非定域量子 Noether 恒等式及应用*

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摘要 基于高阶微商奇异拉氏量系统的相空间生成泛函,导出了定域和非定域变换下的量子正则 Noether 恒等式;对高阶微商规范不变系统,导出了位形空间中定域和非定域变换下的量子 Noether 恒等式. 指出在某些情形下,由量子 Noether 恒等式可导致系统的量子守恒律. 这种求守恒律的程式与量子 Noether (第一)定理不同. 用于高阶微商非 Abel Chern-Simons(CS)理论,求出某些非定域等变换下的量子守恒量.

关键词 高阶微商理论 路径积分 Noether 恒等式 守恒律 CS 理论

1 引言

在经典理论中,相空间定域变换导致的正则 Noether 恒等式,在场论中有重要应用[1,2],例如,分析系统的 Dirac 约束,研究约束乘子的性质[3],讨论 Dirac 猜想等[1,2],在某些情形下,由正则 Noether 恒等式还可导出系统相应的守恒律[4]. 这时在导出守恒律时应用的是经典 Noether 恒等式,结合了量子化后的有效拉氏量[2],是不彻底的量子理论. 近来已建立了整体对称下量子水平的 Noether(第一)定理[5],并导出了定域变换下的量子 Noether 恒等式[6]. 高阶微商理论与引力理论、规范场论、超对称和弦理论等密切相关,受到人们的关注[7]. 在杨-Mills 场论和规范场的共形对称等研究中,均涉及到非定域变换[8-11]. 本文研究高阶微商系统在定域和非定域变换下的量子 Noether 恒等式.

动力系统的路径积分量子化形式中,相空间路径积分比位形空间路径积分更普遍[12]. 在一些特殊情形,当作出前者对动量的路径积分后,可将其化为后者(如杨-Mills 场)[13]. 对于规范不变系统,更直观更简便的路径积分量子化是 Faddeev-Popov(FP)方法. FP 方法给出的结果有时可从约束哈密顿系统路径积分量子化的结果作出对动量的路径积分而得到.

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2 非定域量子正则 Noether 恒等式

设场由高阶微商正规拉氏量(密度)$\mathscr{L}(\varphi^a, \varphi^a_{,\mu}, \cdots \varphi^a_{,\mu(N)})$描述[7],由 Ostrogradsky 变换引入 $\varphi^a_{(s)}$的正则动量 $\pi_a^{(s)}$,系统 Green 函数的相空间生成泛函为[7,13]

$$Z[J,K] = \int \mathscr{D}\varphi^a_{(s)}\mathscr{D}\pi_a^{(s)} \exp\left\{ \mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}^{\mathrm{P}} + J^s_a\varphi^a_{(s)} + K^a_s\pi_a^{(s)}) \right\} \tag{2.1}$$

其中 $\mathscr{L}^{\mathrm{P}} = \pi_a^{(s)}\varphi^a_{(s+1)} - \mathscr{H}_c$,$\mathscr{H}_c$ 为正则哈密顿量密度,J^s_a 和 K^a_s 分别为 $\varphi^a_{(s)}$和 $\pi_a^{(s)}$ 的外源.

考虑扩展相空间中的无穷小定域和非定域变换:

$$\begin{cases} x^{\mu\prime} = x^\mu + \Delta x^\mu = x^\mu + R^\mu_\sigma \varepsilon^\sigma(x), \\ \varphi^a_{(s)}(x') = \varphi^a_{(s)}(x) + \Delta\varphi^a_{(s)}(x) = \varphi^a_{(s)}(x) + S^a_{s\sigma}\varepsilon^\sigma(x) + \int \mathrm{d}^4 y E(x,y) A^a_{s\sigma}(y)\varepsilon^\sigma(y), \\ \pi_a^{(s)\prime}(x') = \pi_a^{(s)}(x) + \Delta\pi_a^{(s)}(x) = \pi_a^{(s)}(x) + T^s_{a\sigma}\varepsilon^\sigma(x) + \int \mathrm{d}^4 y F(x,y) B^s_{a\sigma}(y)\varepsilon^\sigma(y), \end{cases} \tag{2.2}$$

其中 $E(x,y)$和 $F(x,y)$为给定函数,R^μ_σ, $S^a_{s\sigma}$, $T^s_{a\sigma}$, $A^a_{s\sigma}$ 和 $B^s_{a\sigma}$ 为线性微分算符[14],$\varepsilon^\sigma(x)$ $(\sigma = 1,2,\cdots,r)$为无穷小任意函数,它们的值及其微商在时空区域的边界上为零. 在(2.2)式变换下,设正则作用量的变更为

$$\Delta I^{\mathrm{P}} = \Delta\int \mathrm{d}^4 x \mathscr{L}^{\mathrm{P}} = \int \mathrm{d}^4 x U_\sigma \varepsilon^\sigma(x), \tag{2.3}$$

其中 U_σ 为线性微分算符;场量变换的 Jacobi 行列式记为 $\bar{J}[\varphi,\pi,\varepsilon] = 1 + J_1[\varphi,\pi,\varepsilon]$;而生成泛函可写为

$$Z[J,K,\varepsilon] = \int \mathscr{D}\varphi^a_{(s)}\mathscr{D}\pi_a^{(s)}\Big(1 + J_1 + \mathrm{i}\Delta I^{\mathrm{P}} + \mathrm{i}\int \mathrm{d}^4 x \Big\{ J^s_a\delta\varphi^a_{(s)} + K^a_s\delta\pi_a^{(s)} + \partial_\mu[(J^s_a\varphi^a_{(s)} + K^a_s\pi_a^{(s)})\Delta x^\mu] \Big\}\Big) \exp\left\{ \mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}^{\mathrm{P}} + J^s_a\varphi^a_{(s)} + K^a_s\pi_a^{(s)}) \right\}, \tag{2.4}$$

