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Angular Distributions of ψ Radiative Decays*

Sayipjamal Dulat[2,3;1)] ZOU Bing-Song[1,2;2)] WU Ji-Min[1,2;3)]

1 (CCAST (World Laboratory), Beijing 100080, China)

2 (Institute of High Energy Physics, CAS, Beijing 100039, China)

3 (Department of Physics, Xinjiang University, Urumqi 830046, China)

Abstract J/ψ and ψ′ radiative decays to mesons are a good place to look for glueballs, hybrids and for extracting gg-$q\bar{q}$ couplings. Abundant J/ψ and ψ′ events have been collected at the Beijing Electron Positron Collider (BEPC). More data will be collected at upgraded BEPC and CLEO-c. Here we first provide explicit formulae for the angular distribution of photon of the ψ radiative decays in the covariant tensor formalism. Then we discuss helicity formalism of the angular distribution of the ψ radiative decays to two pseudoscalar mesons, and its relation to the covariant tensor formalism.

Key words ψ radiative decay, angular distribution, amplitude

1 Introduction

High statistics data have appeared from BES for J/ψ and ψ′ decays. Further higher statistics data are expected from CLEO-c soon[1]. In order to get more useful information about properties of the resonances such as their J^{PC} quantum numbers, mass, width, production and decay rates, etc., partial wave analysis (PWA) are necessary. Ref.[2] provided PWA formulae in a covariant tensor formalism, which have been used for a number of channels already published by BES[3—8] and are going to be used for more channels. A similar approach has been used in analyzing other reactions[9—11].

Reactions of ψ decays to mesons fall into two categories: non-radiative decays, where final-state particles are pseudoscalars, such as pions and kaons; all polarization information is then available in the form of angular distributions. The second class of reactions consists of radiative decays. For this class, differential cross sections need to be summed over the unmeasured helicities of the photon, incorporating the knowledge that the photon is transverse. Both types of reactions and PWA formulae for many interesting channels in the covariant tensor formalism are discussed in Ref. [2]. But no explicit angular distributions are given. In this paper we present the explicit formulae of the angular distributions of the second class of reactions: radiative decays, which may serve as a reference for people using the covariant tensor formalism for PWA.

Besides the covariant tensor formalism, another commonly used formalism for PWA is the helicity formalism[12,13], which is also the basis of moment analysis[13]. To illustrate the relation between the covariant tensor formalism and the helicity formalism, as an example, we give full amplitude for ψ radiative decay to two pseudoscalars including 0^{++}, 2^{++} and 4^{++} intermediate states in both formalisms.

This paper is organized as follows. In section 2, we briefly review covariant tensor formalism for ψ radiative decays that are relevant to our studies. In section 3, we present the covariant decay amplitudes and corresponding angular distributions of photon for the processes $\psi \to \gamma 0^{++}$, $\gamma 0^{-+}$, $\gamma 1^{++}$, $\gamma 1^{-+}$, $\gamma 2^{++}$, $\gamma 2^{-+}$, $\gamma 4^{++}$. In

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1) sayipjamal@mail.ihep.ac.cn; 2) zoubs@mail.ihep.ac.cn; 3) wujm@mail.ihep.ac.cn

section 4, we first discuss the angular distributions of ψ radiative decay to two pseudoscalar mesons in the helicity formalism. Then we deduce the relation between the helicity amplitudes and covariant tensor amplitudes. The conclusions are given in section 5. The Appendix A deals with the problem of D^J function.

2 Covariant tensor formalism for ψ radiative decay

The general form for the decay amplitude of a vector meson ψ with spin projection of m_J is

$$A(m_J, m_\gamma) = \psi_\mu(m_J) e_\nu^*(m_\gamma) A^{\mu\nu} = \psi_\mu(m_J) e_\nu^*(m_\gamma) \sum_i \Lambda_i U_i^{\mu\nu}, \quad (1)$$

where $\psi_\mu(m_J)$ is the polarization vector of the ψ; $U_i^{\mu\nu}$ is the i-th partial wave amplitude with coupling strength determined by a complex parameter Λ_i. The spin-1 polarization vector for ψ satisfies

$$\sum_{m_J=1}^{3} \psi_\mu(m_J)\psi_\nu^*(m_J) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \equiv -\tilde{g}_{\mu\nu}(p), \quad (2)$$

with $\psi_\mu p^\mu = 0$; the metric tensor has the form

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1).$$

For ψ production from e^+e^- annihilation, the electrons are highly relativistic, with the result that $J_z = \pm 1$ for the ψ spin projection taking the beam direction as the z-axis. This limits m_J to 1 and 2, i.e. components along x and y. Then one has the following relation,

$$\sum_{m_J=1}^{2} \psi_\mu(m_J)\psi_{\mu'}^*(m_J) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2}). \quad (3)$$

