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Medium Effects and Thermal Instability in φ_6^3 Theory*

XIONG Chun HOU De-Fu

(Institute of Particle Physics, Central China Normal University, Wuhan 430079, China)

Abstract We calculate the effective mass and damping rate in φ_6^3 theory at finite temperature by evaluating the real and imaginary parts of the one-loop self energies at the hard thermal loop (HTL) approximation. We show that there is thermal instability above a critical temperature T_c. The effective mass and damping rate are proportional to $\sqrt{g}T$ and gT respectively. We compare our results with those in hot QCD.

Key words hard thermal loop, effective mass, damping rate

1 Introduction

The interaction of particles with a medium can be described by quantum field theory at finite temperature[1,2]. Using perturbation theory, self energies, scattering amplitudes, etc. can be calculated. In this way effective masses, decay rate and other interesting properties can be derived. However, perturbative calculations based on bare propagators and vertices turned out to be inconsistent, i.e. they lead to infrared and mass divergent results. This problem has been solved within the Hard Thermal Loop (HTL) approximation invented by Braaten and Pisarski[3]. In this approximation the momenta of the internal particles of the self energies are assumed to be hard, i.e. of order of the temperature T, while the external momenta are soft, i.e. of the order gT. Those momentum scales are distinguished only in the weak coupling limit $g \ll 1$, on which the HTL approximation is based. The self energies calculated in this way are gauge independent, which guarantee the effective mass and the decay rate gauge independent for gauge theories[3—5]. The HTL approximation has been widely used to study collective effects in various kind theories. In this paper, we are going to study medium effects in ϕ_6^3 theory which is interesting as a toy model of QCD[6]. Due to the interation with the heat bath, a particle at finite temperature obtains an effective mass and damping rate[8,9]. These medium effects can be studied through the self-energy at finite temperature. The one-loop self-energy in ϕ_6^3 theory includes two diagrams: sun-set diagram and the tadpole diagram. The sun-set diagram is the conventional self-energy just like the three-gluon one-loop self-energy in QCD which is supposed to contribute to thermal mass by the order of g^2T^2[7], while the second diagram (tadpole self-energy) is usually overlooked, since it just generates an ultraviolet divergence at zero temperature. We are going to show this tadpole diagram has an nontrivial contribution to the medium effects in ϕ_6^3 theory at finite temperature and leads to the thermal instability above a critical temperature T_c.

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In next section we will present the complete calculation of the one-loop self energies in ϕ_6^3 theory in the HTL approximation. In Section 3 we calculate the effective thermal mass and the decay rate. We also analyse the thermal instability and find out the critical temperature. Finally we give our conclusions in Section 4.

2 One-loop Self Energies at Finite Temperature

In ϕ_6^3 theory, the one loop self-energy contribution comes from two diagrams Fig. 1(a) and (b). We will calculate the self energies from these two diagrams respectively below.

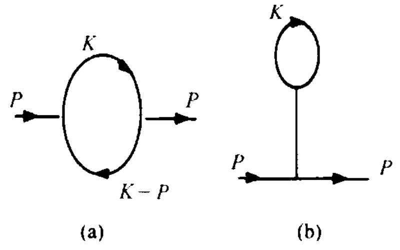

Fig. 1. One-loop self-energy: (a) sun-set diagram; (b) tadpole diagram.

2.1 Sun-set diagram

In the vacuum the self-energy contribution from the sun-set diagram Fig. 1(a) reads

$$\Pi_a(P) = -\mathrm{i}g^2\int\frac{\mathrm{d}^6K}{(2\pi)^6}D_B(K)D_B(Q), \tag{1}$$

where , $K \equiv (k^\Delta, \boldsymbol{K})$, $K = (\boldsymbol{K})$ D_B denotes the Boson propagator and $Q = K - P$.

In order to compute the self-energy at finite temperature we apply the imaginary time formalism. We will work under the hard thermal loop approximation: The temperature is much greater than the mass of ϕ field, $T \gg m$. The internal momentum K is hard, while the external momentum is soft. To perform the sum over the Matsubara frequencies k_0 it is convenient to use the Saclay representation of the bosonic propagators:

$$D_B(K) = -\int_0^\beta \mathrm{e}^{\mathrm{i}k_0\tau}D_B(\tau, k)\mathrm{d}\tau, \tag{2}$$

$$D_B(\tau, k) = \frac{1}{2E_k}[(1 + n_B(k))\mathrm{e}^{-k\tau} - n_B(k)\mathrm{e}^{k\tau}], \tag{3}$$

