

Hopeless to Kinematically Detect the Effective Masses of Muon and Tau Neutrinos*

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Abstract We show that the recent WMAP data can impose a generous upper bound on the effective masses of electron, muon and tau neutrinos defined in the kinematic measurements: $\langle m \rangle_e^2 + \langle m \rangle_\mu^2 + \langle m \rangle_\tau^2 = m_1^2 + m_2^2 + m_3^2 < 0.5\text{eV}^2$, or $\langle m \rangle_\alpha < 0.71\text{eV}$ (for $\alpha = e, \mu, \tau$). When current neutrino oscillation data are taken into account, we obtain $\langle m \rangle_e < 0.24\text{eV}$ and $\langle m \rangle_\mu \approx \langle m \rangle_\tau < 0.24\text{eV}$. Thus there is no hope to kinematically detect $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ in any realistic experiments.

Key words effective neutrino masses, neutrino oscillations

Thanks to the recent Super-Kamiokande^[1], K2K^[2], SNO^[3] and KamLAND^[4] experiments, we are now convinced that the deficit of atmospheric ν_μ neutrinos and the deficit of solar ν_e neutrinos are both due to neutrino oscillations. The oscillation of neutrinos is a quantum phenomenon which can naturally happen if neutrinos are massive and lepton flavors are mixed. Current experimental data indicate that solar and atmospheric neutrino oscillations are dominated respectively by $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ transitions. The neutrino mass-squared differences associated with solar and atmospheric neutrino oscillations are thus defined as

$$\begin{aligned}\Delta m_{\text{sun}}^2 &\equiv |m_2^2 - m_1^2|, \\ \Delta m_{\text{atm}}^2 &\equiv |m_3^2 - m_2^2|,\end{aligned}\quad (1)$$

where m_i (for $i = 1, 2, 3$) denote the physical masses of three neutrinos. Although a strong hierarchy between Δm_{sun}^2 and Δm_{atm}^2 has been observed, the absolute values of m_1 , m_2 and m_3 remain unknown. The kinematic limits on the effective masses of electron, muon and tau neutrinos can be obtained from the tritium β -decay ${}^3_1\text{H} \rightarrow {}^3_2\text{He} + e^- + \nu_e$, the $\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay and the $\tau \rightarrow 5\pi + \nu_\tau$ (or $\tau \rightarrow 3\pi + \nu_\tau$) decay, respectively. Today's results are^[5]

$$\langle m \rangle_e < 3 \text{ eV},$$

$$\begin{aligned}\langle m \rangle_\mu &< 0.19 \text{ MeV}, \\ \langle m \rangle_\tau &< 18.2 \text{ MeV}.\end{aligned}\quad (2)$$

One can see that the experimental sensitivity for $\langle m \rangle_\mu$ is more than four orders of magnitude smaller than that for $\langle m \rangle_e$, and the experimental sensitivity for $\langle m \rangle_\tau$ is two orders of magnitude lower than that for $\langle m \rangle_\mu$. Is there any hope to detect $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ or to constrain them to a meaningful level of sensitivity? The answer to this question relies on how small $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ are.

The main purpose of this paper is to calculate $\langle m \rangle_e$, $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ with the help of the recent WMAP data^[6] and neutrino oscillation data. While $\langle m \rangle_e$ has been extensively studied in the literature^[7], a detailed analysis of $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ has not been done. It is therefore worthwhile to work out the upper bounds on $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ from currently available data, such that one may definitely answer the afore-raised question. Independent of the neutrino oscillation data, a generous upper limit $\langle m \rangle_e^2 + \langle m \rangle_\mu^2 + \langle m \rangle_\tau^2 = m_1^2 + m_2^2 + m_3^2 < 0.5 \text{ eV}^2$ can be achieved from the WMAP observation. Hence $\langle m \rangle_\alpha < 0.71 \text{ eV}$ holds for $\alpha = e, \mu$ or τ . Such an upper bound for $\langle m \rangle_\alpha$ will be reduced by a factor of three, if current neutrino oscillation data are taken into account. In

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particular, $\langle m \rangle_\mu \approx \langle m \rangle_\tau$ is a good approximation and both of them are insensitive to the Dirac CP-violating phase of the lepton flavor mixing matrix. We conclude that there is no hope to kinematically detect the effective masses of muon and tau neutrinos.

