

Quark Mean Field Model for Finite Nuclei and Hypernuclei^{*}

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Abstract The quark mean field model, which describes the baryon using the constituent quark model, is applied to study the properties of finite nuclei and hypernuclei. The meson mean fields couple directly with the quarks and change the properties of baryons in nuclear medium. The quark mean field model provides satisfactory results on the properties of spherical nuclei and hypernuclei. It also predicts an increasing size of the nucleon as well as a reduction of the effective mass in the nuclear environment.

Key words quark mean field model, constituent quark model, finite nuclei, hypernuclei

1 Introduction

One of the most exciting topics in nuclear physics is to study the variations of hadron properties in nuclear medium. Experimentally, the EMC effect reveals the medium modification of the internal structure of the nucleon. Theoretically, much effort has been devoted to the study of hadrons in terms of quarks and gluons. At present, we are still far away from describing nucleons and nuclei in terms of quarks and gluons using quantum chromodynamics (QCD), which is believed to be the fundamental theory of strong interactions. Thus, it is desirable to build models that incorporate quark and gluon degrees of freedom and respect the established theories based on hadronic degrees of freedom. Such models are necessarily crude from various viewpoints since the study of the nuclear many-body systems on the fundamental level is intractable.

The quark-meson coupling (QMC) model proposed by Guichon^[1] provides a simple and attractive framework to incorporate quark degrees of freedom in the study of nuclear many-body systems. The model describes nuclear matter as nonoverlapping MIT bags in which the quarks interact self-consistently with structureless scalar and vector mesons in the mean field approximation. The QMC model has been subsequently extended with reasonable success to various problems of nuclear matter and finite nuclei^[2-5]. The quark mean field (QMF) model took the constituent quark model for the nucle-

on instead of the MIT bag model, so that it could overcome the shortcoming of the QMC model as discussed in Refs. [6, 7]. In the QMF model, the meson fields couple directly with the constituent quarks inside nucleons, and as a result the nucleon properties are modified in nuclear medium. The QMF model has been successfully applied to study the properties of finite nuclei and Λ hypernuclei^[7, 8].

The outline of this paper is as follows. In Section 2 we review the formalism of the QMF model for both nuclear matter and finite system. The parameters in the model are determined by reproducing the properties of nuclear matter. In Section 3 we show the numerical results for finite nuclei and Λ hypernuclei. Section 4 is devoted to the summary.

2 Quark mean field model

In the QMF model, the nucleon and the Λ hyperon as composites of three quarks are described in terms of the constituent quark model. The u and d quarks in baryons, which couple with the non-strange mesons, satisfy the following Dirac equation:

$$[i\gamma_\mu \partial^\mu - m_q - g_\sigma^q \sigma - g_\omega^q \omega \gamma^0 - g_\rho^q \rho \tau_3 \gamma^0 - \chi_c] q(\gamma) = 0. \quad (1)$$

Here the confinement potential is expressed in terms of χ_c , which is the harmonic oscillator potential together with two

Lorentz structures: (1) scalar potential $\chi_c = \frac{1}{2}kr^2$ (2) scalar-vector potential $\chi_c = \frac{1}{2}kr^2(1 + \gamma^0)/2$. We take the mean field approximation for the meson fields, and assume that the meson mean fields σ , ω , and ρ are constant within the small baryon volume. According to the OZI rule, the non-strange mesons couple exclusively to the u and d quarks and not to the s quark, therefore the s quark inside the Λ hyperon satisfies the Dirac equation, in which the terms coupled to the non-strange mesons σ , ω , and ρ , should vanish. The constituent quark masses are taken as $m_q = 313\text{MeV}$ ($q = u, d$) and $m_s = 490\text{MeV}$. The σ meson, which couples directly to the u and d quarks, provides a scalar potential to those quarks and as a consequence reduces the quark mass to $m_q^* = m_q + g_\sigma^q \sigma$. The s quark mass will not be influenced by the σ mean field, since the σ meson doesn't couple to the s quark. We take into account the spin correlations and remove the spurious center of mass motion, and then obtain the effective mass for nucleon as $M_N^* = \sqrt{(3e_q + E_{\text{spin}}^N)^2 - 3\langle p_q^2 \rangle}$, while the effective mass for Λ hyperon is expressed as $M_\Lambda^* = \sqrt{(2e_q + e_s + E_{\text{spin}}^\Lambda)^2 - (2\langle p_q^2 \rangle + \langle p_s^2 \rangle)}$. Here the subscript q denotes the u or d quark. The energies (e_q and e_s) and momenta ($\langle p_q^2 \rangle$ and $\langle p_s^2 \rangle$) can be obtained by solving the Dirac equations. The spin correlations (E_{spin}^N and E_{spin}^Λ) are fixed by fitting the nucleon and Λ masses in free space ($M_N = 939\text{MeV}$, $M_\Lambda = 1116\text{MeV}$).

