Analysis of $D_s \to \phi\pi$ Beyond Naive Factorization *

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Abstract We analyze the decay $D_s \to \phi\pi$ with QCD factorization in the heavy quark limit. The nonfactorizable contributions, including hard spectator contribution, are discussed and numerical results are presented. Our predictions on the branching ratio of the decay are in agreement with the experiment. We also use a pure phenomenological method to estimate the branching ratio for $D_s \to \phi\pi$ with the existing $D^0 \to K^* \pi$ data.

Key words QCD factorization, D_s, nonfactorizable contributions

1 Introduction

Both CLEO[1] and BES[2] have reported their direct model-independent measurements for the $D_s \to \phi\pi$ branching fraction:

$$Br(D_s \to \phi\pi) = \begin{cases} (3.59 \pm 0.77 \pm 0.48) \times 10^{-2} & \text{CLEO}, \\ (3.9^{+5.1+1.8}_{-1.9-1.1}) \times 10^{-2} & \text{BES}. \end{cases}$$

The average branching ratio of $D_s \to \phi\pi$ is $(3.6 \pm 0.9) \times 10^{-2}$[3].

The precise estimation of the branching ratio for the decay $D_s \to \phi\pi$ is very important. First, it is difficult to measure the absolute branching ratio of $D_s \to \phi\pi$ because we do not know the fraction of $D_s^+ D_s^-$ pair production in $e^+ e^-$ annihilation in comparison with $D\bar{D}$ pairs (BES used $e^+ e^- \to D_s^+ D_s^-$ to obtain the first direct model-independent measurement of the $D_s \to \phi\pi$ branching fraction, however, with only two" double-tagged" events). But we need to know the branching ratio for the study of B decays such as $B \to D_s X$ etc. Moreover, most of the measurements of the D_s meson branching fractions are normalized to the clean $D_s \to \phi\pi$ channel. Second, theoretically, the decay of $D_s \to \phi\pi$ is dominated by spectator diagram with external emission of pion. This is easier to handle compared with other exclusive non-leptonic decay channels. Another reason for choosing the Cabibbo-favored decay $D_s \to \phi\pi$ is that, in this decay channel, from isospin analyses[4,5], we find that the final state involves only a single isospin, so there is no interference effects from the elastic final state interactions (FSI) when we calculate the branching ratio of $D_s \to \phi\pi$.

Previous calculations for the branching ratio $Br(D_s \to \phi\pi)$ are based on the naive factorization approach which is proposed by Bauer et al. (BSW)[6]. But in BSW approach, non-factorizable effects can not be calculated, they have to be parameterized by an effective color number N_c^{eff} which is treated as a free parameter. Moreover, results obtained with BSW approach still depend on renormalization scale and scheme. The authors in Ref.[7] examine the $D_s \to \phi\pi$ amplitude through a constituent quark-meson model. With this model, the calculated decay width $\Gamma(D_s \to \phi\pi)$ is larger than the

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experimental data. Paver and Riazuddin[8] studied $D_s\to\phi\pi$ in a valence quark triangle model, incorporating chiral symmetries, the result is compatible with the experimental data. In Refs.[9,10], the authors considered the contribution from the color octet: $\langle \phi\pi^+ \mid H_w^8 \mid D_s^+ \rangle$, where $H_w^{(8)} \equiv \frac{1}{2}\sum_a(\bar{u}\lambda^a c)(s\lambda^a d)$. But they all introduced some new parameters, so they brought new theoretical uncertainties.

In the past years, Beneke et al. developed QCD factorization (QCDF) approach[11] to calculate the hadronic matrix elements of B decays in the heavy quark limit. It has been used for many B decays modes[11,12] with interesting results. In the present paper, we will follow this method to calculate the branching ratio for $D_s\to\phi\pi$. Is it reasonable to apply QCD factorization method to charm decays? Well, firstly, charm quark is heavy($m_c \gg \Lambda_{QCD}$) and D_s is even heavier, although $m_c < m_b$. Secondly, the perturbative QCD (PQCD) is applicable to hard process when the momentum transfer Q^2 is very large. But actually for some processes, when $Q^2 > 1\text{GeV}^2$, PQCD is already applicable. So it is quite possible that QCD factorization is applicable for D_s decays. Thirdly, although ϕ is heavy ($m_\phi \sim 1\text{GeV}$), but ϕ absorbs the spectator quark in D_s, so QCD factorization can still be applied. Considering the above arguments, we should try to use QCD factorization method to study $D_s\to\phi\pi$ in detail and see what will come out.