For the photon polarization four vector e_ν with photon momentum q, there is the usual Lorentz orthogonality condition $e_\nu q^\nu = 0$. This is the same as for a massive vector meson. However, for the photon, there is an additional gauge invariance condition. Here we assume the Coulomb gauge in the ψ rest system, i.e., $e_\nu p^\nu = 0$. Then we have[14]

$$\sum_{m_\gamma} e_\mu^*(m_\gamma) e_\nu(m_\gamma) = -g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q \cdot K} - \frac{K \cdot K}{(q \cdot K)^2} q_\mu q_\nu \equiv -g_{\mu\nu}^{(\perp\perp)} \quad (4)$$

with $K = p - q$ and $e_\nu K^\nu = 0$. Then the differential cross section for the radiative decay to an n-body final state is

$$\frac{d\sigma}{d\Phi_n} = \frac{1}{2}\sum_{m_J=1}^{2}\sum_{m_\gamma=1}^{2} \psi_\mu(m_J) e_\nu^*(m_\gamma) A^{\mu\nu} \psi_{\mu'}^*(m_J) \times e_{\nu'}(m_\gamma) A^{*\mu'\nu'} = -\frac{1}{2}\sum_{m_J=1}^{2} \psi_\mu(m_J)\psi_{\mu'}^*(m_J) \times g_{\nu\nu'}^{(\perp\perp)} A^{\mu\nu} A^{*\mu'\nu'} = -\frac{1}{2}\sum_{\mu=1}^{2} A_{\mu\nu} g_{\nu\nu'}^{(\perp\perp)} A^{*\mu\nu'} = -\frac{1}{2}\sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} U_i^{\mu\nu} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'} \equiv \sum_{i,j} P_{ij} \cdot F_{ij}, \quad (5)$$

where

$$P_{ij} = P_{ji}^* = \Lambda_i \Lambda_j^*, \quad (6)$$

$$F_{ij} = F_{ji}^* = -\frac{1}{2}\sum_{\mu=1}^{2} U_i^{\mu\nu} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'}, \quad (7)$$

$d\Phi_n$ is the standard element of n-body phase space given by

$$d\Phi_n(p; p_1, \cdots p_n) = \delta^4\left(p - \sum_{i=1}^{n} p_i\right) \prod_{i=1}^{n} \frac{d^3 \boldsymbol{p}_i}{(2\pi)^3 2E_i}. \quad (8)$$

The partial wave amplitudes $U_i^{\mu\nu}$ in the covariant Rarita-Schwinger tensor formalism[15] can be constructed by using pure orbital angular momentum covariant tensors $\tilde{t}^{(l)}_{\mu_1\cdots\mu_l}$ and covariant spin wave functions $\phi_{\mu_1\cdots\mu_s}$ together with metric tensor $g^{\mu\nu}$, Levi-Civita tensor $\varepsilon_{\mu\nu\lambda\sigma}$ and momenta of parent particles. For a process a→bc, the covariant tensors $\tilde{t}^{(l)}_{\mu_1\cdots\mu_l}$ for final states of pure orbital angular momentum l are constructed from relevant momenta p_a, p_b and p_c[12],

$$\tilde{t}^{(0)} = 1, \quad (9)$$

$$\tilde{t}^{(1)}_\mu = \tilde{g}_{\mu\nu}(p_a) r^\nu B_1(Q_{abc}) \equiv \tilde{r}_\mu B_1(Q_{abc}), \quad (10)$$

$$\tilde{t}^{(2)}_{\mu\nu} = \left[\tilde{r}_\mu \tilde{r}_\nu - \frac{1}{3}(\tilde{r} \cdot \tilde{r})\tilde{g}_{\mu\nu}(p_a)\right] B_2(Q_{abc}), \quad (11)$$

