Longitudinal Coupled Bunch Instability in Fractionally Filled Storage Rings

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Abstract Longitudinal coupled bunch instability in fractionally filled storage rings is studied in this paper with analytic method to derive the expression for the growth rate of the instability as well as the synchrotron oscillation frequency shift. An interesting phenomenon has been found that imaginary part of impedance makes contribution to the growth rate of coupled bunch instability. This phenomenon is contrary to that in symmetrical bunch filling cases.

Key words longitudinal instability, unsymmetrical, impedance, HOMs

1 Introduction

Most storage rings are fractionally filled with particle bunches by introducing bunch gaps, which is used for ion cleaning in electron machines and for accommodating the kicker rise time in proton machines; also, it is used for synchrotron light source storage rings to produce special time structured radiations. Those beams interact with the HOMs of RF cavity may excite longitudinal coupled bunch instabilities. Although, at the same average current the growth rate produced by such kind of beam is less than that by the symmetric beams[1] which can be calculated by codes BBI and ZAP, however, it is still very useful to analyze this growth rate in detail for designing HOMs coupler and beam feedback system, which are used to damp HOMs and suppress the beam instabilities in storage rings.

The coupled bunch instabilities for the beam with equally populated and unequally spaced bunches are studied by eigen function method[2,3], where Thompson and Ruth solved N_b(N_b is the bunch number) dimensions eigen function to get the solution for every bunch, and a program MULTI which follows Ruth-Thompson Theory has been written by Karl-Bane already. Also a set of formulism given by S.A. Bogacz[4] enables us to have a clear view of its instabilities and phase shift compared with the fully filled cases, Through it we can make insight into various optimizing schemes. And recently a program SLIAB[5] which is based on the beam energy equation via wake forces has been developed to simulate unsymmetrical bunch filling instability. Here in the following, a simple method is introduced for considering the longitudinal dipole instability of storage ring beam with unequqlly populated and unequally spaced bunches. The cavity impedance and beam spectrum are used to characterize the interaction between beam and cavity. By classifying the bunch oscillation into the coupled bunch modes. the growth rate can be easily calculated as that in symmetric case. Unlike the simulation method, a clear physical view can be obtained here.

2 Coupled bunch beam spectrum

B bunches are assumed to be distributed unsymmetrically in storage ring, and each bunch is considered as a rigid body with the same particle distribution, when we consider its dipole oscillation, it can be decomposed to B normal coupled modes (Strictly speaking, it's for symmetrical cases). From the shortest time interval between two bunches in the beam,

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one can find an integer M, the smallest harmonic number of buckets. The buckets are fractionally filled by B bunches, their charges can be described by the normalized filling factor $a_k = Q_k/Q_{\max}, k = 0, 1, 2\cdots, M-1$. We can get $0 \leqslant a_k \leqslant 1$, with $a_k = 0$ representing k'th bucket which is empty. In this way the beam current can be expressed as

$$I'(t) = \sum_{l=-\infty}^{+\infty}\sum_{k=0}^{M-1} a_k I_0(t - lT_0 - kT_0/M - \tau_{lk}), \quad (1)$$

Where $T_0 = 2\pi/\omega_0$ is the bunch revolution time in the ring. $I_0(t)$ is the maximal single bunch current in time domain. τ_{lk} is the time deviation of k'th bunch from synchronous position in l'th turn, here we decompose the B bunches dipole oscillation in to B normal modes with synchrotron frequency ω_{su}. We neglect the time deviation amplitude difference of the same mode among different bunches to avoid solving N_b dimensional eigen equations. So we can write:

$$\tau_{lk} = \sum_{u=0}^{B-1}\hat{\tau}_u\cos\left(\omega_{su}t + k\omega_{su}\frac{T_0}{M} - 2\pi u\frac{k}{M} + \theta_u\right), \quad (2)$$

Where $\hat{\tau}_u$ is the time deviation amplitude of normal mode u, and θ_u is initial phase.