2 $D_s\to\phi\pi$ in QCD Factorization

The low energy effective Hamiltonian for $D_s\to\phi\pi$ can be expressed as follows:

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}[C_1(\mu)Q_1(\mu) + C_2(\mu)Q_2(\mu)] + \text{h.c.}. \tag{1}$$

The four-quark local operators $Q_{1,2}$ are

$$Q_1 = (\bar{s}_\alpha c_\alpha)_{V-A}(u_\beta d_\beta)_{V-A};\quad Q_2 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}, \tag{2}$$

where α,β are the color indices of $SU(3)c$, respectivly. Wilson coefficients $C_i(\mu)$ are calculable with the renormalization group improved perturbation theory. The next-to-leading order (NLO) corrections to $C_i(\mu)$ have been presented in Ref.[13]. In the naive dimensional regularization (NDR) scheme, we give the numerical values for $C_i(\mu)$ at three renormalization scales:

$$\begin{aligned} &C_1(1\text{GeV}) = 1.272, \quad C_2(1\text{GeV}) = -0.500,\\ &C_1(m_c) = 1.200, \quad C_2(m_c) = -0.390,\\ &C_1(2\text{GeV}) = 1.153, \quad C_2(2\text{GeV}) = -0.314. \end{aligned} \tag{3}$$

Under naive factorization, the decay amplitude of $D_s\to\phi\pi$ reads

$$\mathscr{A}(D_s^- \to \phi\pi^-) = \sqrt{2}G_F V_{cs}^* V_{ud} f_\pi m_\phi A_0^{D_s\phi}(m_\pi^2)(\epsilon^* \cdot p_{D_s}) \cdot a_1, \tag{4}$$

where $a_1 = C_1 + \frac{1}{N_c^{\text{eff}}}C_2$, N_c^{eff} is the number of colors. From Eq.(4) we can see that the amplitude depends on the renormalization scale μ, because the Wilson coefficients $C_1(\mu)$, $C_2(\mu)$, and hence a_1, a_2 depend on μ, whereas the decay constant and form factor are independent of μ. So the amplitude $\mathscr{A}(D_s\to\phi\pi)$ is μ-dependent. On the other hand, it does not consider the nonfactorizable effects. If we calculate it with QCD factorization and take all the high order corrections into account, a_i and the amplitude $\mathscr{A}(D_s\to\phi\pi)$ will be μ independent. In our paper, we calculate it only to the order of α_s, so a_i and the amplitude $\mathscr{A}(D_s\to\phi\pi)$ still depend on μ, but the dependence is less sensitive to μ.

In the QCD factorization, the hadronic matrix elements of $D_s \to \phi\pi$ can be represented symbolically as:

$$\langle \pi\phi \mid Q_i(\mu) \mid D_s \rangle = \langle \pi \mid J_1 \mid 0 \rangle \langle \phi \mid J_2 \mid D_s \rangle \cdot \left[1 + \sum r_n \alpha_s^n + O(\Lambda_{QCD}/m_c) \right] . \quad (5)$$

In the $D_s \to \phi\pi$ decay, the emitted meson π is light, the hadronic matrix elements can be written as:

$$\langle \pi\phi \mid Q_i(\mu) \mid D_s \rangle = A_0^{D,\phi} \int_0^1 \mathrm{d}x T_i^{\mathrm{I}}(x) \Phi_\pi(x) + \int_0^1 \mathrm{d}\xi \mathrm{d}x \mathrm{d}y T_i^{\mathrm{II}}(\xi, x, y) \Phi_{D_s}(\xi) \Phi_\pi(x) \Phi_\phi(y) . \quad (6)$$

