



a way as to strongly damp the most prominent HOMs—the main source of coupled beam instabilities. A possibly negative effect of the introduction of these couplers will be detuning of the frequency of the HOMs, possibly on to one of the bunch frequency harmonics. To be able to couple to all HOMs, two HOM couplers using magnetic coupling, located at  $+/-120\text{mm}$  from the cavity (longitudinal) midplane (the total of four couplers is given by installing two pairs at  $90^\circ$ , see Fig.1(b)), will be installed.

Since the calculation of the cavity with HOM couplers and its other accessories is a three-dimension problem, a 3-D numerical method is needed to calculate the damping resulting from the installation of HOM couplers. In this paper, we describe how to calculate the resulting beam coupling impedance, due to the LHC capture cavity with the coaxial HOM couplers installed, using MAFIA<sup>[2]</sup> in the time domain. A complete simulation result for the HOM damping in LHC capture cavity is presented.

## 2 Simulations in Frequency and Time Domain

For the un-damped cavity without HOM couplers, frequency domain simulations for the HOM impedance and quality factor can achieve good results, since damping is caused by the cavity walls exclusively. But for the damped cavity with HOM couplers coupling to the outside, as in the LHC capture cavity, it is difficult to treat mathematically the boundary conditions when in the frequency domain, especially for a strongly damped cavity. Frequency domain modeling is not able to represent the whole real field pattern in the cavity. For this external coupling case, the time domain simulation is better. Consequently, frequency domain modeling is used first to calculate the impedance  $R_{sh}$ , the  $Q$  and the  $(R/Q)$  of HOMs for the LHC capture cavity without HOM couplers. This will provide the naturally damped HOM information. Then the same parameters for the case with the HOM couplers installed are calculated using the time domain simulation to get the damped results.

### 2.1 Frequency Domain Results

In the frequency domain, the “E-solver” of the MAFIA code is used to calculate the eigenvalues of the cavity modes, the impedance  $R_{sh}$  and  $Q$ . First, the cavity structure is modeled using the MAFIA mesh generator. Only 1/8 of the cavity needs to be modeled due to its symmetry. In the E-solver, all boundary conditions are specified as being ideal electric or magnetic surfaces—the beam pipes are considered to be electrically short. All symmetric planes can be specified in the P-module for the calculations of  $Q$ ,  $R_{sh}$  and  $R/Q$ . Several simulations were performed for different boundary conditions at the symmetric planes to obtain all the resonant modes in the frequency range of 2.6 GHz. Meanwhile, for comparison, we used the Urmel code to calculate all possible resonating modes in the same range of frequency for these different boundaries. Results from both codes give a good agreement considering the difference in both mesh size and modeling.

### 2.2 Time Domain Simulation

To study the HOM damping in the LHC capture cavity with the HOM couplers, the frequency domain model is not capable of describing the HOM power coupling to the outside world, whereas the time domain simulation has been shown to give good results<sup>[3,4]</sup>, although it is more complicated.

In the time domain, a full 3-D cavity structure in longitudinal direction (the beam direction) is necessary, to which the four identical HOM couplers are attached. Due to the symmetric distribution

in the longitudinal direction, for simplicity the HOM impedance of the cavity with only two HOM couplers can be calculated, and then analytical methods can be used for the extension to four HOM couplers.

The procedure for simulation in the time domain is as follows.

First, a 3-D model of the LHC capture cavity with HOM damper ports is generated in the mesh module of MAFIA. The HOM output ports are coaxial and are defined as infinitely long waveguide ports with a characteristic impedance of  $50 \Omega$  ( $VSWR = 1$ ). These ports have a wave-guide boundary condition which requires a calculation of the wave-guide modes in 2-D. For this a 2-D slice for coaxial HOM port is used, based on the 3-D model.