By Fourier Transformation of $I'(t)$ we get

$$\tilde{I}'(\omega) = \int_{-\infty}^{+\infty} I'(t)e^{-j\omega t}dt \approx \tilde{I}(\omega) + \Delta\tilde{I}(\omega) =$$

$$\int_{-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}\sum_{k=0}^{M-1}\alpha_k\left[I_0\left(t - lT_0 - \frac{kT_0}{M}\right) - \dot{I}_0\times\tau_{lk}\right]e^{-j\omega t}dt. \quad (3)$$

The beam current in frequency domain can be written as

$$\tilde{I}(\omega) = \omega_0\tilde{I}_0(\omega)\sum_{l=-\infty}^{+\infty}\left(\sum_{k=0}^{M-1}\alpha_k e^{-jk\omega T_0/M}\right)\delta(\omega - l\omega_0), \quad (4)$$

$$\Delta\tilde{I}(\omega) = -\frac{j\omega_0\omega\tilde{I}_0(\omega)}{2}\sum_{l=-\infty}^{+\infty}\sum_{k=0}^{M-1}\sum_{u=0}^{B-1}\hat{\tau}_u\alpha_k\left[e^{-j\theta_{ku}^{+}}\delta(\omega - l\omega_0 - \omega_{su}) + e^{-j\theta_{ku}^{-}}\delta(\omega - l\omega_0 + \omega_{su})\right], \quad (5)$$

$$\theta_{ku}^{\pm} = k(\omega \mp \omega_{su})\frac{T_0}{M} \pm 2\pi u\frac{k}{M} \mp \theta_u, \quad (6)$$

Where $\tilde{I}_0(\omega)$ is the Fourier Transformation of $I_0(t)$. Again by inversing Fourier Transform we can get the beam current in time domain $I'(t) = I(t) + \Delta I(t)$,

$$I(t) = \frac{1}{T_0}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}A_n\tilde{I}(\omega_{pn})e^{j\omega_{pn}t}, \quad (7)$$

$$A_n = \sum_{k=0}^{M-1}a_k e^{-j2\pi nk/M}, \quad (8)$$

$$\omega_{pn} = (pM + n)\omega_0, \quad (9)$$

$$\Delta I(t) = \frac{-j\omega_0}{4\pi}\sum_{u=0}^{B-1}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}\hat{\tau}_u\left[C_{n+u}\omega_{pn}^{+}\tilde{I}_0(\omega_{pn}^{+})e^{j(\omega_{pn}^{+}t+\theta_u)} + C_{n-u}\omega_{pn}^{-}\tilde{I}_0(\omega_{pn}^{-})e^{j(\omega_{pn}^{-}t-\theta_u)}\right], \quad (10)$$

$$C_{n\pm u} = \sum_{k=0}^{M-1}a_k e^{-j2\pi k(n\pm u)/M}, \quad (11)$$

$$\omega_{pn}^{\pm} = (pM + n)\omega_0 \pm \omega_{su}. \quad (12)$$

3 Induced voltage

The induced voltage in RF cavity can be expressed in frequency domain, where the cavity is represented by impedance $Z(\omega)$. The impedance can be decomposed into real part and imaginary part, it can be written as

$$Z(\omega) = Z_r(\omega) + jZ_i(\omega), \quad (13)$$

In this way the induced voltage in frequency domain is

$$\tilde{V}'(\omega) = \tilde{I}'(\omega)Z(\omega). \quad (14)$$

By inversing Fourier Transform we can get

$$V'(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{I}'(\omega)Z(\omega)e^{j\omega t}d\omega = V(t) + \Delta V(t), \quad (15)$$

$$V(t) = \frac{1}{T_0}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}A_n\tilde{I}(\omega_{pn})Z(\omega_{pn})e^{j\omega_{pn}t}, \quad (16)$$

$$\Delta V(t) = \frac{-j\omega_0}{4\pi}\sum_{n=0}^{B-1}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}\hat{\tau}_u\left[C_{n+u}\omega_{pn}^{+}\tilde{I}_0(\omega_{pn}^{+})Z(\omega_{pn}^{+})\times e^{j(\omega_{pn}^{+}t+\theta_u)} + C_{n-u}\omega_{pn}^{-}\tilde{I}_0(\omega_{pn}^{-})Z(\omega_{pn}^{-})e^{j(\omega_{pn}^{-}t-\theta_u)}\right]. \quad (17)$$

Now we consider the reference bunch with $k = 0$, from Eq.(2) the normal mode can be written as

$$\tau_{l0} = \sum_{u=0}^{B-1}\tau_u, \quad (18)$$

$$\tau_u = \hat{\tau}_u\cos(\omega_{su}t + \theta_u), \quad (19)$$

$$\dot{\tau}_u = -\omega_{su}\hat{\tau}_u\sin(\omega_{su}t + \theta_u). \quad (20)$$