where $n_B(k) = \frac{1}{\mathrm{e}^{\beta E_k} - 1}$ denotes the Bose distribution function and $E_k^2 = k^2 + m^2$. By making use of the expression of δ-function

$$\delta(\tau) = \frac{1}{\beta}\sum_n \exp\left(\mathrm{i}\frac{2n\pi\tau}{\beta}\right)$$

and Eqs (2) and (3), we can get

$$\Pi_a(P) = g^2\int\frac{\mathrm{d}^5k}{(2\pi)^5}\frac{1}{2E_k2E_q}\left[\frac{1 + n_B(k) + n_B(q)}{\mathrm{i}p_0 - E_k - E_q} + \frac{n_B(k) - n_B(q)}{\mathrm{i}p_0 + E_k - E_q} + \frac{n_B(q) - n_B(k)}{\mathrm{i}p_0 - E_k + E_q} - \frac{1 + n_B(k) + n_B(q)}{\mathrm{i}p_0 + E_k + E_q}\right]. \tag{5}$$

After performing the analytic continuation $\mathrm{i}p_0 \to p_0 + \mathrm{i}\varepsilon$ we obtain

$$\Pi_a(P) = g^2\int\frac{\mathrm{d}^5\mathrm{k}}{(2\pi)^5}\frac{1}{2E_k2E_q}\Big[(1 + n_B(k) + n_B(q)) \times \left(\frac{1}{p_0 - E_k - E_q + \mathrm{i}\varepsilon} - \frac{1}{p_0 + E_k + E_q + \mathrm{i}\varepsilon}\right) + (n_B(k) - n_B(q))\left(\frac{1}{p_0 + E_k - E_q + \mathrm{i}\varepsilon} - \frac{1}{p_0 - E_k + E_q + \mathrm{i}\varepsilon}\right)\Big]. \tag{6}$$

Where the "1" term which is independent of distribution functions corresponds to the vacuum contribution and contains an ultaviolet divergence. This divergence can be renormalized through appropriate counter-terms as the usual zero temperature field theory. Here we only interested in the temperature dependent part:

$$\Pi_a^{\beta}(P) = g^2\int \frac{d^5k}{(2\pi)^5}\frac{1}{4E_kE_q}\Big[(n_B(k)+n_B(q))\Big(\frac{1}{p_0-E_k-E_q+i\varepsilon}-\frac{1}{p_0+E_k+E_q+i\varepsilon}\Big)+ (n_B(k)-n_B(q))\Big(\frac{1}{p_0+E_k-E_q+i\varepsilon}-\frac{1}{p_0-E_k+E_q+i\varepsilon}\Big)\Big] \tag{7}$$

which contains only linear terms of distribution functions. In the HTL approximation, p_0, $\boldsymbol{P} \ll T$, we have the following relations:

$$E_k \quad E_q \approx 2E_k,$$
$$E_k - E_q \approx E_p\cos\theta,$$
$$n_B(k) + n_B(q) \approx 2n_B(k),$$
$$n_B(k) - n_B(q) \approx -\frac{1}{T}n_B(k)[1+n_B(k)]E_p\cos\theta, \tag{8}$$

where θ is the angle between $\boldsymbol{p}$ and $\boldsymbol{k}$. Using these approximate expressions, we obtain the real part of the self-energy:

$$\mathrm{Re}\Pi_a(p_0,\boldsymbol{p}) \approx -\frac{g^2T^2}{144\pi}[1-2Q_1(p_0/p)], \tag{9}$$

where Q_1 is the second kind of Legendre function and often appear in Hard Thermal Loops. The derivation of Eq.(9) can be found in the appendix.

2.2 The tadpole diagram

The second diagram (tadpole diagram) has been less discussed, because it just generates an ultraviolet divergence which can be regulariged through an appropriate counter-term. At finite temperature, though, we will find there are physical temperature dependent effects. In the vacuum it reads

$$\Pi_b(P) = -ig^2\int\frac{d^6K}{(2\pi)^6}D_B(K)D_B(Q), \tag{10}$$

where $D_B(K) = \frac{1}{K^2}$ with $k_0 = i2n\pi T$ and $D_B(Q) = -\frac{1}{m^2}$ with $Q = 0$. Using the Saclay representation of the bosonic propagator as before and after the Matsubara frequency summation we get

$$\Pi_b(P) = \frac{g^2}{m^2}\int\frac{d^5k}{(2\pi)^5}\frac{1}{2E_k}[1+2n_B(k)]. \tag{11}$$

Here again the n-independent term corresponds to the vacuum contribution and contains an ultravoilet divergence, which can be renormalized by appropriate counter-terms. The temperature dependent contribution of this tadpole diagram turns out to be (in HTL approximation)

$$\Pi_b(P) = \frac{g^2T^4\pi}{180m^2}. \tag{12}$$

The result means that this tadpole diagram does not contain an imaginary part. The derivation of Eq.(12) can be found in the appendix, too.