$A_0^{D,\phi}$ denotes the $D_s \to \phi$ transition form factor, $\Phi_D(\xi)$, $\Phi_\pi(x)$ and $\Phi_\phi(y)$ label lightcone distribution amplitudes (LCDAs) of D_s, π and ϕ meson, respectively. $T_i^{\mathrm{I,II}}$ denote hard-scattering kernels which are calculable in perturbative theory. Neglecting the $O(\Lambda_{QCD}/m_c)$ corrections, $T_i^{\mathrm{I,II}}$ are hard gluon exchange dominant. Other nonperturbative contributions are contained in the LCDAs of mesons or the form factor. The second term in Eq. (6) represents the hard spectator contribution.

We next proceed to calculate the nonfactorizable effects in the $D_s^+ \to \phi\pi^+$ with QCDF approach. Then in heavy quark limit, for simplicity, we will neglect the masses of light quarks and π. We consider the vertex corrections and hard spectator interactions depicted in Fig. 1. The technique is similiar to that of the $B \to \pi\pi/K$ mode, readers can refer to Ref. [11] for details. As in Ref. [11], we obtain the QCD coefficients a_i ($i = 1, 2$) at NLO in NDR scheme. Then the coefficients a_i are given as

$$a_1 = C_1 + \frac{C_2}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_2 F ,$$

$$a_2 = C + \frac{C_1}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_1 F .$$

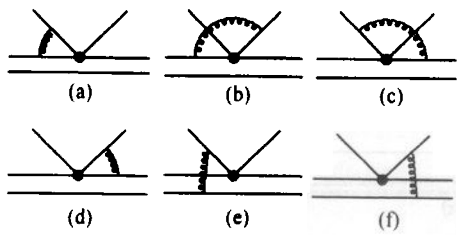

Fig. 1. Order of α_s corrections to the hard scattering kernels T_i^{I} and T_i^{II}. The two lines directed upwards represent the two quarks that make up π. These diagrams are called vertex corrections for Figs (a)—(d) and hard spectator diagrams for Figs. (e), (f), respectively.

Here $N = 3$ ($f = 4$) is the number of colors (flavors), and $C_F = \frac{N^2 - 1}{2N}$ is the factor of color. We define the symbols in the above expressions as the same as Beneke's, which are

$$F = -18 - 12 \ln \frac{\mu}{m_c} + f_{\mathrm{I}} + f_{\mathrm{II}} ,$$

$$f_{\mathrm{I}} = \int_0^1 \mathrm{d}x g(x) \Phi_\pi(x) , \quad (8)$$

with the hard-scattering function $g(x) = 3 \frac{1 - 2x}{1 - x} \ln x - 3i$. The hard spectator scattering contribution is given by

$$f_{\mathrm{II}} = \frac{4\pi^2}{N} \frac{f_\phi f_{D_s}}{A_0^{D,\phi}(0) m_{D_s}^2} \int_0^1 \mathrm{d}\xi \frac{\Phi_{D_s}(\xi)}{\xi} \int_0^1 \mathrm{d}x \frac{\Phi_\pi(x)}{x} \int_0^1 \mathrm{d}y \frac{\Phi_\phi(y)}{y} , \quad (9)$$

where $f_\phi(f_{D_s})$ is the $\phi(D_s)$ meson decay constant, $A_0^{D_s\phi}(0)$ the $D_s \to \phi$ transition form factor at zero momentum transfer, and ξ the light-cone momentum fraction of the spectator quark in the D_s meson, $f_{\rm II}$ depends on the wave function Φ_{D_s} through the integral $\int_0^1 d\xi \Phi_{D_s}(\xi)/\xi = m_{D_s}/\lambda_D$. The quantity λ_D parameterizes our ignorance about the D-meson wave-function, it is expected to be of order Λ_{QCD} (the value of the QCD scale Λ_{QCD} for flavour number $f = 4$ is $\Lambda_{\overline{MS}}^{(4)} = 335$MeV with the $\overline{\rm MS}$ scheme[13]). For the D-meson, most of its momentum is carried by the heavy quark c, the momentum of the spectator quark in the D_s meson is very small. So its wave function is very asymmetric. For convenience, we take $\lambda_D/m_{D_s} \approx \Lambda_{QCD}/m_{D_s} \approx 0.2$ in our calculation.