The E-solver of MAFIA calculates the 2-D eigenvalues for the wave-guide modes at each port. These coaxial ports are attached to a loop corresponding to the HOM coupler to be used (from the old SPS cavities) and penetrates into the cavity volume. Since the four HOM couplers have the same structure, only one port is calculated and the result can be used for others.

Next, the calculation procedure moves into the time domain by first loading correctly the wave-guide modes calculated in 2-D to all ports, including two coaxial ports and two beam pipe ports. Then a longitudinal excitation by a bunch with a Gaussian distribution is defined, which excites either the wake field along the  $z$ -axis in the cavity for the longitudinal impedance calculation or an off-set wake field for transverse impedance calculation (vertical offset of this exciting bunch). In fact, since the cavity with HOM ports is not a symmetric structure, dipole modes will also be excited even if the excitation bunch travels on the axis.

In our calculation, the exciting beam (line beam) used is given by

$$I(t) = \frac{cq}{\sqrt{2\pi}\sigma_s} e^{-\frac{(ct)^2}{2\sigma_s^2}}, \quad (1)$$

where  $q$  is the total charge of the beam bunch,  $\sigma_s$  the rms bunch length and  $c$  the speed of light. In our case,  $\sigma_s = 4\text{cm}$ .

The Fourier transform of the above line current is given by

$$I(\omega) = qe^{-\frac{(\sigma_s\omega)^2}{2c^2}}. \quad (2)$$

In the time domain solver (T3), MAFIA calculates and records the induced wake fields in the bunch coordinate system  $s = ct$  in a given range behind the excitation bunch. The longitudinal impedance can be obtained by a fast Fourier transform on the calculated wake function  $W(s)$

$$Z(\omega) = \frac{1}{c} \int_0^{\infty} W(s) e^{-i\frac{\omega}{c}s} ds. \quad (3)$$

The impedance given by Eq. (3), divided by the bunch spectrum  $I(\omega)$  gives the normalized beam impedance.

In our calculation, 100, 200 and 400 m wake fields lengths have been used. The observed wake fields depend on the  $Q$  of the resonance. By comparing and extrapolating the results from different wake field lengths, the impedance of each HOM can be estimated with better accuracy. Due to the computer memory limitations, the minimum mesh size, which can be chosen in the calculation, is only 0.005 m (in the  $z$ -direction, the  $x$  and  $y$  directions have to be bigger), so a Gaussian bunch rms length  $\sigma_s = 0.04\text{m}$ , with total charge  $q = 1\text{C}$ , was put on the  $z$ -axis, traveling with the speed of light.

In principle, the longer the calculation time, the more accurate the impedance obtained. However, it is impossible to calculate an infinitely long wake, hence two MAFIA runs were made with the second having twice the calculation time (or calculation wake length) of the first run. The two

calculated HOM impedance values were then used to obtain the HOM impedance for the damped modes for an infinitely long wake field using the method in Ref. [1].

The calculated impedance (peak values) for every HOM mode in both runs can be accurately measured by using enough sampling points. But in most cases a careful search point by point has to be made, and sometimes some fitting has to be done to get the peak values. This is due to the limited resolution of the impedance spectrum in the frequency domain that has a range of 0 to 2.6GHz in our case.

### 3 Simulation Results

As mentioned in the section 2.1, to compare the HOM damping effects before and after the installation of HOM couplers, the un-damped modes were first calculated in the frequency domain, which gives better results than in the time domain. The calculation result shows that there are a lot of HOMs with very high impedance in the LHC capture cavity below cut-off frequency. These must be coupled outside for stable operation.

The un-damped mono-and dipole-modes in the LHC capture cavity are displayed in Fig.2.