其中[14]

$$\Delta I^{\mathrm{P}} = \int \mathrm{d}^4 x \left\{ \frac{\delta I^{\mathrm{P}}}{\delta\varphi^a_{(s)}}\delta\varphi^a_{(s)} + \frac{\delta I^{\mathrm{P}}}{\delta\pi_a^{(s)}}\delta\pi_a^{(s)} + \mathrm{D}(\pi_a^{(s)}\delta\varphi^a_{(s)}) + \partial_\mu[(\pi_a^{(s)}\varphi^a_{(s+1)} - \mathscr{H}_c)\Delta x^\mu] \right\}, \tag{2.5}$$

$$\frac{\delta I^{\mathrm{P}}}{\delta\varphi^a_{(s)}} = -\dot{\pi}_a^{(s)} - \frac{\delta H_c}{\delta\varphi^a_{(s)}}, \quad \frac{\delta I^{\mathrm{P}}}{\delta\pi_a^{(s)}} = \dot{\varphi}^a_{(s)} - \frac{\delta H_c}{\delta\pi_a^{(s)}}, \tag{2.6}$$

$$\delta\varphi^a_{(s)} = \Delta\varphi^a_{(s)} - \varphi^a_{(s),\mu}\Delta x^\mu, \quad \delta\pi_a^{(s)} = \Delta\pi_a^{(s)} - \pi^{(s)}_{a,\mu}\Delta x^\mu \tag{2.7}$$

$\mathrm{D} = \mathrm{d}/\mathrm{d}t$,$H_c$ 为正则哈密顿量. 根据 $\varepsilon^\sigma(x)$的边界条件,由(2.3)—(2.5)式,有

$$\int \mathscr{D}\varphi^a_{(s)}\mathscr{D}\pi_a^{(s)}\int \mathrm{d}^4 x\left[\frac{\delta I^{\mathrm{P}}}{\delta\varphi^a_{(s)}}\delta\varphi^a_{(s)} + \frac{\delta I^{\mathrm{P}}}{\delta\pi_a^{(s)}}\delta\pi_a^{(s)} + \mathrm{D}(\pi_a^{(s)}\delta\varphi^a_{(s)}) - U_\sigma\varepsilon^\sigma(x) \right] \cdot \exp\left\{ \mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}^{\mathrm{P}} + J^s_a\varphi^a_{(s)} + K^a_s\pi_a^{(s)}) \right\} = 0. \tag{2.8}$$

将(2.2)式和(2.7)式代入(2.8)式,对与微分算符有关的项作分部积分,利用 $\varepsilon^\sigma(x)$的边界条件,并对 $\varepsilon^\sigma(x)$求泛函微商,得

$$\int \mathscr{D}\varphi^a_{(s)}\mathscr{D}\pi^{(s)}_a \Big\{\tilde{S}^a_{s\sigma}(x)\frac{\delta I^P}{\delta\varphi^a_{(s)}(x)} + \tilde{T}^s_{a\sigma}\frac{\delta I^P}{\delta\pi^{(s)}_a(x)} - \tilde{R}^\mu_\sigma(x)\Big[\varphi^a_{(s),\mu}(x)\frac{\delta I^P}{\delta\varphi^a_{(s)}(x)} + \pi^{(s)}_{a,\mu}(x)\frac{\delta I^P}{\delta\pi^{(s)}_a(x)}\Big] + \int d^4y\Big[\tilde{A}^a_{s\sigma}(y)\Big(E(y,x)\frac{\delta I^P}{\delta\varphi^a_{(s)}(y)} + D(\pi^{(s)}_a(y)E(y,x))\Big) + \tilde{B}^s_{a\sigma}\Big(F(y,x)\frac{\delta I^P}{\delta\pi^{(s)}_a(y)}\Big)\Big] - \tilde{U}_\sigma(1)\Big\}\cdot \exp\Big\{i\int d^4x(\mathscr{L}^P + J^s_a\varphi^a_{(s)} + K^a_s\pi^{(s)}_a)\Big\} = 0, \tag{2.9}$$

其中 $\tilde{S}^a_{s\sigma}, \tilde{T}^s_{a\sigma}, \tilde{R}^\mu_\sigma, \tilde{A}^a_{s\sigma}, \tilde{B}^s_{a\sigma}$ 和 $\tilde{U}_\sigma$ 分别为 $S^a_{s\sigma}, T^s_{a\sigma}, R^\mu_\sigma, A^a_{s\sigma}, B^s_{a\sigma}$ 和 U_σ 的伴随算符[15].

将(2.9)式关于 $J^0_a(x)$ 求 n 次泛函微商后,让 $J^s_a = K^a_s = 0$,得

$$\langle O|T^*\Big\{\tilde{S}^a_{s\sigma}\frac{\delta I^P}{\delta\varphi^a_{(s)}} + \tilde{T}^s_{a\sigma}\frac{\delta I^P}{\delta\pi^{(s)}_a} - \tilde{R}^\mu_\sigma\Big[\varphi^a_{(s),\mu}\frac{\delta I^P}{\delta\varphi^a_{(s)}} + \pi^{(s)}_{a,\mu}\frac{\delta I^P}{\delta\pi^{(s)}_a}\Big] + \int d^4y\Big[\tilde{A}^a_{s\sigma}\Big(E(y,x)\frac{\delta I^P}{\delta\varphi^a_{(s)}} + D(\pi^{(s)}_a E(y,x))\Big) + \tilde{B}^s_{a\sigma}\Big(F(y,x)\frac{\delta I^P}{\delta\pi^{(s)}_a}\Big)\Big] - \tilde{U}_\sigma(1)\Big\} \varphi^a(x_1)\varphi^a(x_2)\cdots\varphi^a(x_n)|O\rangle \tag{2.10}$$