For a many-body system including many nucleons and a single Λ hyperon, the effective Lagrangian within the mean field approximation can be written as

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left[i\gamma_\mu \partial^\mu - M_N^*(\sigma) \right. \\ & \left. - g_\omega \omega \gamma^0 - g_\rho \rho \tau_3 \gamma^0 - e \frac{(1 + \tau_3)}{2} A \gamma^0 \right] \psi + \\ & \bar{\psi}_\Lambda \left[i\gamma_\mu \partial^\mu - M_\Lambda^*(\sigma) - g_\omega^\Lambda \omega \gamma^0 \right] \psi_\Lambda - \\ & \frac{1}{2} (\nabla \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} (\nabla \omega)^2 + \frac{1}{2} m_\omega^2 \omega^2 + \\ & \frac{1}{4} c_3 \omega^4 + \frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} (\nabla A)^2, \end{aligned} \quad (2)$$

where ψ and ψ_Λ are the Dirac spinors for the nucleon and the Λ hyperon. The mean field values of the σ , ω , and ρ mesons are denoted by σ , ω , and ρ , respectively. m_σ , m_ω , and m_ρ are the meson masses. A is the electromagnetic field which couples to the protons. The effective masses of nucleons and Λ hyperon change under the influence of the scalar mean field, which is evidently expressed in the effective masses

$M_n^*(\sigma)$ and $M_\Lambda^*(\sigma)$ as a function of the σ mean field. The effective masses (M_N^* and M_Λ^*) have been worked out in the constituent quark model. The ω and ρ mean fields do not cause any change in the baryon properties, and they appear merely as the energy shift. These contributions are carried over as the nucleon-meson coupling terms with the replacement of the quark-meson couplings as $g_\omega = 3g_\omega^q$ and $g_\rho = g_\rho^q$. Since the Λ hyperon is neutral and isoscalar, it only couples to the σ and ω mesons. The ω meson couples to the Λ hyperon with the coupling constant $g_\omega^\Lambda = 2g_\omega^q$.

In the QMF model, the basic parameters are the quark-meson couplings (g_σ^q , g_ω^q , and g_ρ^q), the nonlinear self-coupling constants (g_3 and c_3), and the meson masses. We take $m_\omega = 783\text{MeV}$ and $m_\rho = 770\text{MeV}$, while m_σ is chosen to reproduce the charge radius of ^{40}Ca to be around 3.45fm. The parameters, g_σ^q , g_ω^q , g_ρ^q , g_3 , and c_3 , can be determined by reproducing the equilibrium properties of nuclear matter, which are listed in Table 1^[3,7]. In the calculations for finite nuclei and Λ hypernuclei, there is no more adjustable parameters. From the effective Lagrangian, we can obtain the Euler-Lagrange equations, and then solve those equations self-consistently.

Table 1. The nuclear matter properties used to determine the parameters in the QMF model. The saturation density and the energy are denoted by ρ_0 and E/A , and the incompressibility by k , the effective mass by M_n^* and the symmetry energy by a_{sym} .

ρ_0/fm^{-3}	$(E/A)/\text{MeV}$	k/MeV	M_n^*/M_n	$a_{\text{sym}}/\text{MeV}$
0.145	-16.3	280	0.63	35

3 Results

We restrict our consideration to spherically symmetric nuclei. In Table 2, the calculated results for the binding energies per nucleon E/A and the rms charge radii R_c are compared with experimental values^[9]. For the case of the scalar-vector confinement potential, the calculated results for E/A and R_c are almost same as the RMF(TM1) results^[9]. On the other hand, for the case of the scalar confinement potential, E/A are somewhat underestimated.

For several hypernuclei consisting of a closed-shell nuclear core and a single Λ hyperon, we present the calculated Λ single particle energies in Fig. 1, while the results in the QMC model^[4] and the experimental values^[10,11] are also

Table 2. The binding energies per nucleon E/A and the rms charge radii R_c in the QMF model compared with the results in the RMF(TM1) model and the experimental values.

