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Dependence of Entropy Index on the Width and Shape of Multiplicity Distributions

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Abstract The dependence of entropy index μ_2 on the width and shape of multiplicity distributions are studied in detail by using Monte Carlo method and comparing with the results from NA22 experiment. It is found that the entropy index is insensitive to the shape of multiplicity distribution but decreases with the increase of the distribution width. The latter observation contradicts the usually expected role of the index, indicating that μ_q is not an appropriate parameter for measuring event-by-event fluctuation.

Key words erraticity, event space fluctuation, entropy index, multiplicity distribution

Since discovery of large local fluctuations in a high multiplicity event by JACEE collaboration[1], The study of local fluctuations in multiparticle production has attracted much attention of both theorelicians and experimentalists.

Cao and Hwa studied the event-by-event fluctuation of multiplicity fluctuation, called erraticity, and defined an entropy index μ_q[2] as a measure. They suggested that this index is adequate to describe fluctuation degree of spatial patterns[2,3].

The erraticity method has been studied both theoretically[4,5] and experimentally[6-8]. The basic idea of this method is the following. In contrast to the sample factorial moment, the event factorial moment is defined by

$$F_q^{(e)} = \frac{\frac{1}{M}\sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1)}{\left(\frac{1}{M}\sum_{m=1}^{M} n_m\right)^q}. \quad (1)$$

Consider a certain pseudorapidity region Δ, and divide it into M bins. The n_m is the number of particles falling into the mth bin for event e. In order to measure the fluctuation in $F_q^{(e)}$, the normalized factorial moment in event space is then defined as:

$$C_{p,q} = \langle \phi_q^p \rangle, \quad \phi_q = \frac{F_q^{(e)}}{\langle F_q^{(e)} \rangle}, \quad (2)$$

where the average $\langle \cdots \rangle$ is carried out over all the events and p is a positive real number. The erraticity moment in the event space is defined by:

$$\Sigma_q = \left.\frac{\mathrm{d}C_{p,q}}{\mathrm{d}p}\right|_{p=1} = \langle \phi_q \ln \phi_q \rangle. \quad (3)$$

If $C_{p,q}$ has a power-law behavior in M, i.e.

$$C_{p,q}(M) \propto M^{\psi_q(p)}, \quad (4)$$

then it is called erraticity[9], and $\psi_q(p)$ is corresponding erraticity exponent. The entropy index is defined as:

$$\mu_q = \left.\frac{\mathrm{d}\psi_q(p)}{\mathrm{d}p}\right|_{p=1} = \frac{\partial \Sigma_q}{\partial \ln M}. \quad (5)$$

Cao and Hwa suggested that the entropy index μ_q can be a measure of the event-by-event fluctuation of $F_q^{(e)}$ and showed that $\mu_q > 0$ is a criterion for chaos[4,10].

Based on the fact that the erraticity quantities defined above are measurable, the erraticity was applied to analyze the NA22 data at $\sqrt{s} = 22\text{GeV}$[11]. In the present letter, a cascade model, called α model, is used to study multiplicity fluctuations and a sample with non-zero entropy index[12,13] is generated. In this model, the phase space region Δ is firstly divided into two equal parts. The each part is further divided into two equal parts. After ν dividings, the number of partitions of Δ is $M = 2^\nu$. The probability of a particle falling into a given partition is defined as:

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$$\omega_{\nu,2j-1} = \frac{1}{2}(1+\alpha\gamma),$$
$$\omega_{\nu,2j} = \frac{1}{2}(1-\alpha\gamma), \quad (j=1,\cdots,2^{\nu-1}), \tag{6}$$

where j is the number of the partition at the ν-th step, γ is a random number ranged from -1 to 1, α is a positive number smaller than unity. To get pure statistical fluctuations, we set $\alpha=0$. By using Monte Carlo method, the result of α model with $q=2$, $\nu=6$ is obtained and the erraticity moment in one-dimensional pseudorapidity space is calculated.

What interests us is whether or how the entropy index depends on the width and the shape of the multiplicity distribution. Thus, we carried out a simulation of α model with the number of particles in each event obtained from various multiplicity distributions. The first step is to generate a 60,000-event sample of Poisson distributed n with average multiplicity $\bar{n}=6.15$, which corresponds to that of the NA22 experiment. For comparison, a 60,000-event sample with fixed $n=6$[5,11] is used. In either concerned sample, only statistical fluctuations exist in the events of a certain multiplicity n. However, an additional multiplicity fluctuation, included in the Poisson sample, attributes to the event-to-event fluctuations.