For the decay $D_s \to \pi\phi$, the annihilation diagrams are suppressed according to the Okubo-Zweig-Iizuka(OZI) rule because the ϕ meson is nearly a pure $s\bar{s}$ state. Moreover, according to the power counting in Ref. [11], the contributions from the weak annihilation are a factor of Λ_{QCD}/m_Q smaller than those from the lowest-order diagram, the vertex correction diagrams and so on. However, those terms that are power suppressed can not be calculated with QCDF approach. Though the authors of Ref. [11] simulated the calculations of hard scattering kernels to estimate the annihilation contributions which are from single gluon exchange, there are large uncertainties in their predictions. In our calculations, the annihilation contributions are of order α_s^3 because of triple gluon fusion to ϕ, so they can be neglected in the decay $D_s \to \phi\pi$.

From the expression (7) of the QCD coefficients a_i, with the renormalization group equation for Wilson coefficients $C_i(\mu)$ at leading order logarithm approximation[13]: $\frac{dC_i(\mu)}{d\ln\mu} = \frac{\alpha_s}{4\pi}\gamma_{ij}^T C_i(\mu)$, where γ is the anomalous dimension matrix, we find $\frac{da_i}{d\ln\mu} = 0(i = 1, 2)$ at the order of α_s. This makes the μ-dependence of the decay amplitude calculated with QCDF approach less sensitive than that calculated with naive factorization. This point can also be seen roughly from the data in Table 1 and Fig.2.

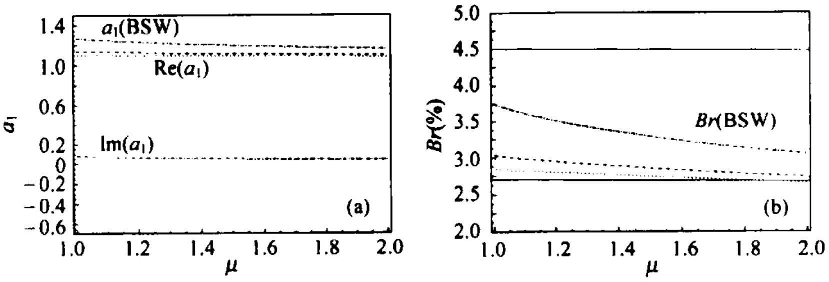

Fig.2. Dependence of a_1 and Br on the renormalization scale μ in BSW and QCDF. The dotted and dashed lines correspond to the values obtained with $\Phi_\pi(x) = 6x(1-x)$ and $\Phi_\pi(x) = \delta\left(x - \frac{1}{2}\right)$, respectively, in the QCDF approach. The dash-dotted line corresponds to those obtained with BSW approach.

In the D_s rest frame, the two body decay width is

$$\Gamma(D_s \to \phi\pi) = \frac{1}{8\pi}|\mathcal{A}(D_s \to \phi\pi)|^2 \frac{|p|}{m_{D_s}^2}, \tag{10}$$

where

Table 1. The values of a_1 and Br at μ = 1GeV, m_c and 2GeV calculated with QCDF and BSW approach ($N^{eff} = \infty$). For QCDF, we calculate the spectator contribution with two different wave functions of π. In the QCDF columns, the values in the parentheses are those with $\Phi_\pi(x) = 6x(1-x)$, the values in the brackets are those with $\Phi_\pi(x) = \delta\left(x - \frac{1}{2}\right)$.

