i\alpha}L_{j,-\alpha}, \tag{B.9a}$$

$$\frac{2}{\alpha^2}\delta_{\alpha+\beta,0} = \sum_{jl}\frac{\alpha_l^2}{2}K_{jl}^{-1}L_{\alpha l}L_{\beta l} + \sum_{\gamma\in\Phi}\frac{\gamma^2}{2}L_{\alpha,-\gamma}L_{\beta\gamma}, \tag{B.9b}$$

$$0 = \sum_{jl}\frac{\alpha_l^2}{2}K_{jl}^{-1}L_{il}L_{\alpha j} + \sum_{\gamma\in\Phi}\frac{\gamma^2}{2}L_{i\gamma}L_{\alpha,-\gamma}, \tag{B.9c}$$

又由 $\mathrm{Tr}([A,B], gcg^{-1}) = \mathrm{Tr}(g^{-1}Ag[g^{-1}Bg, c])$，知:

$$0 = \sum_{\alpha\in\Phi}\frac{\alpha^2}{2}K_{\alpha l}L_{i\alpha}L_{j}, \tag{B.10a}$$

$$0 = \sum_{kl}\frac{\alpha_l^2}{2}K_{kl}^{-1}K_{\alpha l}(L_{il}L_{jk} - L_{ik}L_{j\alpha}) + \sum_{\beta\in\Phi}\frac{\beta^2}{2}N_{\alpha,\beta}L_{i,-\beta}L_{j,\alpha+\beta}, \tag{B.10b}$$

$$K_{\alpha i}L_{\alpha j} = \sum_{\beta\in\Phi}\frac{\beta^2}{2}K_{\beta i}L_{i,-\beta}L_{\alpha\beta}, \tag{B10.c}$$

$$K_{\alpha i}L_{\alpha\beta} = \sum_{jl}\frac{\alpha_l^2}{2}K_{jl}^{-1}K_{\beta l}(L_{i\beta}L_{\alpha j} - L_{ij}L_{\alpha\beta}) + \sum_{\gamma\in\Phi}\frac{\gamma^l}{2}N_{\gamma,\beta}L_{\alpha,-\gamma}L_{i,\gamma+\beta}) \qquad \text{(B10.d)}$$

$$\delta_{\alpha+\beta,0}\sum_{kl}K_{kl}^{-1}K_{l\alpha}L_{kj} + N_{\alpha,\beta}L_{\alpha+\beta,j} = \sum_{\gamma\in\Phi}\frac{\gamma^2}{2}K_{\gamma j}L_{\alpha,-\gamma}L_{\beta,\gamma}, \qquad \text{(B10.e)}$$

$$\delta_{\alpha+\beta,0}\sum_{kl}K_{kl}^{-1}K_{l\alpha}L_{k\gamma} + N_{\alpha,\beta}L_{\alpha+\beta,\gamma} = \sum_{jl}\frac{\alpha_l^2}{2}K_{jl}^{-1}K_{\gamma l}(L_{\alpha\gamma}L_{\beta j} - L_{\alpha j}L_{\beta\gamma})$$
$$+ \sum_{\sigma\in\Phi}\frac{\sigma^2}{2}N_{\sigma,\gamma}L_{\alpha,\sigma+\gamma}L_{\beta,-\sigma}. \qquad \text{(B.10f)}$$

5.根据恒等式 $\partial_a L_{AB} = \mathrm{Tr}([A, \partial_a g g^{-1}]gBg^{-1})$ 知:

$$\partial_a L_{jk} = \sum_{\beta\in\Phi}K_{\beta j}\Omega_a^{-\beta}L_{\beta k}, \qquad \text{(B.11a)}$$

$$\partial_a L_{j\alpha} = \sum_{\beta\in\Phi}K_{\beta j}\Omega_a^{-\beta}L_{\beta\alpha}, \qquad \text{(B.11b)}$$

$$\partial_a L_{\alpha j} = -\sum_i K_{\alpha i}\Omega_a^i L_{\alpha j} + \sum_{kl}K_{kl}^{-1}K_{l\alpha}\Omega_a^\alpha L_{kj} + \sum_{\gamma\in\Phi}N_{\alpha,\gamma}\Omega_a^{-\gamma}L_{\alpha+\gamma,j}, \qquad \text{(B.11c)}$$

$$\partial_a L_{\alpha\beta} = -\sum_i K_{\alpha i}\Omega_a^i L_{\alpha\beta} + \sum_{kl}K_{kl}^{-1}K_{l\alpha}\Omega_a^\alpha L_{k\beta} + \sum_{\gamma\in\Phi}N_{\alpha,\gamma}\Omega_a^{-\gamma}L_{\alpha+\gamma,\beta}. \qquad \text{(B.11d)}$$

参　考　文　献

[1] P. Goddard & D. olive, *Int. J. Mod. Phys.*, **A1**(1986), 303.
[2] E. Witten, *Comm. Math. Phys.*, **92**(1984), 455.
[3] P. Bowcock, *Nucl. Phys.*, **B316**(1989), 80.
[4] J. Balog, L. Fehér, L. O'Raifeartaigh, P. Forgács and A Wipf, *Ann Phys.*, **203**(1990), 76.
[5] P Forgács, A Wipf, J. Balog, L. Fehér & L. O'Raifeartaigh, *Phys Lett.*, **B227**(1989), 214, **B244** (1990), 435.
[6] V G Knizhnik & A B Zamolodchikov, *Nucl. Phys.*, **B247**(1984), 83.
[7] 侯伯宇、赵柳、杨焕雄,高能物理与核物理, **15**(1991),701.
[8] K. Bardakci, E. Rabinovici & B. Säring, *Nucl. Phys.*, **B299**(1989), 151.
[9] L. O'Raifeartaigh & A Wipf, *Phys. Lett.*, **B251**(1990), 361.
[10] L. O'Raifeartaigh, P. Ruelle & I. Tsutsui, *Phys. Lett.*, **B258**(1991), 359.

Hamiltonian Formalism of WZNW Field Under Chevalley Basis

Yang Huanxiong

(*Inst of Mod. Phys., Northwest University, Xian* 710069)

Abstract

The Hamiltonian canonical formalism of two dimensional WZNW theory based on arbitrary semi-simple Lie algebras is given under Chevalley basis. The Poisson brackets of conserved chiral currents are calculated, which turn out to be the classical Kac-Moody current algebras.