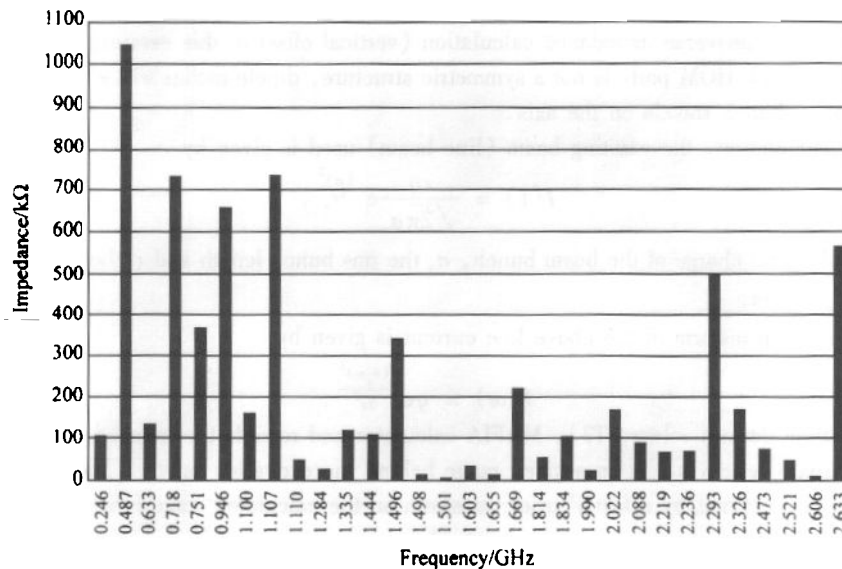


Fig.2. Amplitude of the un-damped mode impedance in the LHC capture cavity without HOM couplers.

The procedure described above was then used to find the HOM impedance in the cavity with two HOM couplers located at  $90^\circ$  to each other on one side of the cavity, supposing that there is not much influence on the  $R/Q$  of cavity by leaving out the other two HOM couplers. The two pairs of HOM couplers have a symmetric location along  $z$ -axis and it is very difficult for the MAFIA time domain simulation to process such a case where there are also two coaxial ports in one plane. Consequently, we adopted an analytical method, based on the 2-coupler case and the naturally damped case, to derive the HOM impedance for the case of the four HOM couplers. Figs. 3 and 4 display clearly the strengths of the HOM damping for the cases of two and four HOM couplers, respectively.

The simulation suggests that the impedance of most of the modes is strongly damped to below 5

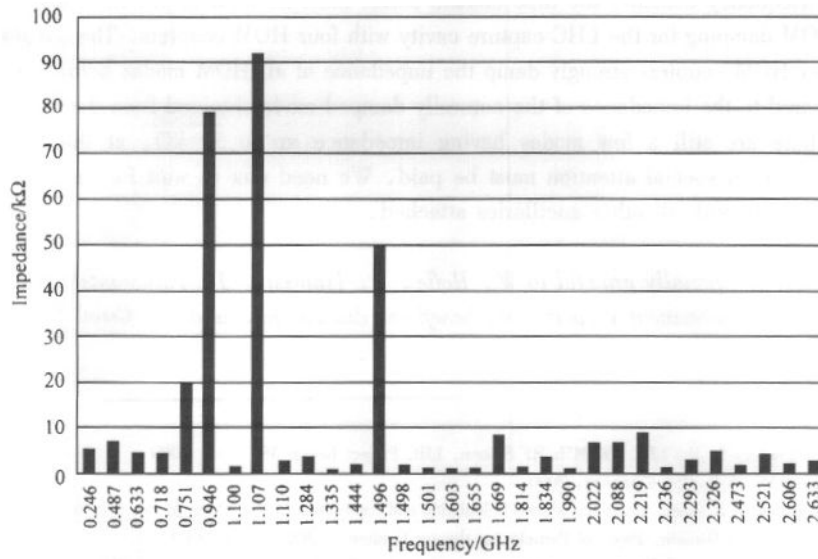


Fig. 3. Amplitude of the damped mode impedance in the LHC capture cavity with two HOM couplers.