3 Medium Effects

We have got the final results of the self energies for ϕ_6^3 theory in the HTL approximation, from which the effective mass and the decay rate etc. can be calculated. The effective mass is related to the real part of self-energies. When external momenta $p_0 = 0$, $\boldsymbol{P}\rightarrow 0$, the real part is just opposite to the thermal mass square

$$m_a^2 = -\operatorname{Re}\Pi_a(p_0, \boldsymbol{p} \to 0) = \frac{g^2 T^2}{144\pi}, \tag{13}$$

$$m_b^2 = -\operatorname{Re}\Pi_b(p_0, \boldsymbol{p} \to 0) = -\frac{g^2 T^4 \pi}{180 m^2}. \tag{14}$$

The total thermal mass square is $m_{total}^2 = m^2 + m_a^2 + m_b^2$ and thus,

$$m_{total} = (m_a^2 + m^2)^{1/2} \sqrt{1 - \frac{g^2 T^4 \pi}{180 m^2 (m^2 + m_a^2)}}. \tag{15}$$

The above formula indicates that the effective mass is well-defined if only the following condition is satisfied

$$T \leqslant T_c = \sqrt[4]{\frac{180 m^2 (m^2 + m_a^2)}{\pi g^2}}. \tag{16}$$

If the temperature is higher than T_c, there will be an imaginary thermal effective mass, which will lead to thermal instability. This agrees qualitatively with Ref. [10], where the analysis was based on the effective potential. The critical condition Eq. (16) indicates that for temperature of order $T_c \approx m/\sqrt{g}$, m is of order $\sqrt{g}T$. Therefore the total effective mass is of order $\sqrt{g}T$, while the effective gluon mass in hot QCD is of order gT.

The decay rate, however, is related to the imaginary part of the self energy. The concretly analytic expression is

$$\gamma = -\frac{\operatorname{Im}\Pi_R}{2p_0}. \tag{17}$$

Now let's calculate the imaginary part of the self-energe. Since the tadpole diagram has no any imaginary part, we only need to look at Fig. 1 (a). By making use of

$$\frac{1}{x + i\epsilon} = Pr\frac{1}{x} - i\pi\delta(x) \tag{18}$$

the imaginary part of the retarded self-energy from Fig. 1(a) consists of two parts C and D,

$$\operatorname{Im}\Pi_R(P) = C + D, \tag{19}$$

where

$$C = -\pi g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{4k^2} 2n_B(k)[\delta(p_0 - E_k - E_q) - \delta(p_0 + E_k + E_q)], \tag{20}$$

$$\begin{aligned} D = & -\pi g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{4k^2}[n_B(k) - n_B(q)][\delta(p_0 + E_k - E_q) - \delta(p_0 - E_k + E_q)] = \\ & -\pi g^2 \int_0^\infty dk \frac{k^4}{4k^2 T} n_B(k)[1 + n_B(k)] \int \frac{d\Omega_5}{(2\pi)^5} E_p \cos\theta \times \\ & [\delta(p_0 + E_p\cos\theta) - \delta(p_0 - E_p\cos\theta)]. \end{aligned} \tag{21}$$

In HTL approximation, $E_k \gg p_0$, there is no contribution from C due to kinematical constraints. After using the HTL approximation and evaluating the integrals in D, we obtain the imaginary part of the self-energy

$$\operatorname{Im}\Pi_R \approx -\frac{g^2 T^2 p_0}{144 E_p}, \tag{22}$$

thus the decay rate reads

$$\gamma = \frac{g^2 T^2}{288 E_p}. \tag{23}$$

Since E_p is of order $\sqrt{g}T$, the damping rate turns out to be order of gT which is larger than the gluon damping rate of order $g^2 T$[4]

4 Conclusion

We have studied the medium effects and thermal instability in ϕ_6^3 theory. By evaluating the one-loop self-energy in HTL approximation including both sun-set and tadpole diagrams, we calculate the effective mass as well as the damping rate of the particle at finite temperature. We show that the tadpole diagram dominates over the sun-set diagram for the effective mass contribution, whereas only the latter contributes to the damping rate. Further more, we show that there is a critical temperate T_c above which the thermal instability arises in ϕ_6^3 theory at finite temperature. The damping rate is proportional to $g^2 T^2 / E_p$. For temperature of order $T_c \sim m/\sqrt{g}$ the effective mass is of order $\sqrt{g}T$ and the damping rate turns out to be of order gT in ϕ_6^3 theory. While in hot QCD, the effective gluon mass is of order gT and its damping rate is of order $g^2 T$.