其中 T^* 是一种特定的编时乘积[16]. 固定 t,让

$$t_1, t_2, \cdots, t_m \to \infty, t_{m+1}, t_{m+2}, \cdots, t_n \to -\infty.$$

于是(2.10)式化为[16]

$$\langle \text{out}, m\Big|\Big\{\tilde{S}^a_{s\sigma}\frac{\delta I^P}{\delta\varphi^a_{(s)}} + \tilde{T}^s_{a\sigma}\frac{\delta I^P}{\delta\pi^{(s)}_a} - \tilde{R}^\mu_\sigma\Big[\varphi^a_{(s),\mu}\frac{\delta I^P}{\delta\varphi^a_{(s)}} + \pi^{(s)}_{a,\mu}\frac{\delta I^P}{\delta\pi^{(s)}_a}\Big] + \int d^4y\Big[\tilde{A}^a_{s\sigma}\Big(E(y,x)\cdot\frac{\delta I^P}{\delta\varphi^a_{(s)}} + D(\pi^{(s)}_a E(y,x))\Big) + \tilde{B}^s_{a\sigma}\Big(F(y,x)\frac{\delta I^P}{\delta\pi^{(s)}_a}\Big)\Big] - \tilde{U}_\sigma(1)\Big\}\Big|n - m, \text{in}\rangle = 0. \tag{2.11}$$

由于 m, n 任意,从(2.11)式,得

$$\tilde{S}^a_{s\sigma}\frac{\delta I^P}{\delta\varphi^a_{(s)}} + \tilde{T}^s_{a\sigma}\frac{\delta I^P}{\delta\pi^{(s)}_a} - \tilde{R}^\mu_\sigma\Big[\varphi^a_{(s),\mu}\frac{\delta I^P}{\delta\varphi^a_{(s)}} + \pi^{(s)}_{a,\mu}\frac{\delta I^P}{\delta\pi^{(s)}_a}\Big] + \int d^4y\Big[\tilde{A}^a_{s\sigma}\Big(E(y,x)\frac{\delta I^P}{\delta\varphi^a_{(s)}} + D(\pi^{(s)}_a E(y,x))\Big) + \tilde{B}^s_{a\sigma}\Big(F(y,x)\frac{\delta I^P}{\delta\pi^{(s)}_a}\Big)\Big] - \tilde{U}_\sigma(1) = 0. \tag{2.12}$$

(2.12)式为正规拉氏量系统在定域和非定域变换下的量子正则 Noether 恒等式.

对于含高阶微商的奇异拉氏量系统,其 Green 函数的相空间生成泛函可写为[7]

$$Z[J,K] = \int \mathscr{D}\varphi^a_{(s)}\mathscr{D}\pi^{(s)}_a \exp\Big\{i\int d^4x(\mathscr{L}^P_{\text{eff}} + J^s_a\varphi^a_{(s)} + K^a_s\pi^{(s)}_a)\Big\}, \tag{2.13}$$

其中

$$\mathscr{L}^P_{\text{eff}} = \mathscr{L}^P + \lambda_k\Phi_k + \frac{1}{2}\int d^4y\bar{C}_l(x)\{\Phi_l(x), \Phi_m(y)\}C_m(y), \tag{2.14}$$

而 $\varphi^a_{(s)}$ 代表$(\varphi^a_{(s)}, \bar{C}_l, C_m, \lambda_k)$. 对含第二类约束系统, $\{\Phi\}$ 代表所有第二类约束;对含第一类约束系统, $\{\Phi_k\}$ 包括所有第一类约束和规范条件[13], $\{\cdot,\cdot\}$ 代表场的广义 Poisson 括

号,$C_l(x)$ 和 $C_m(x)$ 为 Grassmann 变量场,$\lambda_k(x)$ 为乘子场. 上述对正规拉氏量系统的讨论,也适用于奇异拉氏量系统,并同样可得量子正则 Noether 恒等式(2.12),只要将相应的 I^P 改为 I^P_{eff} 就行了. 无论变换(2.2)的 Jacobi 行列式是否为 1,量子正则 Noether 恒等式均成立.

3 规范不变系统

高阶微商规范不变系统为广义约束哈密顿系统[17],该系统的路径积分量子化,可用 FP 方法来实现. 设系统规范不变的拉氏量密度为 $\mathscr{L}=\mathscr{L}(\varphi,\varphi_{,\mu},\cdots,\varphi_{,\mu(N)})$,选取适当的规范条件,用 FP 方法得到系统位形空间中 Green 函数的生成泛函为[13]

$$Z[J]=\int\mathscr{D}\varphi\exp\left\{i\int d^4x(\mathscr{L}_{\text{eff}}+J\varphi)\right\},\tag{3.1}$$

其中 J 为 φ 的外源,$\mathscr{L}_{\text{eff}}=\mathscr{L}+\mathscr{L}_f+\mathscr{L}_{gh}$,$\mathscr{L}_f$ 为规范固定项,它与规范条件有关,$\mathscr{L}_{gh}$ 为鬼粒子项.