μ	BSW	QCDF	BSW	QCDF	BSW	QCDF
1GeV	1.272	(1.103 + 0.084i)	−0.500	(−0.071 − 0.215i)	3.75	(2.84)
		[1.141 + 0.084i]		[−0.168 − 0.215i]		[3.03]
m_c	1.200	(1.092 + 0.049i)	−0.390	(−0.058 − 0.150i)	3.33	(2.77)
		[1.112 + 0.049i]		[−0.125 − 0.150i]		[2.88]
2GeV	1.153	(1.077 + 0.033i)	−0.314	(−0.032 − 0.119i)	3.08	(2.69)
		[1.091 + 0.033i]		[−0.086 − 0.119i]		[2.76]

$$|p| = \frac{\sqrt{[m_{D_s}^2 - (m_\phi + m_\pi)^2][m_{D_s}^2 - (m_\phi - m_\pi)^2]}}{2m_{D_s}}$$

is the magnitude of the momentum of ϕ meson. The corresponding branching ratio is given by

$$Br(D_s \to \phi\pi) = \frac{\Gamma(D_s \to \phi\pi)}{\Gamma_{total}}, \quad \Gamma_{total} = \frac{1}{\tau_{D_s}}. \tag{11}$$

In our numerical calculations, we will take the following values for the relevant input parameters[3]: $|V_{cs}| = |V_{ud}| = 0.975$, $f_\pi = 131$MeV, $f_\phi = 233$MeV, $m_c = 1.45$GeV, $f_{D_s} = 280$MeV. As for the form factor $A_0^{D_s\phi}(0)$, for lack of experimental data, we use the value taken from the Ref. [6] $A_0^{D_s\phi}(0) = 0.70$.

For distribution amplitude of π, two kinds of the wave functions are used, one is the asymptonic form[11] $\Phi_\pi(x) = 6x(1-x)$, the other is a delta-function $\Phi_\pi(x) = \delta\left(x - \frac{1}{2}\right)$. In Table.1 we list the values of a_1, a_2 and branching ratio(Br) at μ = 1GeV, m_c, and 2GeV with different wave functions of π. The numerical results which are calculated with BSW approach (where we take $N_c^{eff} = \infty$ because the experimental data of MARK Ⅲ for charm decays do not show color suppression[14]) are also listed for comparison.

It is necessary to note that the QCDF approach gives a_i (i = 1, 2) an imaginary part, which comes from the gluon exchange between the quarks u and d in π and the s quark in ϕ (see Fig.1 (c), (d)). From the numerical values summarized in Table 1, we find that the contributions of the vertex corrections and the hard-spectator diagrams in Fig.1 can reduce over 10 % of the values obtained with BSW approach. And the coefficients a_1, a_2 are less sensitive to the choice of the wave functions. In Fig.2, we depict the dependence of a_1 and Br on scale μ, we also show the results calculated by BSW approach for comparison. The horizontal solid lines in Fig.2(b) show the experimental branching ratio at 1σ level. It is clear that the scale dependence of the values calculated with QCDF approach are milder than that calculated with BSW approach. But the μ dependence still exists, the reason is that we calculate a_i only at one-loop level, the source of μ dependence is from the high order effects. When considering the contributions from the high order corrections in α_s or Λ_{QCD}/m_c, the μ dependence of our predictions will be further reduced.

In Fig.2, we also show the results which are calculated with different wave functions of π in the calculations of the hard-spectator contribution in QCDF. It shows again that a_1 and Br are less sensitive to the selection of the wave function of π. Moreover, we find that the results obtained with QCD factorization approach fall in the 1σ allowed region from the central experimental value 3.6×10^{-2}, regardless of the selection of the wave function of π-meson. Though the branching ratios with BSW approach are also within the 1σ region, this approach takes $N_c^{eff}=\infty$ in order to fit the experimental data, so it is more phenomenological in comparison with QCDF approach. From Fig. 2, we can see apparently that our predictions with QCDF approach are small compared with the values obtained with BSW approach.

3 Direct estimation of $Br(D_s\to\phi\pi)$

Now we estimate $Br(D_s\to\phi\pi)$ directly from the existed data of $D^0\to K^{*-}\pi^+$. Assuming spectator diagram dominance, $D_s^+\to\phi\pi^+$ can go through quark decay diagram depicted in Fig. 3(a) with the decay width in Eq. (10). Using the experimental data listed in Sec. 2, we get $|p|=0.720$.