Direct neutrino mass measurements are based on the analysis of the kinematics of charged particles produced together with neutrino flavor eigenstates $|\nu_\alpha\rangle$ (for $\alpha = e, \mu, \tau$), which are superpositions of neutrino mass eigenstates $|\nu_i\rangle$ (for $i = 1, 2, 3$):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (3)$$

The unitary matrix V is just the lepton flavor mixing matrix. The effective masses of electron, muon and tau neutrinos in the kinematic measurements can then be defined,

$$\langle m \rangle_\alpha^2 \equiv |V_{\alpha1}|^2 m_1^2 + |V_{\alpha2}|^2 m_2^2 + |V_{\alpha3}|^2 m_3^2, \quad (4)$$

for $\alpha = e, \mu$ and τ . The unitarity of V leads straightforwardly to a simple sum rule between $\langle m \rangle_\alpha^2$ and m_i^2 :

$$\langle m \rangle_e^2 + \langle m \rangle_\mu^2 + \langle m \rangle_\tau^2 = m_1^2 + m_2^2 + m_3^2. \quad (5)$$

Note that this sum rule allows us to derive an upper bound on $\langle m \rangle_\alpha^2$ independent of any neutrino oscillation data.

This point can clearly be seen from the inequality

$$m_1^2 + m_2^2 + m_3^2 < (m_1 + m_2 + m_3)^2, \quad (6)$$

where the sum of three neutrino masses has well be constrained by the recent WMAP data^[6]: $m_1 + m_2 + m_3 < 0.71$ eV at the 95 % confidence level. Therefore,

$$\langle m \rangle_e^2 + \langle m \rangle_\mu^2 + \langle m \rangle_\tau^2 < 0.50 \text{ eV}^2. \quad (7)$$

This generous upper bound implies that $\langle m \rangle_\alpha^2 < 0.50 \text{ eV}^2$ or $\langle m \rangle_\alpha < 0.71$ eV holds for $\alpha = e, \mu$ and τ . Two comments are then in order.

1. The cosmological upper bound of $\langle m \rangle_\mu$ is more than five orders of magnitude smaller than its kinematic upper bound given in Eq. (2). In comparison, the upper limit of $\langle m \rangle_\tau$ set by the WMAP data is more than seven orders of magnitude smaller than its kinematic upper limit. It seems hopeless to improve the sensitivity of the kinematic measurements to the level of 0.71 eV.

2. The cosmological upper bound of $\langle m \rangle_e$ is about four times smaller than its kinematic upper bound given in Eq. (2). The former may be accessible in the future

KATRIN experiment^[8], whose sensitivity is expected to be about 0.3 eV.

If current data on neutrino oscillations are taken into account, however, more stringent upper limits can be obtained for the effective neutrino masses $\langle m \rangle_e$, $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$.

To see how $\langle m \rangle_\alpha$ may be related to the parameters of neutrino oscillations, we make use of Eq. (1) to express m_1 and m_2 in terms of m_3 , Δm_{sun}^2 and Δm_{atm}^2 . The results are

$$\begin{aligned} m_1 &= \sqrt{m_3^2 + p\Delta m_{\text{atm}}^2 + q\Delta m_{\text{sun}}^2}, \\ m_2 &= \sqrt{m_3^2 + p\Delta m_{\text{atm}}^2}, \end{aligned} \quad (8)$$

where $p = \pm 1$ and $q = \pm 1$ stand for four possible patterns of the neutrino mass spectrum. For example, $p = q = +1$ corresponds to $m_1 > m_2 > m_3$, and so on. The present solar neutrino oscillation data favor $q = -1$ or $m_1 < m_2$ ^[9], but the sign of p has not been fixed. Substituting Eq. (8) into Eq. (4), we obtain

$$\langle m \rangle_\alpha = \sqrt{m_3^2 + p(1 - |V_{\alpha3}|^2)\Delta m_{\text{atm}}^2 + q|V_{\alpha1}|^2\Delta m_{\text{sun}}^2}. \quad (9)$$