考虑位形空间中的无穷小定域和非定域变换

$$\begin{cases} x'^{\mu}=x^{\mu}+\Delta x^{\mu}=x^{\mu}+R^{\mu}_{\sigma}\varepsilon^{\sigma}(x),\\ \varphi'(x')=\varphi(x)+\Delta\varphi(x)=\varphi(x)+S_{\sigma}\varepsilon^{\sigma}_{(x)}+\int d^4yE(x,y)A_{\sigma}(y)\varepsilon^{\sigma}(y),\end{cases}\tag{3.2}$$

其中 R^{μ}_{σ},S_{σ} 和 A_{σ} 为线性微分算符,$\varepsilon^{\sigma}(x)$ 为无穷小任意函数. 在(3.2)式变换下,设有效作用量的变更为

$$\Delta I_{\text{eff}}=\Delta\int d^4x\mathscr{L}_{\text{eff}}=\int d^4xV_{\sigma}\varepsilon^{\sigma}(x),\tag{3.3}$$

其中 V_{σ} 为线性微分算符;场量变换的 Jacobi 行列式记为 $J[\varphi,\varepsilon]=1+J_1[\varphi,\varepsilon]$. 在(3.2)式变换下,生成泛函为

$$Z[J,\varepsilon]=\int\mathscr{D}\varphi\left\{1+J_1+i\Delta I_{\text{eff}}+i\int d^4x[J\delta\varphi+\partial_{\mu}(J\varphi\Delta x^{\mu})]\right\}\cdot \exp\left\{i\int d^4x(\mathscr{L}_{\text{eff}}+J\varphi)\right\},\tag{3.4}$$

其中[18]

$$\Delta I_{\text{eff}}=\int d^4x\left\{\frac{\delta I_{\text{eff}}}{\delta\varphi}\left[(S_{\sigma}-\varphi_{,\mu}R^{\mu}_{\sigma})\varepsilon^{\sigma}(x)+\int d^4yE(x,y)A_{\sigma}(y)\varepsilon^{\sigma}(y)\right]+\partial_{\mu}(j^{\mu}_{\sigma}\varepsilon^{\sigma}(x))+\partial_{\mu}\left[\sum_{m=0}^{N-1}\prod{}^{\mu\nu(m)}_{\text{eff}}\partial_{\nu(m)}\int d^4yE(x,y)A_{\sigma}(y)\varepsilon^{\sigma}(y)\right]\right\},\tag{3.5}$$

$$\frac{\delta I_{\text{eff}}}{\delta\varphi}=(-1)^m\partial_{\mu(m)}\mathscr{L}^{\mu(m)}_{\text{eff}},\quad \mathscr{L}^{\mu(m)}_{\text{eff}}=\frac{1}{m!}\sum_{\text{指标}m\text{的所有排列}}\frac{\partial\mathscr{L}_{\text{eff}}}{\partial\varphi_{,\mu(m)}},\tag{3.6}$$

$$j^{\mu}_{\sigma}=\mathscr{L}_{\text{eff}}R^{\mu}_{\sigma}+\sum_{m=0}^{N-1}\prod{}^{\mu\nu(m)}_{\text{eff}}\partial_{\nu(m)}(S_{\sigma}-\varphi_{,\mu}R^{\mu}_{\sigma}),\tag{3.7}$$

$$\prod{}^{\mu\nu(m)}_{\text{eff}}=\sum_{l=0}^{N-(m+1)}(-1)^l\partial_{\lambda(l)}\mathscr{L}^{\mu\nu(m)\lambda(l)}_{\text{eff}}.\tag{3.8}$$

由(3.3)—(3.7)式,注意到 $\varepsilon^\sigma(x)$ 的边界条件,有

$$\int \mathscr{D}\varphi \int \mathrm{d}^4 x \left\{ \frac{\delta I_{\mathrm{eff}}}{\delta\varphi}\left[(S_\sigma - \varphi_{,\mu} R^\mu_\sigma)\varepsilon^\sigma(x) + \int \mathrm{d}^4 y E(x,y) A_\sigma(y)\varepsilon^\sigma(y)\right] - V_\sigma \varepsilon^\sigma(x) + \partial_\mu\left[\sum_{m=0}^{N-1}\prod\nolimits_{\mathrm{eff}}^{\mu\nu(m)}\partial_{\nu(m)}\int \mathrm{d}^4 y E(x,y)A_\sigma(y)\varepsilon^\sigma(y)\right]\right\}\exp\left\{\mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}_{\mathrm{eff}} + J\varphi)\right\} = 0 , \quad (3.9)$$

将(3.9)式中与微分算符作用的项做分部积分,利用 $\varepsilon^\sigma(x)$ 的边界条件,然后对 $\varepsilon^\sigma(x)$ 求泛函微商,得

$$\int \mathscr{D}\varphi \left\{ \tilde{S}_\sigma(x)\frac{\delta I_{\mathrm{eff}}}{\delta\varphi(x)} - \tilde{R}^\mu_\sigma(x)\left(\varphi_{,\mu}(x)\frac{\delta I_{\mathrm{eff}}}{\delta\varphi(x)}\right) + \int \mathrm{d}^4 y \tilde{A}_\sigma(y)\left[E(y,x)\frac{\delta I_{\mathrm{eff}}}{\delta\varphi(y)} + \partial_\mu\left(\sum_{m=0}^{N-1}\prod\nolimits_{\mathrm{eff}}^{\mu\nu(m)}\partial_{\nu(m)}E(y,x)\right)\right] - \tilde{V}_\sigma(1)\right\}\exp\left\{\mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}_{\mathrm{eff}} + J\varphi)\right\} = 0 , \quad (3.10)$$

其中 $\tilde{S}_\sigma$, $\tilde{R}^\mu_\sigma$, $\tilde{A}_\sigma$ 和 $\tilde{V}_\sigma$ 分别为 S_σ, R^μ_σ, A_σ 和 V_σ 的伴随算符.