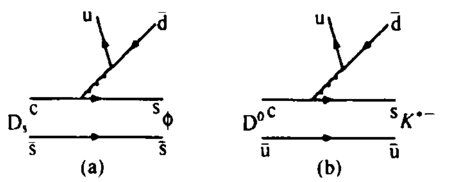

Fig. 3. Diagrams for the decays of $D_s\to\phi\pi$ and $D^0\to K^*\pi$.

Consider the decay $D^0\to K^{*-}\pi^+$ which proceeds dominantly through diagram Fig. 3(b). The contribution of its annihilation diagram is very small so that it can be neglected. Obviously, in Fig. 3, diagrams (a) and (b) are very similiar. If $\bar{s}$ in (a) is replaced by u, we will get (b). In addition, the particle decay width of $D^0\to K^*\pi$ is

$$\Gamma(D^0\to K^*\pi)=\frac{1}{8\pi}|\mathscr{A}(D^0\to K^*\pi)|^2\frac{|p'|}{m_{D^0}^2}, \tag{12}$$

where $|p'|=0.719$. The momentum of K^* in the D^0 rest frame is almost the same as that of ϕ in $D_s^+\to\phi\pi^+$. So the Lorentz contraction effects of the wave functions of ϕ and K^* are nearly the same. We know that the decay amplitudes $\mathscr{A}(D_s\to\phi\pi)$ and $\mathscr{A}(D^0\to K^*\pi)$ are proportional to the wave function overlap integrals of D_s^+-ϕ and D^0-K^*, respectively. Moreover, $|p|=0.720$ and $|p'|=0.719$ mean that these overlap integrals are almost the same under the condition of $SU(3)$ symmetry. The $SU(3)$ symmetry breaking effects in the cases of $D_s^+\to\phi\pi^+$ and $D^0\to K^{*-}\pi^+$ should be fairly small. As an approximation, we can take

$$|\mathscr{A}(D_s\to\phi\pi)|\approx|\mathscr{A}(D^0\to K^{*-}\pi^+)|. \tag{13}$$

Using the experimental data listed in Ref. [3]: $\tau(D^0)=(0.4126\pm0.0028)\times10^{-12}$s, $Br(D^0\to K^{*-}\pi^+)=(5.0\pm0.4)\%$, $\tau(D_s)=(0.496^{+0.010}_{-0.009})\times10^{-12}$s and Eqs. (10)—(13), we obtain $Br(D_s\to\phi\pi)\approx(5.40\pm0.45)\%$, where the error comes from the data of $\tau(D^0)$, $Br(D^0\to K^*\pi)$ and $\tau(D_s)$. It is a little outside the one σ allowed region from the central experimental value 3.6×10^{-2}. But in our estimation we did not include the $SU(3)$ breaking effect which is roughly a few percent. So considering this effect, the estimation would be compatible with the data.

4 Conclusions

We have analyzed the decay of $D_s\to\phi\pi$ with QCD factorization in the heavy quark limit. We calculate the nonfactorizable contributions, including vertex correction, and hard-spectator contribution. According to our calculation, the branching ratios with QCDF approach is not sensitive to the choice of the wave function of pion. Our predictions are in agreement with the present experimental data. The direct estimation of $Br(D_s \to \phi\pi)$ from $D^0 \to K^* \pi$ data gives a bit larger result comparing with the present data. But the measured data on $Br(D_s \to \phi\pi)$ are still rough, we need more data for drawing our final conclusion.

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$D_s \to \phi\pi$ 的理论分析*

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摘要　在重夸克极限下，用 QCD 因子化方法分析 $D_s \to \phi\pi$ 的衰变．讨论了硬旁观者散射等非因子化贡献，并给出了数值结果，理论预言的分支比与实验相符．最后，结合 $D^0 \to K^* \pi$ 的测量值，采用纯唯象的方法估计 $D_s \to \phi\pi$ 的分支比．

关键词　QCD 因子化　D_s　非因子化贡献

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