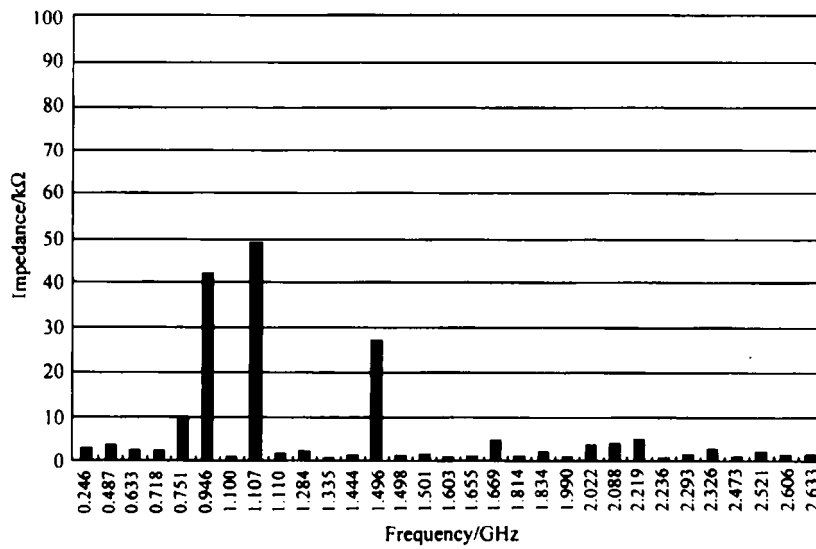


Fig. 4. Amplitude of the damped mode impedance in the LHC capture cavity with four HOM couplers.

kΩ. But the modes at 751, 946, 1100 and 1496 MHz have higher impedance values up to 50 kΩ (although they are nonetheless strongly damped by the introduction of HOM couplers).

#### 4 Conclusion

Due to the lack of an efficient method for calculating the HOM impedance in asymmetric 3-D

structures in frequency domain, the time domain solver must be used to provide the essential information on HOM damping for the LHC capture cavity with four HOM couplers. The simulation results show that four HOM couplers strongly damp the impedance of all HOM modes below the cut-off frequency compared to the impedance of the naturally damped mode obtained from the frequency domain solver. There are still a few modes having impedance up to 50 k $\Omega$ , at the limit for beam stability<sup>[5]</sup>, to which special attention must be paid. We need now to wait for the actual measurements on the cavity with all other ancillaries attached.

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### References

- 1 Design Considerations for the LHC 200 MHz RF System, LHC Project Report 386, Jan. 2000
- 2 MAFIA Manual PC V.4, CST Company, 2000
- 3 Li De-Run, Robert A Rimmer. Beam Impedance Calculation and Analysis of Higher Order Modes (HOMS) Damped RF Cavities Using MAFIA in Time Domain. Proc. of Particle Accelerator Conference 2001. June, 2001
- 4 Marhauser F et al. Numerical Simulations of HOM Damped Cavity, Proc. of Particle Accelerator Conference 2001. June, 2001
- 5 Shaposhnikova E. Longitudinal Beam Parameters During Acceleration in the LHC. LHC Project Note 242, Dec. 8, 2000

## LHC 俘获腔高次模阻抗及抑制研究

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**摘要** 为研究 LHC 俘获腔高次模阻抗的抑制结果,将频域和时域的方法应用于此腔的高次模阻抗的计算中,对装有 4 个磁耦合高次模抑制器 LHC 俘获腔(三维情况)做了抑制前后高次模阻抗的计算,得到了详细的腔的高次模频谱和阻抗抑制结果,由此可判断高次模吸收器的吸收效果和是否满足束流稳定性要求以及证实其设计的合理性.同时,模拟结果也显示引入高次模抑制器后腔中仍有一些阻抗值较高、具有潜在危险的高次模,这已引起高度重视并为今后的实验研究提供了重要信息.本文详细论述了用于 LHC 俘获腔的高次模阻抗计算的三维方法和高次模抑制的计算结果.

**关键词** 高次模 阻抗抑制 MAFIA 时域 LHC 俘获腔