将(3.10)式关于 $J(x)$ 求 n 次泛函微商,让 $J=0$,固定 t,并让 $t_1,t_2,\cdots,t_m\to\infty$, t_{m+1}, $t_{m+2},\cdots,t_n\to-\infty$,类似于第2节中的推导,可得

$$\tilde{S}_\sigma\left(\frac{\delta I_{\mathrm{eff}}}{\delta\varphi}\right) - \tilde{R}^\mu_\sigma\left(\varphi_{,\mu}\frac{\delta I_{\mathrm{eff}}}{\delta\varphi}\right) + \int \mathrm{d}^4 y\tilde{A}_\sigma\left[E(y,x)\frac{\delta I_{\mathrm{eff}}}{\delta\varphi} + \partial_\mu\left(\sum_{m=0}^{N-1}\prod\nolimits_{\mathrm{eff}}^{\mu\nu(m)}\partial_{\nu(m)}E(y,x)\right)\right] = \tilde{V}_\sigma(1) \quad (3.11)$$

(3.10)式为高阶微商规范不变系统在位形空间中定域和非定域变换下的量子 Noether 恒等式,出现在该式中是量子化后的有效作用量 I_{eff},而不是经典作用量 I.

4 量子守恒律

由量子 Noether 恒等式,在某些情形下,可导致量子守恒律.为明确起见,考虑变换(2.2)式中 $\Delta x^\mu=0$ 的情形,而

$$\begin{cases} S^a_\sigma = a^a_{s\sigma} + a^{a\mu}_{s\sigma}\partial_\mu + a^{a\mu\nu}_{s\sigma}\partial_\mu\partial_\nu , \\ T^s_{a\sigma} = b^s_{a\sigma} + b^{s\mu}_{a\sigma}\partial_\mu + b^{s\mu\nu}_{a\sigma}\partial_\mu\partial_\nu , \end{cases} \quad (4.1)$$

其中系统 a 和 b 等均是 x, $\varphi^a_{(s)}$ 和 $\pi^{(s)}_a$ 的函数.在(2.2)和(4.1)式的变换下,设(2.3)式中的 U_σ 为

$$U_\sigma = u_\sigma + u^\mu_\sigma\partial_\mu + u^{\mu\nu}_\sigma\partial_\mu\partial_\nu , \quad (4.2)$$

其中 u_σ, u^μ_σ 和 $u^{\mu\nu}_\sigma$ 均为 x, $\varphi^a_{(s)}$ 和 $\pi^{(s)}_a$ 的函数.此时量子正则 Noether 恒等式(2.12)成为

$$a^a_{s\sigma}\frac{\delta I^{\mathrm{P}}_{\mathrm{eff}}}{\delta\varphi^a_{(s)}} - \partial_\mu\left(a^{a\mu}_{s\sigma}\frac{\delta I^{\mathrm{P}}_{\mathrm{eff}}}{\delta\varphi^a_{(s)}}\right) + \partial_\mu\partial_\nu\left(a^{a\mu\nu}_{s\sigma}\frac{\delta I^{\mathrm{P}}_{\mathrm{eff}}}{\delta\varphi^a_{(s)}}\right) + b^s_{a\sigma}\frac{\delta I^{\mathrm{P}}_{\mathrm{eff}}}{\delta\pi^{(s)}_a} - \partial_\mu\left(b^{s\mu}_{a\sigma}\frac{\delta I^{\mathrm{P}}_{\mathrm{eff}}}{\delta\pi^{(s)}_a}\right) + \partial_\mu\partial_\nu\left(b^{s\mu\nu}_{a\sigma}\frac{\delta I^{\mathrm{P}}_{\mathrm{eff}}}{\delta\pi^{(s)}_a}\right) = u_\sigma - \partial_\mu u^\mu_\sigma + \partial_\mu\partial_\nu u^{\mu\nu}_\sigma . \quad (4.3)$$

在(2.2)和(4.1)式变换下,由有效正则作用量的变更,有基本恒等式

$$\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}(a_{s\sigma}^{a}+a_{s\sigma}^{a\mu}\partial_{\mu}+a_{s\sigma}^{a\mu\nu}\partial_{\mu}\partial_{\nu})\varepsilon^{\sigma}(x)+\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}(b_{a\sigma}^{s}+b_{a\sigma}^{s\mu}\partial_{\mu}+b_{a\sigma}^{s\mu\nu}\partial_{\mu}\partial_{\nu})\varepsilon^{\sigma}(x)+$$

$$\mathrm{D}[\pi_{a}^{(s)}S_{s\sigma}^{a}\varepsilon^{\sigma}(x)]=(u_{\sigma}+u_{\sigma}^{\mu}\partial_{\mu}+u_{\sigma}^{\mu\nu}\partial_{\mu}\partial_{\nu})\varepsilon^{\sigma}(x) \tag{4.4}$$

将(4.3)式乘 $\varepsilon^{\sigma}(x)$，对 σ 求和后与(4.4)式相减，得

$$\partial_{\mu}\left\{\left[a_{s\sigma}^{a\mu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}+b_{a\sigma}^{s\mu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}+\partial_{\nu}\left(a_{s\sigma}^{a\mu\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}\right)-\left(a_{s\sigma}^{a\mu\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}\right)\partial_{\nu}+\partial_{\nu}\left(b_{a\sigma}^{s\mu\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}\right)-\right.\right.$$

$$\left.\left.\left(b_{a\sigma}^{s\mu\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}\right)\partial_{\nu}-u_{\sigma}^{\mu}+\partial_{\nu}u_{\sigma}^{\mu\nu}-u_{\sigma}^{\mu\nu}\partial_{\nu}\right]\varepsilon^{\sigma}(x)+\mathrm{D}\left[\pi_{a}^{(s)}S_{s\sigma}^{a}\varepsilon^{\sigma}(x)\right]\right\}=0, \tag{4.5}$$