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Appendix (A)

We will give here the explicit derivations of Eqs. (9), (12).

The real part of diagram (a)

We start our calculation from Eq. (7). The unintegreted real part can be divided into two parts as below

$$\mathrm{Re}\Pi_a(P) = A + B, \tag{A1}$$

$$A = g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{4E_k E_q} [n_B(k) + n_B(q)] \left(\frac{1}{p_0 - E_k - E_q} - \frac{1}{p_0 + E_k + E_q} \right), \tag{A2}$$

$$B = g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{4E_k E_q} [n_B(k) - n_B(q)] \left(\frac{1}{p_0 + E_k - E_q} - \frac{1}{p_0 - E_k + E_q} \right). \tag{A3}$$

approximation, i.e. assuming $p_0, P \ll T, q \approx k$, Eqs. (25), (26) now become the following:

$$A = g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{4k^2} \frac{2}{e^{\beta k} - 1} \left(-\frac{1}{k} \right) = -g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{2k^3 (e^{\beta k} - 1)} =$$
$$-g^2 \int \frac{d\Omega_5}{(2\pi)^5} \int_0^\infty dk \frac{k}{2[e^{\beta k} - 1]} = -g^2 T^2 / 144,$$

$$B = -g^2 \int \frac{d^5 k}{(2\pi)^5} \frac{1}{4k^2 T} n_B(k)[1 + n_B(k)] E_p \cos\theta \left[\frac{1}{p_0 + E_p \cos\theta} - \frac{1}{p_0 - E_p \cos\theta} \right] =$$
$$g^2 \int_0^\infty dk \frac{k^4 n_B(k)[1 + n_B(k)]}{4k^2 T} \int \frac{d\Omega_5}{(2\pi)^5} \left(\frac{-E_p \cos\theta}{p_0 + E_p \cos\theta} + \frac{E_p \cos\theta}{p_0 - E_p \cos\theta} \right) =$$
$$\frac{\pi^2 g^2 T^2}{12(2\pi)^5} \frac{4\pi^2}{3} \int_{-1}^1 d(\cos\theta) \left(\frac{-E_p \cos\theta}{p_0 + E_p \cos\theta} + \frac{E_p \cos\theta}{p_0 - E_p \cos\theta} \right).$$

Now we define $\cos\theta = x$, then we obtain

$$B = \frac{g^2 T^2}{288\pi}\int_{-1}^{1} \mathrm{d}x \, \frac{2E_p x}{p_0 - E_p x} = \frac{g^2 T^2}{72} Q_1(p_0/E_p).$$

So the total real part from diagram (a) is

$$\mathrm{Re}\Pi_a(P) = A + B = \frac{g^2 T^2}{144\pi}[2Q_1(p_0/E_p) - 1].$$

The self energy of diagram (b)

We start from Eq. (12) and neglect the vacuum part which does not contain any distribution function

$$\Pi_b(P) = \frac{g^2}{m^2}\int \frac{\mathrm{d}^5 k}{(2\pi)^5} \frac{2}{2E_k(\mathrm{e}^{\beta k} - 1)} \approx \frac{g^2}{m^2}\int \frac{\mathrm{d}\Omega_5}{(2\pi)^5}\int_0^\infty \mathrm{d}k \, \frac{k^3}{\mathrm{e}^{\beta k} - 1} = \frac{g^2 T^4 \pi}{180 m^2}.$$

φ_6^3 的媒质效应和热不稳定性*

熊春 侯德富

(华中师范大学粒子物理研究所 武汉 430079)

摘要 在有限温度下运用硬热圈近似求得 φ_6^3 理论单圈自能,进而求得有限质量和衰变率. 发现在临界温度 T_c 以上存在热不稳定性,而有效温度和衰变率则分别正比于$\sqrt{g}T$ 和 gT. 最后将所得结果与热 QCD 比较.

关键词 硬热圈 有效质量 衰变率

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