Taking account of $|V_{e3}|^2 \ll 1$ ^[10] and $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2$, we arrive at the following approximation for $\langle m \rangle_e$:

$$\langle m \rangle_e \approx \sqrt{m_3^2 + p\Delta m_{\text{atm}}^2}. \quad (10)$$

Taking account of the observed (nearly) maximal mixing factor of atmospheric neutrino oscillations^[1], which is equivalent to $|V_{\mu3}| \approx |V_{\tau3}| \approx 1/\sqrt{2}$ for $|V_{e3}| \ll 1$, we obtain

$$\begin{aligned} \langle m \rangle_\mu &\approx \sqrt{m_3^2 + \frac{p}{2}\Delta m_{\text{atm}}^2}, \\ \langle m \rangle_\tau &\approx \sqrt{m_3^2 + \frac{p}{2}\Delta m_{\text{atm}}^2} \end{aligned} \quad (11)$$

One can see that $\langle m \rangle_\mu \approx \langle m \rangle_\tau$ is a natural consequence of current neutrino oscillation data. In addition, Eqs. (10) and (11) tell us that $\langle m \rangle_e$ is slightly larger than $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ for $p = +1$ or $m_2 > m_3$; and it is slightly smaller than $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ for $p = -1$ or $m_2 < m_3$.

In general, one may analyze the correlation between $\langle m \rangle_\alpha$ and m_3 by use of Eq. (9), only if $|V_{e1}|$ and $|V_{e3}|$ are already measured. Since the mixing angles of solar, atmospheric and CHOOZ (or Palo Verde) re-

actor^[10] neutrino oscillations read as

$$\begin{aligned} \sin^2 2\theta_{\text{sun}} &= 4 |V_{e1}|^2 |V_{e2}|^2, \\ \sin^2 2\theta_{\text{atm}} &= 4 |V_{\mu3}|^2 (1 - |V_{\mu3}|^2), \\ \sin^2 2\theta_{\text{chr}} &= 4 |V_{e3}|^2 (1 - |V_{e3}|^2), \end{aligned} \quad (12)$$

we reversely obtain^[11]

$$\begin{aligned} |V_{e1}| &= \frac{1}{\sqrt{2}} \sqrt{\cos^2 \theta_{\text{chr}} + \sqrt{\cos^4 \theta_{\text{chr}} - \sin^2 2\theta_{\text{sun}}}}, \\ |V_{e2}| &= \frac{1}{\sqrt{2}} \sqrt{\cos^2 \theta_{\text{chr}} - \sqrt{\cos^4 \theta_{\text{chr}} - \sin^2 2\theta_{\text{sun}}}}, \\ |V_{e3}| &= \sin \theta_{\text{chr}}, \\ |V_{\mu3}| &= \sin \theta_{\text{atm}}, \\ |V_{\tau3}| &= \sqrt{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}}}. \end{aligned} \quad (13)$$

The other four matrix elements of V (i. e., $|V_{\mu1}|$, $|V_{\mu2}|$, $|V_{\tau1}|$ and $|V_{\tau2}|$) are unable to be determined from the afore-mentioned neutrino oscillation experiments. They can be derived from Eq. (13), however, if the Dirac CP-violating phase in the standard parametrization of V ^[12] is taken into account. The explicit expressions of $|V_{\mu1}|$, $|V_{\mu2}|$, $|V_{\tau1}|$ and $|V_{\tau2}|$ are given in Appendix. We see that $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ depend on the Dirac phase δ in the chosen parametrization of V . However, the sensitivity of $\langle m \rangle_\mu$ or $\langle m \rangle_\tau$ to δ is negligibly weak. The reason is simply that the contribution of $\cos \delta$ to $|V_{\mu1}|$ (or $|V_{\tau1}|$) is suppressed by $|V_{e3}|$ or $\sin \theta_{\text{chr}}$, and the contribution of $|V_{\mu1}|$ (or $|V_{\tau1}|$) to $\langle m \rangle_\mu$ (or $\langle m \rangle_\tau$) is further suppressed by Δm_{sun}^2 . It is therefore hopeless to probe the CP-violating phase δ , even if $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ could be measured.