model		$(E/A)/\text{MeV}$				R_c/fm			
		^{40}Ca	^{48}Ca	^{90}Zr	^{208}Pb	^{40}Ca	^{48}Ca	^{90}Zr	^{208}Pb
QMF	$k = 700$	7.53	7.66	7.92	7.36	3.45	3.46	4.28	5.53
$\chi_c = \frac{1}{2}kr^2$	$k = 1000$	7.32	7.46	7.75	7.24	3.45	3.46	4.28	5.53
QMF	$k = 700$	8.35	8.43	8.54	7.81	3.44	3.46	4.28	5.54
$\chi_c = \frac{1}{2}kr^2(1 + \gamma^0)/2$	$k = = 1000$	8.21	8.30	8.43	7.73	3.44	3.46	4.27	5.53
RMF (TM1)		8.62	8.65	8.71	7.87	3.44	3.45	4.27	5.53
Expt.		8.55	8.67	8.71	7.87	3.45	3.45	4.26	5.50

shown for comparison. Here the QMC results do not contain the effect of the Pauli blocking, which has been included phenomenologically in Ref. [5] in order to reproduce the experimental single particle energies. The QMF^I and QMF^{II} denote the models with confinements $\chi_c = \frac{1}{2}kr^2(1 + \gamma^0)/2$ and $\chi_c = \frac{1}{2}kr^2$, respectively. The single particle energies in the present model seem to be slightly underestimated, which is opposite to the tendency in the QMC model. It is well known that the properties of Λ hypernuclei are very sensitive to the

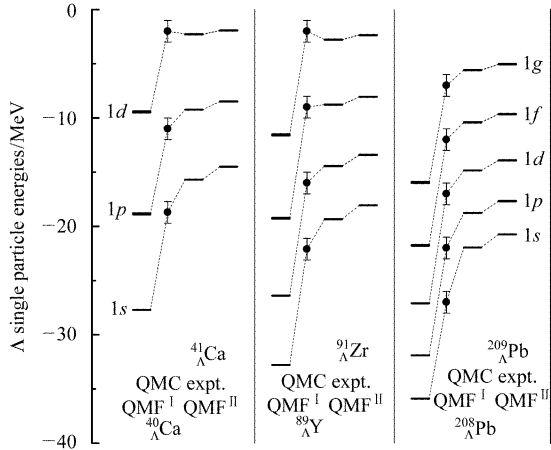


Fig. 1. Λ Single particle energies in $^{41}_{\Lambda}\text{Ca}$, $^{91}_{\Lambda}\text{Zr}$, and $^{209}_{\Lambda}\text{Pb}$.

QMF^I and QMF^{II} denote the models with

$\chi_c = \frac{1}{2}kr^2(1 + \gamma^0)/2$ and $\chi_c = \frac{1}{2}kr^2$, respectively. The

results in the QMC model^[4] are also shown for comparison.

The experimental data are taken from Refs. [10, 11].

effective coupling constants on the hadronic level, especially the two relative couplings $R_\sigma = g_\sigma^\Lambda/g_\sigma$ and $R_\omega = g_\omega^\Lambda/g_\omega$ ^[12]. The quark model value, $R_\sigma = R_\omega = 2/3$, usually gives large overbinding of Λ single particle energies. Most studies of the hypernuclei in the RMF models are performed by treating both R_σ and R_ω (or only one of them) as phenomenological parameters, which are fitted by using experimental data^[12–14]. In the present model, $R_\sigma = g_\sigma^\Lambda/g_\sigma = \left[\frac{\partial M_\Lambda^*}{\partial \sigma} \right] / \left[\frac{\partial M_N^*}{\partial \sigma} \right]$ must be calculated self-consistently on the quark level, while $R_\omega = 2/3$ is based on the quark model. Comparing with the RMF models, R_σ in the QMF model depends on the σ mean field, and could not be a constant again. The resulting Λ single particle energies are slightly underestimated in comparison with the experimental values as shown in Fig. 1. The results can be largely improved if we use the scaled coupling constant $0.97 \times g_\omega^\Lambda$.

4 Conclusions

In summary, we have reported the results of the application of the QMF model to finite nuclei and Λ hypernuclei. In the QMF model, we have used the constituent quark model for the descriptions of baryons. The QMF model can provide a significantly swollen nucleon radius in nuclear medium. At the normal matter density, the nucleon radius increases by about 5%–9%. With the parameters determined by the properties of nuclear matter, the calculated results for finite nuclei and Λ hypernuclei are satisfactory.

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夸克平均场模型的有限核及超核研究*

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摘要 夸克平均场模型采用组分夸克模型描述重子,已被用于研究有限核及超核的性质.介子平均场直接与核子内的组分夸克相互作用,从而改变了核介质内重子的性质.夸克平均场模型能够给出令人满意的球型核及超核的性质,该模型也预言了核介质中核子体积的膨胀及核子有效质量的降低.

关键词 夸克平均场模型 组分夸克模型 有限核 超核