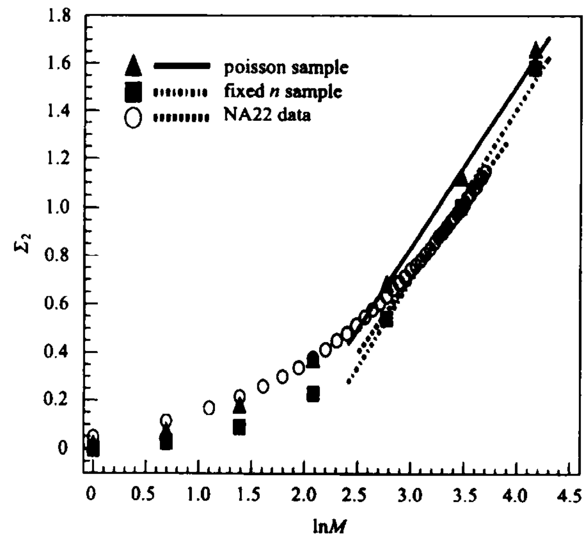

Fig.1. Σ_2 vs. $\ln M$ in rapidity space for Poissonian and fixed n samples, compared with the experimental result from NA22[11]. The straight lines are linear fits.

Calculated for the two MC samples in the $\alpha=0$ model, The resulting erraticity moment Σ_2 as a function of $\ln M$ for two MC samples in the $\alpha=0$ model are shown in Fig.1. For comparison, the result from NA22 data[11] is also plotted in the figure. The slopes (entropy indices) in these three cases are close to each other, confirming that the erraticity behavior is dominated by statistical fluctuations in the low-multiplicity sample[5,13].

On the other hand, if we consider the fixed-n sample as a sample with the multiplicity distribution width $\sigma=0$, and the Poissonian sample as that with a positive value of σ, both the Poissonian and the experimental samples have additional multiplicity fluctuations in comparison with the fixed-n sample. The influence due to the $\bar{n}$ differences of three samples is negligible. From the slopes by the linear fitting shown in Fig.1, one sees that Σ_2 of the fixed n sample increases faster than that in the other cases. That is to say, cf. Eq.(5), the entropy index μ_2 is the largest one in the fixed n sample, and the additional multiplicity fluctuations in the Poissonian and experimental samples made the entropy index μ_2 decreasing, which is in contradiction with the expectation that μ_q measures the event-by-event fluctuation. This observation motivates a further investigation.

For this purpose, a function that can generate the n distribution with the different width σ at a given n. The usual choice, say Gaussian function, is however inadequate in the present case, because after omitting the unphysical negative-n, the distribution is no longer in the Gaussian form and the average multiplicity deviates from the input value of $\bar{n}$. The larger the width is, the more the deviation from the given value of $\bar{n}$ would be.

To avoid this complication, we propose a new function

$$f(n) = Ane^{-a(n-b)^2}, \quad (a>0), \tag{7}$$

where A is the normalization constant. The characteristic shapes of $f(n)$ for various values of a and b are exhibited in Fig.2. The values of a and b for given values of average n and width σ are available in Table 1. The values of a and b for other values of n and σ in the ranges of $3\leqslant n\leqslant 7$ and $0.5\leqslant\sigma\leqslant 3$ can be obtained by intrapolation.

Fixing the average multiplicity at $\bar{n}=6.15$, 20 event samples are generated with a multiplicity distribution width σ ranging from 0.35 to 4.21. The resulting entropy indices μ_2 versus σ are plotted by solid triangles in Fig.3. The shadow band is the entropy index for the

Table 1. Parameters a and b for some given values of $\bar{n}$ and σ.