To numerically determine the upper bound of $\langle m \rangle_\alpha$, we need first of all work out the upper limit of m_3 set by the WMAP and neutrino oscillation data. In Fig. 1, we show the dependence of $m_1 + m_2 + m_3$ on m_3 , where the best-fit values $\Delta m_{\text{sun}}^2 = 7.3 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ^[9] have typically been input. Note that only the $m_1 < m_2$ case, which is supported by current solar neutrino oscillation data, is taken into account. For the $m_2 < m_3$ case, m_3 has an lower bound $m_3 \geq \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sun}}^2} \approx 0.051 \text{ eV}$; but for the $m_2 > m_3$ case, even $m_3 = 0$ is allowed (inverted hierarchy). We see that these two cases become indistinguishable for $m_3 \geq 0.2 \text{ eV}$, implying the near degeneracy of three neutrino

masses. Once the WMAP limit $m_1 + m_2 + m_3 < 0.71 \text{ eV}$ is included, we immediately get $m_3 < 0.24 \text{ eV}$. Then we have $m_i < 0.24 \text{ eV}$ for $i = 1, 2$ or 3 . This result is apparently consistent with those obtained before^[13].

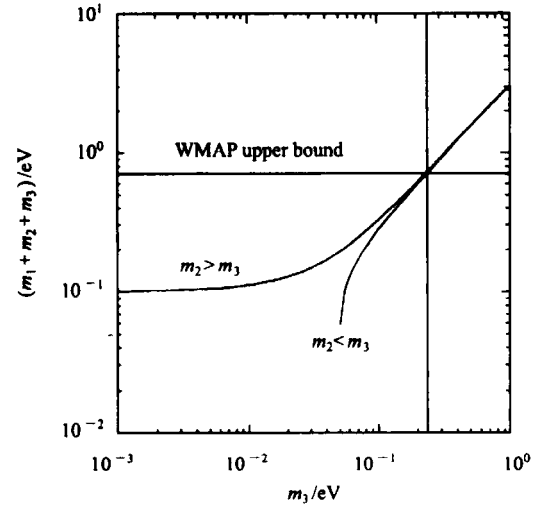


Fig. 1. Illustrative dependence of $m_1 + m_2 + m_3$ on m_3 . The WMAP result sets an upper limit on m_3 ; i. e., $m_3 < 0.24 \text{ eV}$ for both $m_3 > m_2$ and $m_3 < m_2$ cases.

Next we evaluate $\langle m \rangle_\alpha$ by using the best-fit values $\theta_{\text{sun}} \approx 33^\circ$ and $\theta_{\text{atm}} \approx 45^\circ$ ^[9] in addition to taking $\theta_{\text{chr}} \approx 5^\circ$ as a typical input, which is compatible with $\sin^2 2\theta_{\text{chr}} < 0.1$ extracted from the CHOOZ reactor neutrino experiment^[10] (note that $\theta_{\text{chr}} \sim 5^\circ$ is also favored in a number of phenomenological models of lepton mass

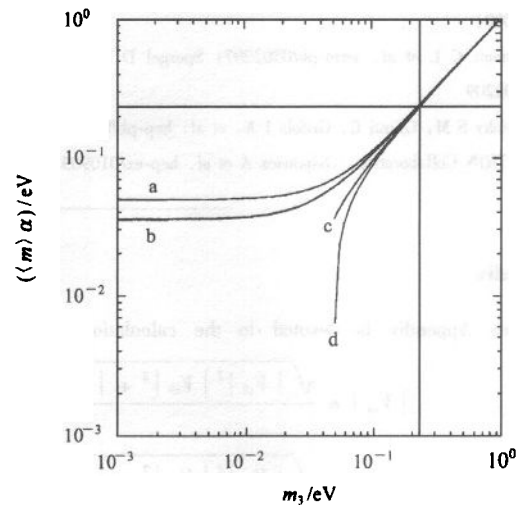


Fig. 2. Illustrative dependence of $\langle m \rangle_\alpha$ (for $\alpha = e, \mu, \tau$) on m_3 .