从而得强守恒律

$$Q=\int\mathrm{d}^{3}x j_{\sigma}\varepsilon^{\sigma}(x)=\mathrm{const} \tag{4.6a}$$

$$j_{\sigma}=a_{s\sigma}^{a0}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}+b_{a\sigma}^{s0}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}+\partial_{\nu}\left(a_{s\sigma}^{a0\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}\right)-\left(a_{s\sigma}^{a0\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\varphi_{(s)}^{a}}\right)\partial_{\nu}+$$

$$\partial_{\nu}\left(b_{a\sigma}^{s0\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}\right)-\left(b_{a\sigma}^{s0\nu}\frac{\delta I_{\mathrm{eff}}^{\mathrm{P}}}{\delta\pi_{a}^{(s)}}\right)\partial_{\nu}-u_{\sigma}^{0}+\partial_{\nu}u_{\sigma}^{0\nu}-u_{\sigma}^{0\nu}\partial_{\nu}+\pi_{a}^{(s)}S_{s\sigma}^{a}. \tag{4.6b}$$

当变换群有子群，且 $\varepsilon^{\sigma}(x)=\varepsilon_{0}^{\rho}\zeta_{\rho}^{\sigma}(x)$，其中 ε_{0}^{ρ} 为参数，$\zeta_{\rho}^{\sigma}(x)$ 为给定函数，此时强守恒律为

$$Q_{\rho}=\int\mathrm{d}^{3}x j_{\sigma}\zeta_{\rho}^{\sigma}=\mathrm{const}\ . \tag{4.7}$$

导出(4.6)和(4.7)式时，未利用系统的量子运动方程．根据系统的量子运动方程[18] $\delta I_{\mathrm{eff}}^{\mathrm{P}}/\delta\varphi_{(s)}^{\sigma}=0$，$\delta I_{\mathrm{eff}}^{\mathrm{P}}/\delta\pi_{a}^{(s)}=0$，由(4.7)式可得系统的(弱)量子守恒律．当系统的有效正则作用量 $I_{\mathrm{eff}}^{\mathrm{P}}$ 在上述变换下不变时，此时相应的(弱)量子守恒律为

$$Q_{\rho}^{*}=\int\mathrm{d}^{3}x\pi_{a}^{(s)}S_{s\sigma}^{a}\zeta_{\rho}^{\sigma}=\mathrm{const}\ , \tag{4.8}$$

此结果恰为有限李群对称(整体对称)下的量子守恒律[5]．这里给出的求量子守恒律的程式与量子正则 Noether(第一)定理完全不同．

对高阶微商规范不变系统，在(3.2)式中 $\Delta x^{\mu}=0$，$E=0$，而 $S_{\sigma}=a_{\sigma}+a_{\sigma}^{\mu}\partial_{\mu}+a_{\sigma}^{\mu\nu}\partial_{\mu}\partial_{\nu}$；在(3.3)式中，$V_{\sigma}=v_{\sigma}+v_{\sigma}^{\mu}\partial_{\mu}+v_{\sigma}^{\mu\nu}\partial_{\mu}\partial_{\nu}$，从(3.11)式出发，类似地可得强守恒律

$$\partial_{\mu}\left\{\left[\sum_{m=0}^{N-1}\prod_{\mathrm{eff}}^{\mu\nu(m)}\partial_{\nu(m)}S_{\sigma}+a_{\sigma}^{\mu}\frac{\delta I_{\mathrm{eff}}}{\delta\varphi}+\partial_{\nu}\left(a_{\sigma}^{\mu\nu}\frac{\delta I_{\mathrm{eff}}}{\delta\varphi}\right)-\left(a_{\sigma}^{\mu\nu}\frac{\delta I_{\mathrm{eff}}}{\delta\varphi}\right)\partial_{\nu}\right.\right.$$

$$\left.\left.-v_{\sigma}^{\mu}+\partial_{\nu}v_{\sigma}^{\mu\nu}-v_{\sigma}^{\mu\nu}\partial_{\nu}\right]\varepsilon^{\sigma}(x)\right\}=0 \tag{4.9}$$

当 $\varepsilon^{\sigma}(x)=\varepsilon_{0}^{\rho}\xi_{\rho}^{\sigma}$，$\varepsilon_{0}^{\rho}$ 为参数，利用系统的量子运动方程 $\delta I_{\mathrm{eff}}/\delta\varphi=0$，由(4.9)式可得系统的(弱)量子守恒律．

5　高阶微商非 Abel CS 理论

CS 理论在分数量子 Hall 效应乃至高温超导中有重要应用．高阶微商(2+1)维非

Abel CS规范场 A_μ^a 与标量场 φ 耦合的拉氏量密度为[18]

$$\mathscr{L} = -\frac{C^2}{4\pi}D_\rho F_{\mu\nu}^a D^\rho F^{a\mu\nu} - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}\left(\partial_\mu A_\nu^a A_\rho^a + \frac{1}{3}f_{bc}^a A_\mu^a A_\nu^b A_\rho^c\right) + (D_\mu\varphi)^+(D^\mu\varphi), \tag{5.1}$$

其中 $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{bc}^a A_\mu^b A_\nu^c$,$D_\mu$ 为协变微商,而 f_{bc}^a 为规范群的结构常数. 量子水平下理论规范不变要求 $\kappa = \frac{n}{4\pi}$(n 为整数)[19]. 根据 Faddeev-Senjanovic 量子化方案系统 Green 函数的相空间生成泛函可写为[18]

$$Z[J] = \int \mathscr{D}A_\mu^a \mathscr{D}P^{a\mu} \mathscr{D}B_\mu^a \mathscr{D}Q^{a\mu} \mathscr{D}\varphi \mathscr{D}\pi^+ \mathscr{D}\varphi^+ \mathscr{D}\pi \mathscr{D}\lambda \mathscr{D}C \mathscr{D}\bar{C} \cdot$$

$$\exp\left\{i\int d^3x\left(\mathscr{L}_{eff}^P + J_{1a}^\mu A_\mu^a + J_{2a}^\mu B_\mu^a + J_1^+\varphi + \varphi^+ J_1 + \bar{J}_{3a}C^a + \bar{C}^a J_{3a}\right)\right\}, \tag{5.2}$$