	Average multiplicity $\bar{n}$						
	3	4	5	5.5	6	6.5	7
$\sigma = 0.5$	$a = 1.9248$	$a = 1.9639$	$a = 1.9762$	$a = 1.9822$	$a = 1.9826$	$a = 1.9839$	$a = 1.99$
	$b = 2.9115$	$b = 3.935$	$b = 4.9485$	$b = 5.4535$	$b = 5.9575$	$b = 6.4605$	$b = 6.9635$
$\sigma = 1$	$a = 0.4135$	$a = 0.4602$	$a = 0.4767$	$a = 0.4812$	$a = 0.4842$	$a = 0.4867$	$a = 0.4886$
	$b = 2.5275$	$b = 3.707$	$b = 4.7804$	$b = 5.304$	$b = 5.8225$	$b = 6.3375$	$b = 6.8505$
$\sigma = 1.5$	$a = 0.1078$	$a = 0.1687$	$a = 0.1937$	$a = 0.2001$	$a = 0.2046$	$a = 0.2078$	$a = 0.2101$
	$b = 0.657$	$b = 3.0805$	$b = 4.417$	$b = 5.0005$	$b = 5.5605$	$b = 6.1055$	$b = 6.6415$
$\sigma = 2$	$a = 0.0102$	$a = 0.0606$	$a = 0.089$	$a = 0.0974$	$a = 0.1034$	$a = 0.1078$	$a = 0.1109$
	$b = -28.3185$	$b = 0.877$	$b = 3.5525$	$b = 4.361$	$b = 5.0555$	$b = 5.688$	$b = 6.2835$
$\sigma = 2.5$		$a = 0.0141$	$a = 0.0388$	$a = 0.0475$	$a = 0.0542$	$a = 0.0593$	$a = 0.0632$
		$b = -12.182$	$b = 1.1$	$b = 2.811$	$b = 3.9685$	$b = 4.8685$	$b = 5.633$
$\sigma = 3$			$a = 0.0131$	$a = 0.0207$	$a = 0.027$	$a = 0.0321$	$a = 0.0363$
			$b = -8.445$	$b = -1.6645$	$b = 1.32$	$b = 3.0931$	$b = 4.3445$

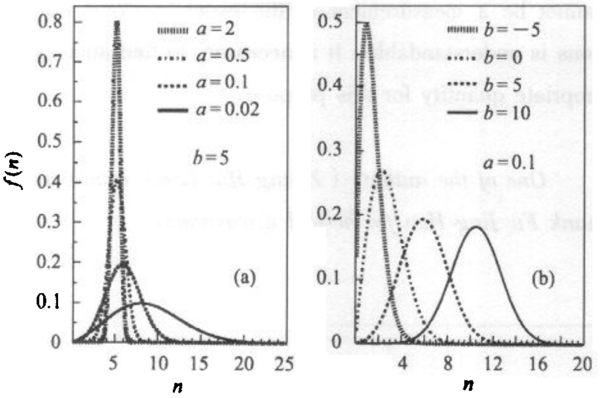

Fig.2. $f(n)$ vs. n.
(a) with various a at fixed $b = 5$; (b) with various b at $a = 0.1$.

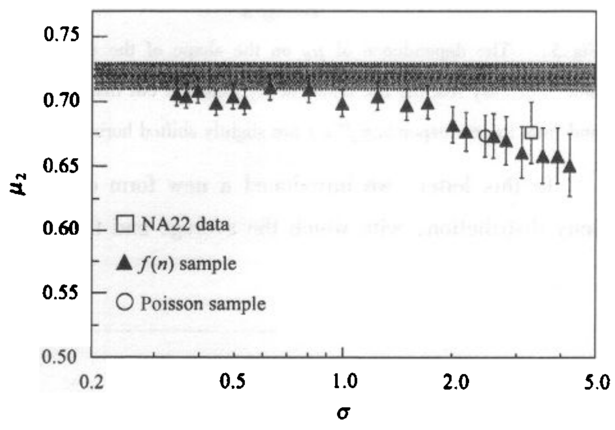

Fig.3. The dependence of entropy index μ_2 on width σ. The shadow region represents the result in the fixed-n sample case.

fixed-n sample. It can be seen from the figure that μ_2 decreases with the increase of σ, further confirming the unexpected trend seen in Fig.1. Note that the entropy indices of the Poissonian sample and the NA22 experimental data are consistent with the curve obtained from $f(n)$.