Curve a: $\langle m \rangle_e$ with $m_3 < m_2$; Curve b: $\langle m \rangle_\mu \approx \langle m \rangle_\tau$ with $m_3 < m_2$; Curve c: $\langle m \rangle_\mu \approx \langle m \rangle_\tau$ with $m_3 > m_2$; and Curve d: $\langle m \rangle_e$ with $m_3 > m_2$. The WMAP result leads to $\langle m \rangle_\alpha < 0.24 \text{ eV}$.

matrices^[14]). Our numerical results for $\langle m \rangle_e$, $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ changing with m_3 are shown in Fig.2. Once again, it is impossible to distinguish between the $m_2 < m_3$ case and the $m_2 > m_3$ case for $m_3 \geq 0.2\text{eV}$. We find that the dependence of $\langle m \rangle_\alpha$ on m_3 is very similar to the dependence of $m_1 + m_2 + m_3$ on m_3 . As expected in Eqs. (10) and (11), $\langle m \rangle_e > \langle m \rangle_\mu \approx \langle m \rangle_\tau$ holds for $m_3 < m_2$ (curves a and b in Fig.2); and $\langle m \rangle_e < \langle m \rangle_\mu \approx \langle m \rangle_\tau$ holds for $m_3 > m_2$ (curves c and d in Fig.2). In view of the upper limit $m_3 \leq 0.24\text{eV}$ obtained above, we arrive at $\langle m \rangle_\alpha \leq 0.24\text{eV}$ for $\alpha = e, \mu$ or τ . This upper bound is suppressed by a factor of three, compared to the upper bound obtained from Eq. (7) which is independent of the neutrino oscillation data.

The result $\langle m \rangle_\mu \approx \langle m \rangle_\tau < 0.24\text{eV}$ implies that there is no hope to kinematically detect the effective masses of muon and tau neutrinos. As the WMAP upper bound is in general valid for a sum of the masses of all possible relativistic neutrinos (no matter whether they are active or sterile), it seems unlikely to loosen the upper limit of $\langle m \rangle_\alpha$ obtained above in the assumption of only active neutrinos. Therefore, the kinematic measurements of $\langle m \rangle_\mu$ and $\langle m \rangle_\tau$ have little chance to reveal the existence of any exotic neutral particles with masses much larger than the light neutrino masses.

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Appendix

This Appendix is devoted to the calculation of $|V_{\mu i}|$,

$|V_{\mu 2}|$, $|V_{\tau 1}|$ and $|V_{\tau 2}|$, which are entirely unrestricted by current neutrino oscillation data. Taking account of the Dirac CP-violating phase in the standard parametrization of $V^{[12]}$, we find

$$\begin{aligned}
 |V_{\mu 1}| &= \frac{\sqrt{|V_{e2}|^2 |V_{\tau 3}|^2 + |V_{e1}|^2 |V_{\tau 3}|^2 |V_{\mu 3}|^2 + 2|V_{e1}| |V_{e2}| |V_{\tau 3}| |V_{\mu 3}| |V_{\tau 3}| \cos \delta}}{1 - |V_{e3}|^2}, \\
 |V_{\mu 2}| &= \frac{\sqrt{|V_{e1}|^2 |V_{\tau 3}|^2 + |V_{e2}|^2 |V_{\tau 3}|^2 |V_{\mu 3}|^2 - 2|V_{e1}| |V_{e2}| |V_{\tau 3}| |V_{\mu 3}| |V_{\tau 3}| \cos \delta}}{1 - |V_{e3}|^2}, \\
 |V_{\tau 1}| &= \frac{\sqrt{|V_{e2}|^2 |V_{\mu 3}|^2 + |V_{e1}|^2 |V_{\tau 3}|^2 |V_{\mu 3}|^2 - 2|V_{e1}| |V_{e2}| |V_{\tau 3}| |V_{\mu 3}| |V_{\tau 3}| \cos \delta}}{1 - |V_{e3}|^2}, \\
 |V_{\tau 2}| &= \frac{\sqrt{|V_{e1}|^2 |V_{\mu 3}|^2 + |V_{e2}|^2 |V_{\tau 3}|^2 |V_{\mu 3}|^2 + 2|V_{e1}| |V_{e2}| |V_{\tau 3}| |V_{\mu 3}| |V_{\tau 3}| \cos \delta}}{1 - |V_{e3}|^2}.
 \end{aligned} \tag{A1}$$