其中 $P^{a\mu}$,$Q^{a\mu}$,π^+ 和 π 分别为 A_μ^a,$B_\mu^a = \dot{A}_\mu^a$,φ 和 φ^+ 的正则动量,$\lambda(x)$ 为乘子场,$C(x)$ 和 $\bar{C}(x)$ 为 Grassmann 变量鬼场,而

$$\mathscr{L}_{eff}^P = \mathscr{L}^P + \mathscr{L}_g + \mathscr{L}_{gh} + \mathscr{L}_m, \tag{5.3}$$

$$\mathscr{L}^P = B_\mu^a P^{a\mu} + \dot{B}_\mu^a Q^{a\mu} + \dot{\varphi}^+\pi + \pi^+\dot{\varphi} - \mathscr{H}_c \tag{5.4}$$

$$\mathscr{L}_g = -\frac{1}{2\alpha_2}(\Omega_2^a)^2 = -\frac{1}{2\alpha_2}(\partial^\mu A_\mu^a)^2, \tag{5.5}$$

$$\mathscr{L}_{gh} = -\partial^\mu \bar{C}^a D_{b\mu}^a C^b, \tag{5.6}$$

$$\mathscr{L}_m = \lambda_0^a \Lambda^{(0)a} + \lambda_1^a \Lambda^{(1)a} + \lambda_2^a \Lambda^{(2)a} - \frac{1}{2\alpha_0}(\Omega_0^a)^2 - \frac{1}{2\alpha_1}(\Omega_1^a)^2 \tag{5.7}$$

其中 $\Lambda^{(i)a} \approx 0$ 为第一类约束,Ω_i^a 为规范条件.

考虑 BRS 变换

$$\begin{cases} \delta A_\mu^a = -\tau D_{b\mu}^a C^b, & (5.8a) \\ \delta\varphi = -i\tau T^a C^a \varphi, \delta\varphi^+ = i\tau\varphi^+ T^a C^a, & (5.8b) \\ \delta C^a = \frac{1}{2}f_{bc}^a C^b C^c, \delta\bar{C}^a = \frac{1}{\alpha_2}\partial^\mu A_\mu^a, & (5.8c) \end{cases}$$

其中 τ 为 Grassmann 参数($\varepsilon^a(x) = \tau C^a(x)$),$T^a$ 为规范群的生成元. 在 BRS 变换下,由 $\mathscr{L}^P + \mathscr{L}_g + \mathscr{L}_{gh}$ 决定的作用量在理论中是不变的,而第一类约束在 A_μ^a 的变换下,不离开约束超曲面,$\delta\mathscr{L}_m \approx 0$,因此在 BRS 变换下,$I_{eff}$ 决定的理论是不变的. 在约束超曲面内,由(4.8)式,得量子 BRS 守恒荷

$$Q_B = \int d^2x\left(P_a^\mu \delta A_\mu^a + Q_a^\mu \delta B_\mu^a + \pi^+\delta\varphi + \delta\varphi^+\pi + R_a\delta C^a + \delta\bar{C}^a R_a\right) \tag{5.9}$$

其中 $\bar{R}_a$,R_a 分别为 C^a,$\bar{C}^a$ 的正则动量. 此结果也可由量子水平的正则 Noether(第一)定理导出[18].

如果仅对 A_μ^a,φ 和 φ^+ 作(5.8a)和(5.8b)的变换,而鬼场 C^a 和 $\bar{C}^a$ 不变. 在约束超曲面内,$\mathscr{L}_{eff}^P$ 的变更

$$\delta\mathscr{L}_{eff}^P = W_a\varepsilon^a(x) + f_{bc}^a \partial^\mu \bar{C}^a C^b \partial_\mu \varepsilon^c(x), \tag{5.10}$$

其中 W_a 不依赖于 $\varepsilon^a(x)$ 的微商. 由(4.7)式,得量子 PBRS 守恒荷

$$Q' = \int d^2 x (P^\mu_a \delta A^a_\mu + Q^\mu_a \delta B^a_\mu + \pi^+ \delta\varphi + \delta\varphi^+ \pi - f^a_{bc} \dot{\bar{C}}^a C^b C^c) \tag{5.11}$$

此守恒荷与(5.9)式不同.

上述结果也可用第 3 节中的方法得到. 按 FP 方法，在 Lorentz 规范下，位形空间中的有效拉氏量为

$$\mathscr{L}_{\text{eff}} = \mathscr{L} - \partial^\mu \bar{C}^a D^a_{b\mu} C^b - \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2 . \tag{5.12}$$

不难验证，上式右端前两项之和对应的作用量其理论在下列变换下不变[8,9]

$$\begin{cases} \delta A^a_\mu = D^a_{\sigma\mu} \varepsilon^\sigma(x) , \\ \delta\varphi = - i T^\sigma \varphi \varepsilon^\sigma(x), \ \delta\varphi^+ = i\varphi^+ T^\sigma \varepsilon^\sigma(x) , \\ \delta C^a = i(T_\sigma)^a_b C^b \varepsilon^\sigma(x) , \\ \delta \bar{C}^a = i\bar{C}^b (T_\sigma)^a_b \varepsilon^\sigma(x) - i\int d^3 y \Delta_0(x,y) \partial_\mu [\bar{C}^b(y)(T_\sigma)^a_b \partial^\mu \varepsilon^\sigma(y)] \end{cases} \tag{5.13}$$