The $k = 0$ bunch traverses the cavity at time $t = lT_0 + \tau_{l0}, l = 0, 1, 2, \ldots$, under linear approximation, the induced voltage is

$$V'(lT_0 + \tau_{l0}) \approx V'(lT_0) + \dot{V}'(lT_0)\tau_{l0} \approx V(lT_0) + \Delta V(lT_0) + \dot{V}(lT_0)\tau_{l0}. \quad (21)$$

The terms can be expressed as following

$$V(lT_0) = \frac{1}{T_0}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}A_n\tilde{I}(\omega_{pn})Z(\omega_{pn}), \quad (22)$$

$$\dot{V}(lT_0) = \frac{j}{T_0}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}A_n\omega_{pn}\tilde{I}(\omega_{pn})Z(\omega_{pn}), \quad (23)$$

$$\Delta V(lT_0) = \frac{-j\omega_0}{4\pi}\sum_{u=0}^{B-1}\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{M-1}\left[(\tau_u - j\dot{\tau}_u/\omega_{su})C_{n+u}\omega_{pn}^{+}\tilde{I}_0\times(\omega_{pn}^{+})Z(\omega_{pn}^{+}) + (\tau_u + j\dot{\tau}_u/\omega_{su})\times C_{n-u}\omega_{pn}^{-}\tilde{I}_0(\omega_{pn}^{-})Z(\omega_{pn}^{-})\right]. \quad (24)$$

In real notation

$$V(lT_0) = \frac{2}{T_0}\sum_{p=0}^{+\infty}\sum_{n=1}^{M}\tilde{I}_0(\omega_{pn})[A_{n,r}Z_r(\omega_{pn}) - A_{n,i}Z_i(\omega_{pn})], \tag{25}$$

$$\dot{V}(lT_0) = -\frac{2}{T_0}\sum_{p=0}^{+\infty}\sum_{n=1}^{M}\omega_{pn}\tilde{I}_0(\omega_{pn})[A_{n,i}Z_r(\omega_{pn}) + A_{n,r}Z_i(\omega_{pn})], \tag{26}$$

$$\Delta V(lT_0) = \sum_{u=0}^{B-1}(K_u\tau_u + D_u\dot{\tau}_u), \tag{27}$$

$$K_u = \frac{1}{2T_0}\Big\{\omega_{su}\tilde{I}_0(\omega_{su})\Big[C_{0+u,r}Z_i(\omega_{su}) + C_{0+u,i}Z_r(\omega_{su})\Big] + 2\sum_{p=0}^{+\infty}\sum_{n=1}^{M}\Big[\omega_{pn}^{+}\tilde{I}_0(\omega_{pn}^{+})(C_{n+u,r}Z_i(\omega_{pn}^{+}) + C_{n+u,i}Z_r(\omega_{pn}^{+})) + \omega_{pn}^{-}\tilde{I}_0(\omega_{pn}^{-})(C_{n-u,r}Z_i(\omega_{pn}^{-}) + C_{n-u,i}Z_r(\omega_{pn}^{-}))\Big]\Big\}, \tag{28}$$

$$D_u = \frac{1}{2\omega_{su}T_0}\Big\{\omega_{su}\tilde{I}_0(\omega_{su})\Big[C_{0+u,r}Z_r(\omega_{su}) - C_{0+u,i}Z_i(\omega_{su})\Big] + 2\sum_{p=0}^{+\infty}\sum_{n=1}^{M}\Big[\tilde{I}_0(\omega_{pn}^{+})\omega_{pn}^{+}(C_{n+u,r}Z_r(\omega_{pn}^{+}) - C_{n+u,i}Z_i(\omega_{pn}^{+})) - \tilde{I}_0(\omega_{pn}^{-})\omega_{pn}^{-}(C_{n-u,r}Z_r(\omega_{pn}^{-}) - C_{n-u,i}Z_i(\omega_{pn}^{-}))\Big]\Big\}, \tag{29}$$

Here $C_{n+/-u,r}(A_{n,r})$ and $C_{n+/-u,i}(A_{n,2})$ are real and imaginary part of $C_{n+/-u}(A_n)$ respectively, and $\tau_u = \tau_u(lT_0)$.