The explicit expressions of $|V_{\mu 1}|$, $|V_{\mu 2}|$, $|V_{\tau 1}|$ and $|V_{\tau 2}|$ in terms of θ_{sun} , θ_{atm} , θ_{chr} and δ can then be obtained from Eq. (13) and Eq. (A1):

$$\begin{aligned}
 |V_{\mu 1}| &= \left[\frac{\cos^2 \theta_{\text{atm}}}{2} - \frac{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}} (1 + \sin^2 \theta_{\text{chr}})}{2 \cos^4 \theta_{\text{chr}}} \sqrt{\cos^4 \theta_{\text{chr}} - \sin^2 2\theta_{\text{sun}}} + \right. \\
 &\quad \left. \frac{\sin 2\theta_{\text{sun}} \sin \theta_{\text{atm}} \sin \theta_{\text{chr}} \cos \delta}{\cos^4 \theta_{\text{chr}}} \sqrt{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}}} \right]^{1/2}, \\
 |V_{\mu 2}| &= \left[\frac{\cos^2 \theta_{\text{atm}}}{2} + \frac{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}} (1 + \sin^2 \theta_{\text{chr}})}{2 \cos^4 \theta_{\text{chr}}} \sqrt{\cos^4 \theta_{\text{chr}} - \sin^2 2\theta_{\text{sun}}} - \right. \\
 &\quad \left. \frac{\sin 2\theta_{\text{sun}} \sin \theta_{\text{atm}} \sin \theta_{\text{chr}} \cos \delta}{\cos^4 \theta_{\text{chr}}} \sqrt{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}}} \right]^{1/2} \\
 |V_{\tau 1}| &= \left[\frac{\sin^2 \theta_{\text{atm}} + \sin^2 \theta_{\text{chr}}}{2} - \frac{4 \sin^2 \theta_{\text{atm}} (1 + \sin^2 \theta_{\text{chr}}) - \sin^2 2\theta_{\text{chr}}}{8 \cos^4 \theta_{\text{chr}}} \sqrt{\cos^4 \theta_{\text{chr}} - \sin^2 2\theta_{\text{sun}}} - \right. \\
 &\quad \left. \frac{\sin 2\theta_{\text{sun}} \sin \theta_{\text{atm}} \sin \theta_{\text{chr}} \cos \delta}{\cos^4 \theta_{\text{chr}}} \sqrt{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}}} \right]^{1/2}, \\
 |V_{\tau 2}| &= \left[\frac{\sin^2 \theta_{\text{atm}} + \sin^2 \theta_{\text{chr}}}{2} + \frac{4 \sin^2 \theta_{\text{atm}} (1 + \sin^2 \theta_{\text{chr}}) - \sin^2 2\theta_{\text{chr}}}{8 \cos^4 \theta_{\text{chr}}} \sqrt{\cos^4 \theta_{\text{chr}} - \sin^2 2\theta_{\text{sun}}} + \right. \\
 &\quad \left. \frac{\sin 2\theta_{\text{sun}} \sin \theta_{\text{atm}} \sin \theta_{\text{chr}} \cos \delta}{\cos^4 \theta_{\text{chr}}} \sqrt{\cos^2 \theta_{\text{chr}} - \sin^2 \theta_{\text{atm}}} \right]^{1/2}.
 \end{aligned}$$

These results are useful for a numerical analysis of V by use of current experimental data on neutrino oscillations, if δ is allowed to vary from 0 to π .

Muon 与 Tau 中微子的运动学有效质量及其不可探测性*

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摘要 利用最新的 WMAP 观测数据推导出电子、Muon 和 Tau 中微子的运动学有效质量的一般上限: $\langle m \rangle_e^2 + \langle m \rangle_\mu^2 + \langle m \rangle_\tau^2 = m_1^2 + m_2^2 + m_3^2 < 0.5 \text{eV}^2$, 或 $\langle m \rangle_\alpha < 0.71 \text{eV}$ (其中 $\alpha = e, \mu, \tau$). 考虑现有中微子振荡的实验数据, 进一步得到 $\langle m \rangle_e < 0.24 \text{eV}$ 以及 $\langle m \rangle_\mu \approx \langle m \rangle_\tau < 0.24 \text{eV}$. 因此有效质量 $\langle m \rangle_\mu$ 和 $\langle m \rangle_\tau$ 太小而无法被探测.

关键词 中微子振荡 有效质量

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