其中 $\Delta_0(x,y)$ 适合

$$\Box \Delta_0(x,y) = i\delta(x-y) \tag{5.14}$$

由量子 Noether 恒等式(3.11)，有

$$- i\left(\frac{\delta I_{\text{eff}}}{\delta\varphi(x)}\right) T^\sigma \varphi(x) + i\varphi^+(x) T^\sigma \left(\frac{\delta I_{\text{eff}}}{\delta\varphi^+(x)}\right) + \tilde{D}^a_{\sigma\mu}\left(\frac{\delta I_{\text{eff}}}{\delta A^a_\mu(x)}\right) + i(T_\sigma)^a_b \frac{\delta I_{\text{eff}}}{\delta C^a(x)} C^b(x) - $$
$$ i\bar{C}^b(x)(T_\sigma)^a_b \frac{\delta I_{\text{eff}}}{\delta \bar{C}^a(x)} + \int d^3 y \tilde{N}^a_\sigma(x) \left[\partial_\mu \left(\frac{\partial \mathscr{L}_{\text{eff}}}{\partial \bar{C}^a_{,\mu}}\right) \Delta_0(y,x)\right] = \frac{1}{\alpha}\tilde{D}^a_{\sigma\mu} \partial^\mu (\partial^\nu A^a_\nu), \tag{5.15}$$

其中

$$N^a_\sigma(x) = \partial_\mu [\bar{C}^b(x)(T_\sigma)^a_b \partial^\mu] , \tag{5.16}$$

$$\tilde{D}^a_{\sigma\mu} = - \delta^a_\sigma \partial_\mu + f^a_{\sigma c} A^c_\mu . \tag{5.17}$$

在规范约束下，利用系统的量子运动方程，由(5.15)式得量子守恒荷

$$Q = \int d^2 x \int d^3 y \bar{C}^b(x)(T_\sigma)^a_b \partial_{x_0} \left[\partial_{y_\nu} \left(\frac{\partial \mathscr{L}_{\text{eff}}}{\partial \bar{C}^a_{,y_\nu}}\right) \Delta_0(y,x)\right] . \tag{5.18}$$

将(5.12)代入(5.18)式，得

$$Q = \int d^2 x \int d^3 y \bar{C}^b(x)(T_\sigma)^a_b (\partial^\nu D^a_{c\nu} C^c) \partial_{x_0} \Delta_0(y,x) . \tag{5.19}$$

导出此量子守恒荷的方法有别于量子 Noether(第一)定理.

参考文献(References)

1 LI Z P. J. Phys. A: Math. Gen., 1991, **24**: 4261

2 LI Z P. Phys. Rev., 1994, **E50**: 876

3 LI Z P. Int. J. Theor. Phys., 1993, **32**: 201; Acta Physica Sinica, 1992, **41**: 710 (in Chinese)
(李子平. 物理学报, 1992, **41**: 710)

4 LI Z P. Int. J. Theor. Phys., 1994, **33**: 1207

5 LI Z P. Science in China (Scienta Sinica), Series A, 1996, **39**: 739; Z. Phys., 1997, **C76**: 181

6 LI Z P. High Energy Phys. and Nucl. Phys., 2002, **26**(3): 230 (in Chinese)
(李子平. 高能物理与核物理, 2002, **26**(3): 230)

7 LI Z P. Acta Physica Sinica, 1996, **45**: 1255(in Chinese)
(李子平. 物理学报, 1996, **45**: 1255)
8 KUANG Y P, YI Y P. High Energy Phys. and Nucl. Phys., 1980, **4**: 286(in Chinese)
(邝宇平, 易余萍. 高能物理与核物理, 1980, **4**: 286)
9 LI Z P. Int. J. Theor. Phys., 1995, **34**: 523
10 Fradkin E S, Ya M. Palchik, Phys. Lett., 1984, **B147**: 86
11 Rabello S J, Gaete P. Phys. Rev., 1995, **D52**: 7205
12 Mizrahi M M. J. Math. Phys., 1978, **19**: 298
13 Gitman D M, Tyutin T V. Quantization of Field with Constraints, Berlin: Springer-Verlag. 1990
14 LI Z P. Science in China(Scienta Sinica), Series A, 1993, **36**: 1212
15 LI Z P. Int. J. Theor. Phys., 1987, **26**: 853
16 Suura H, Young B L. Phys. Rev., 1973, **D8**: 875
17 LI Z P, JIANG J H. Symmetries in Constrained Canonical Systems. Beijing: Science press, 2002
18 LI Z P, LONG Z W. J. Phys. A: Math. Gen., 1999, **32**: 6391
19 Deser S, Jackiw R, Templeton S. Ann. Phys., 1982, **140**: 372

Non-local Quantal Noether Identities and Their Applications

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Abstract Based on the phase-space generating functional for a system with a singular higher-order Lagrangian, the quantal canonical Noether identities under the local and non-local transformation in phase space for such system have been derived. For a gauge-invariant system with a higher-order Lagrangian, the quantal Noether identities under the local and non-local transformation in configuration space have also been derived. It has been pointed out that in certain cases the quantal Noether identities may be converted to the conservation laws at the quantum level. This algorithm to derive the quantal conservation laws is significantly different from the first quantal Noether theorem. The applications to the non-Abelian CS theories with higher-order derivatives are given. The conserved quantities at the quantum level for some local and non-local transformation are found respectively.

Key words theory of higher-order derivatives, path integral, Noether identities, conservation laws, CS theories

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