4 Longitudinal dynamics

4.1 Incoherent synchrotron frequency shift

When the nonsymmetrical beam is stable, the beam induced field will modify the RF longitudinal focusing force, then introduce an incoherent synchrotron frequency shift to the particle. Assuming τ_0 is the time displacement of the reference bunch with $k=0$, its energy gain per revolution is

$$\delta E = eV_{\mathrm{RF}}(lT_0 + \tau_0) - eV(lT_0 + \tau_0) - U(E) + \mathrm{e}k_l\alpha_k Q_0. \tag{30}$$

Where, $U(E)$ is the synchrotron radiation energy per revolution and E is the particle energy, K_l is the longitudinal loss factor, $V_{\mathrm{RF}}(t) = \hat{V}_{\mathrm{RF}}\sin(\omega_{\mathrm{RF}}t + \phi_{s0})$ is the accelerating voltage provided by RF system, and Q_0 is the nominal single bunch charge. In linear approximation the relative energy change can be expressed as

$$\dot{\varepsilon} = \frac{\delta E}{ET_0} = \frac{e}{ET_0}[\dot{V}_{\mathrm{RF}}(lT_0) - \dot{V}(lT_0)]\tau_0 - \frac{1}{T_0}\frac{\mathrm{d}U}{\mathrm{d}E}\varepsilon, \tag{31}$$

with slippage factor $\eta(\dot{\tau} = -\eta\varepsilon)$ the oscillation equation can be written as

$$\ddot{\tau}_0 + \frac{1}{T_0}\frac{\mathrm{d}U}{\mathrm{d}E}\dot{\tau}_0 + \frac{e\eta}{ET_0}[\dot{V}_{\mathrm{RF}}(lT_0) - \dot{V}(lT_0)]\tau_0 = 0. \tag{32}$$

Solving the equation, we can get the incoherent synchrotron frequency

$$\omega_{si,0} = \omega_{s,0} + \Delta\omega_{i,0}, \tag{33}$$

$$\omega_{s,0} = \sqrt{\frac{e\eta}{ET_0}\dot{V}_{\mathrm{RF}}(lT_0)}, \tag{34}$$

$$\Delta\omega_{i,0} = -\frac{e\eta}{2\omega_{s,0}ET_0}\dot{V}(lT_0). \tag{35}$$

From Eq.(26) the incoherent synchrotron frequency shift can be written as

$$\Delta\omega_{i,0} = \frac{e\eta}{\omega_{s,0}ET_0^2}\sum_{p=0}^{+\infty}\sum_{n=1}^{M}\omega_{pn}\tilde{I}_0(\omega_{pn})[A_{n,ji}Z_r(\omega_{pn}) + A_{n,r}Z_i(\omega_{pn})]. \tag{36}$$

Here we can see that the real part impedance has the contribution to the incoherent synchrotron frequency shift in unsymmetrical bunch case. It is different from the symmetric bunch situation.

4.2 Growth rate of coherent synchrotron oscillation

When the unsymmetrical bunches execute dipole oscillation, the coherent synchrotron frequency shift is introduced to the beam longitudinal motion. The imaginary part of the coherent frequency is the growth rate of bunch dipole oscillation. Assuming τ_0 is the time displacement of the reference bunch with $k=0$, and ε_{l0} is its relative energy deviation, from Eqs.(21) and (30) (with $V(lT_0+\tau_0)$ replaced by $V'(lT_0+\tau_0)$) we get

$$\dot{\varepsilon}_{l0} = \frac{e}{ET_0}\{[\dot{V}_{\mathrm{RF}}(lT_0) - \dot{V}(lT_0)]\tau_0 - \Delta V(lT_0)\} - \frac{1}{T_0}\frac{\mathrm{d}U}{\mathrm{d}E}\varepsilon_{l0}. \tag{37}$$

From Eqs.(18),(28),(29), the oscillation equation can be written as

$$\sum_{u=0}^{B-1}\Big\{\ddot{\tau}_u + \Big(\frac{1}{T_0}\frac{\mathrm{d}U}{\mathrm{d}E} - \frac{e\eta}{ET_0}D_u\Big)\dot{\tau}_u + \frac{e\eta}{ET_0}\Big[\dot{V}_{\mathrm{RF}}(lT_0) - \dot{V}(lT_0) - K_u\Big]\tau_u\Big\} = 0. \tag{38}$$