Let us now turn to the relationship between erraticity and the shape of multiplicity distribution. Dividing the whole experimental sample into two subsamples according to the conditions of $n < \bar{n}$ and $n > \bar{n}$, respectively, as done in Ref.[11], we obtain two samples with $\bar{n}_1 = 3.94$, $\sigma_1 = 1.56$ and $n_2 = 9.36$, $\sigma_2 = 2.41$, respectively. The multiplicity distributions of these two subsamples (referred to "cut distributions" in the following), together with the $f(n)$ distributions with the same value of $\bar{n}$ and σ in these two subsamples, are shown in Fig.4(a) and (b), respectively. It is seen that the shapes of cut distributions and corresponding $f(n)$ distributions differ considerably.

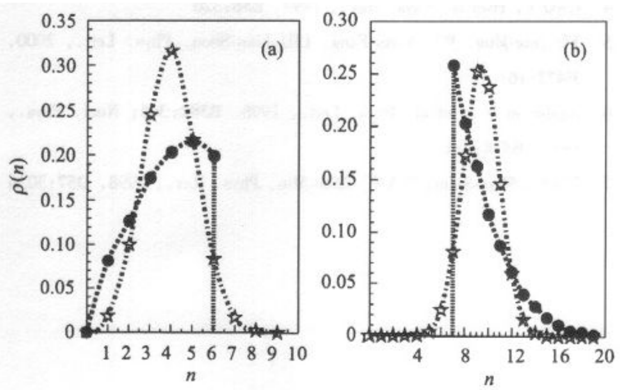

Fig.4. Comparison of the shapes of the cut multiplicity distribution and the distribution $f(n)$ with (a) $\bar{n}_1 = 3.94$, $\sigma_1 = 1.56$ and (b) $n_2 = 9.36$, $\sigma_2 = 2.41$. (the solid dots with dashed line denote the cut distribution, the open stars with dotted-dashed line represent the distribution $f(n)$.)

The erraticity analysis is then performed for all these cases. The corresponding entropy indices are shown in Fig.5. It is found that the entropy index μ_2 is insensitive to the shape of the multiplicity distribution. Therefore, it is natural that the results of the Poissonian sample fits NA22 data so well in Fig.1.

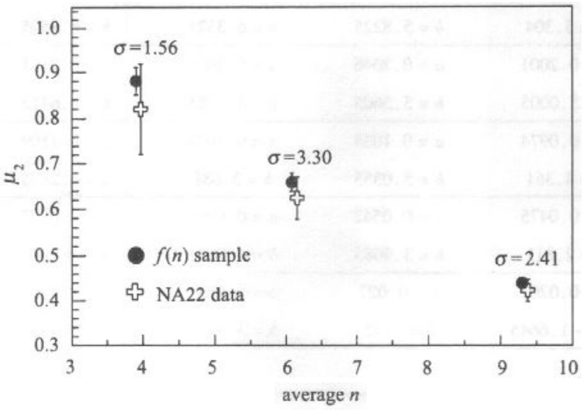

Fig.5. The dependence of μ_2 on the shape of the distribution. For easy reading the result of μ_2 from the cut distribution and from the corresponding $f(n)$ are slightly shifted horizontally.

In this letter, we introduced a new form of multiplicity distribution, with which the average and the width can be freely changed by two parameters. In terms of this distribution, the dependence of entropy index on the width and the shape of the multiplicity distribution are studied in detail by using the Monte Carlo method and compared with the result from the NA22 experiment. It is found that the entropy index is insensitive to the shape of the multiplicity distribution, but decreases with the increase of the distribution width. The latter observation contradicts the expectation that entropy index μ_q measures the event-by-event fluctuations of factorial moments. The reason is the following. According to the definition of μ_q, Eq.(5), μ_q is equal to the derivative of $\psi_q(p)$ at a certain point $p=1$. Thus μ_q might not be a monotonically increasing function of ψ_q, consequently, the fact that μ_q cannot be a measurement of the event-by-event fluctuations is understandable. It is necessary to find another appropriate quantity for this purpose.

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熵指数对多重数分布的宽度和形状的依赖性*

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摘要　通过运用蒙特卡罗方法,研究了熵指数对多重数分布的宽度和形状的依赖性,并和 NA22 实验结果作比较.发现,熵指数对多重数分布的形状变化不敏感,但是却随着分布宽度的增加而减小.这一发现和通常基于熵指数的物理意义的期待是矛盾的.

关键词　反常无规性　事件空间起伏　熵指数　多重数分布

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