For normal modes we have

$$\ddot{\tau}_u + \Big(\frac{1}{T_0}\frac{\mathrm{d}U}{\mathrm{d}E} - \frac{e\eta}{ET_0}D_u\Big)\dot{\tau}_u + \frac{e\eta}{ET_0}\Big[\dot{V}_{\mathrm{RF}}(lT_0) - \dot{V}(lT_0) - K_u\Big]\tau_u = 0. \tag{39}$$

The growth rate and coherent frequency shift due to HOM of RF cavity are

$$\alpha_u = \frac{e\eta}{2ET_0}D_u, \tag{40}$$

$$\Delta\omega_{cu} = -\frac{e\eta}{2\omega_{si,0}ET_0}K_u, \tag{41}$$

Here $\omega_{si,0}$ is the incoherent synchrotron frequency. ω_{su} is the coherent synchrotron frequency of reference bunch. It can be approximately written as[7]:

$$\omega_{su} = (u+1)\omega_{si,0} + \Delta\omega_{cu}. \tag{42}$$

From Eqs.(28),(29),(40) and Eq.(41) we can see that both the imaginary and real parts of impedance make contributions to the growth rate and coherent frequency shift.

5 Some discussions

In unsymmetrical case $C_{n\pm u}$ is a complex number, But in symmetrical case ($a_k \equiv 1$),

$$\begin{cases} C_{n\pm u,i} \equiv 0, \\ C_{n\pm u,r} = 0 \text{ if } n \pm u \neq H\times M \ (H \text{ is an integer}), \\ C_{n\pm u,r} = M \text{ when } n\pm u = H\times M, \end{cases}$$

Substitute the value to Eqs.(28) and (29), we get

$$D_u = \frac{M}{T_0\omega_{su}}\Big\{\frac{\omega_{su}\tilde{I}_0(\omega_{su})Z_r(\omega_{su})\delta(u)}{2} - \omega^-_{p=0,u}\tilde{I}(\omega^-_{p=0,u}) \times Z_r(\omega^-_{p=0,u})\delta(\delta(u)) + \sum_{p=1}^{+\infty}\Big[\omega^+_{p,-u}\tilde{I}_0(\omega^+_{p,-u}) Z_r(\omega^+_{p,-u}) - \omega^-_{p,u}\tilde{I}_0(\omega^-_{p,u})Z_r(\omega^-_{p,u}), \tag{43}$$

$$K_u = \frac{M}{T_0}\Big\{\frac{\omega_{su}\tilde{I}_0(\omega_{su})Z_i(\omega_{su})\delta(u)}{2} + \omega^-_{p=0,u}\tilde{I}(\omega^-_{p=0,u}) \times Z_i(\omega^-_{p=0,u})\delta(\delta(u)) + \sum_{p=1}^{+\infty}\Big[\omega^+_{p,-u}\tilde{I}_0(\omega^+_{p,-u}) Z_i(\omega^+_{p,-u}) + \omega^-_{p,u}\tilde{I}_0(\omega^-_{p,u})Z_i(\omega^-_{p,u})\Big]\Big\}, \tag{44}$$

$$\omega^+_{p,-u} = (pM-u)\omega_0 + \omega_{su}, \tag{45}$$

$$\omega^-_{p,u} = (pM+u)\omega_0 - \omega_{su}. \tag{46}$$

When considering a narrow-band cavity, the impedance peaks at $h\omega_0$, drop all the terms in Eq.(43) except one with $pM = h$, $u=0$, and the result degenerates to Robinson Instability growth rate

$$\alpha_0 = \frac{Me\eta}{2ET_0^2\omega_{s0}}[(h\omega_0+\omega_{s0})I_0(h\omega_0+\omega_{s0})Z_r(h\omega_0+\omega_{s0}) - (h\omega_0-\omega_{s0})I_0(h\omega_0-\omega_{s0})Z_r(h\omega_0-\omega_{s0})], \tag{47}$$

$$\Delta\omega_{cu} = -\frac{Me\eta}{2\omega_{si,0}ET_0}[(h\omega_0+\omega_{s0})I_0(h\omega_0+\omega_{s0})Z_i(h\omega_0+\omega_{s0}) + (h\omega_0-\omega_{s0})I_0(h\omega_0-\omega_{s0})Z_i(h\omega_0-\omega_{s0})], \tag{48}$$

Which does well agree with Alex Wu Chao's conclusion[8].

As mentioned above, both the real and imaginary parts of impedance make contribution to the growth rate, we should take a further look in physical view. Here we decompose the wake potential into an odd function and an even function,

$\boldsymbol{W}_{/\!/}(z) = \boldsymbol{W}_{/\!/\text{odd}}(z) + \boldsymbol{W}_{/\!/\text{even}}(z)$.

$$W_{/\!/\text{odd}}(z<0) = W_{/\!/\text{even}}(z<0) = \frac{1}{2}W_{/\!/}(z<0), \tag{49}$$

$$-W_{/\!/\text{odd}}(z>0) = W_{/\!/\text{even}}(z>0) = \frac{1}{2}W_{/\!/}(-z<0). \tag{50}$$

After doing this, $\boldsymbol{W}_{/\!/\text{odd}}(z)$, $\boldsymbol{W}_{/\!/\text{even}}(z)$ can sample all the z region, which means the following bunches can also affect the leading bunches (of course the total effect of $\boldsymbol{W}_{/\!/\text{odd}}(z) + \boldsymbol{W}_{/\!/\text{even}}(z)$ is zero). The imaginary part of impedance comes absolutely from $\boldsymbol{W}_{/\!/\text{odd}}(z)$, and the real part absolutely from $\boldsymbol{W}_{/\!/\text{even}}(z)$ ($Z(\omega) = \frac{\tilde{W}(\omega)}{\tilde{I}(\omega)}$, as Gauss bunch $\tilde{I}(\omega)$ is real). When considering bunch with $k=0$ in mode u, the bunch ahead has an oscillation amplitude $\hat{\tau}\cos\left(-\omega_{su}\frac{kT_0}{M}+\frac{\pi}{2}\right)$ and the bunch behind has the amplitude $\hat{\tau}\cos\left(\omega_{su}\frac{kT_0}{M}+\frac{\pi}{2}\right)$. As illustrated below in Fig.2, their total $\boldsymbol{W}_{/\!/\text{odd}}(z)$ effect on bunch with $k=0$ just compensated, makes no contribution to bunch momentum change. Then we can easily deduce that in symmetrically placed and equally populated bunch filling conditions. The imaginary part of impedance does not contribute to the growth rate. In unsymmetrically or unequally populated bunch filling conditions, we get the reversal conclusion.

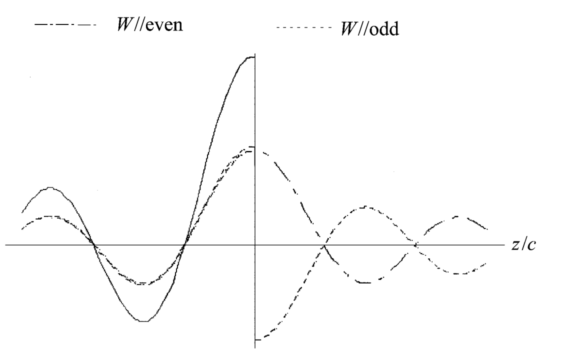

Fig.1. Decompose of wake potential.

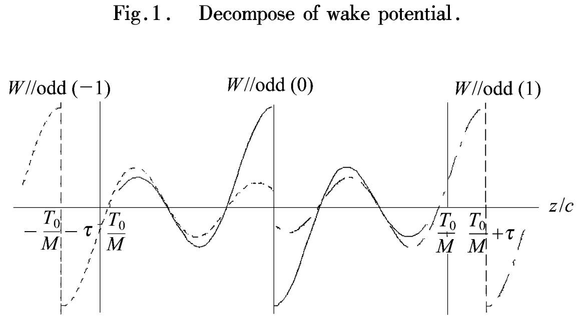

Fig.2. Odd parts of wake potential act on reference bunch.

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储存环部分填充情况下的纵向耦合不稳定性

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摘要　通过解析的方法分析了储存环在部分填充情况下的纵向耦合不稳定性,给出了不稳定性增长率和同步振荡频移的表达式.得到了一个有趣的结果:虚部阻抗对不稳定性的增长率也有贡献.这一点和均匀填充情况是不一样的.

关键词　纵向-不稳定性　非均匀-阻抗　高阶模

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