$$\tilde{t}^{(3)}_{\mu\nu\lambda} = \left[\tilde{r}_\mu \tilde{r}_\nu \tilde{r}_\lambda - \frac{1}{5}(\tilde{r} \cdot \tilde{r})(\tilde{g}_{\mu\nu}(p_a)\tilde{r}_\lambda + \tilde{g}_{\nu\lambda}(p_a)\tilde{r}_\mu + \tilde{g}_{\lambda\mu}(p_a)\tilde{r}_\nu)\right] B_3(Q_{abc}), \quad (12)$$

$$\tilde{t}^{(4)}_{\mu\nu\lambda\sigma} = \left[\tilde{r}_\mu \tilde{r}_\nu \tilde{r}_\lambda \tilde{r}_\sigma - \frac{1}{7}(\tilde{r} \cdot \tilde{r})(\tilde{g}_{\mu\nu}(p_a)\tilde{r}_\lambda \tilde{r}_\sigma + \tilde{g}_{\nu\lambda}(p_a)\tilde{r}_\mu \tilde{r}_\sigma + \tilde{g}_{\lambda\mu}(p_a)\tilde{r}_\nu \tilde{r}_\sigma + \tilde{g}_{\mu\sigma}(p_a)\tilde{r}_\nu \tilde{r}_\lambda + \tilde{g}_{\nu\sigma}(p_a)\tilde{r}_\lambda \tilde{r}_\nu + \tilde{g}_{\lambda\sigma}(p_a)\tilde{r}_\mu \tilde{r}_\nu) + \frac{1}{35}(\tilde{r} \cdot \tilde{r})^2(\tilde{g}_{\mu\nu}(p_a)\tilde{g}_{\lambda\sigma}(p_a) + \tilde{g}_{\nu\lambda}(p_a)\tilde{g}_{\mu\sigma}(p_a) + \tilde{g}_{\lambda\mu}(p_a)\tilde{g}_{\nu\sigma}(p_a))\right] B_4(Q_{abc}) \quad (13)$$

⋮

with relative momentum $r = p_b - p_c$; $(\tilde{r}\cdot\tilde{r}) = -\boldsymbol{r}^2$. Q_{abc} is the magnitude of $\boldsymbol{p}_b$ or $\boldsymbol{p}_c$ in the rest system of a, where

$$Q^2_{abc} = \frac{(s_a + s_b - s_c)^2}{4s_a} - s_b \qquad (14)$$

with $s_a = E^2_a - \boldsymbol{p}^2_a$. Then $\tilde{t}^{(l)}_{\mu_1\cdots\mu_l}$ contains the angular distribution function multiplied by a Blatt-Weisskopf barrier factor[12,16] $Q^l_{abc}B_l(Q_{abc})$. Explicitly

$$B_1(Q_{abc}) = \sqrt{\frac{2}{Q^2_{abc} + Q^2_0}}, \qquad (15)$$

$$B_2(Q_{abc}) = \sqrt{\frac{13}{Q^4_{abc} + 3Q^2_{abc}Q^2_0 + 9Q^4_0}}, \qquad (16)$$

$$B_3(Q_{abc}) = \sqrt{\frac{277}{Q^6_{abc} + 6Q^4_{abc}Q^2_0 + 45Q^2_{abc}Q^4_0 + 225Q^6_0}}, \qquad (17)$$

$$B_4(Q_{abc}) = \sqrt{\frac{12746}{Q^8_{abc} + 10Q^6_{abc}Q^2_0 + 135Q^4_{abc}Q^4_0 + 1575Q^2_{abc}Q^6_0 + 11025Q^8_0}}, \qquad (18)$$

here Q_0 is a hadron "scale" parameter, $Q_0 = 0.197321/R$ GeV/c, where R is the radius of the centrifugal barrier in fm. We remark that in these Blatt-Weisskopf factors, we make the approximation that the centrifugal barrier may be replaced by a square well of radius R.

Projection operators are the useful general tool in constructing covariant amplitudes. For a meson a with spin S and corresponding spin wave function $\phi_{\mu_1\cdots\mu_S}(p_a, m)$, what we usually need to use in constructing amplitudes is its spin projection operator $P^{(S)}_{\mu_1\cdots\mu_S\mu'_1\cdots\mu'_S}(p_a)$.

$$P^{(1)}_{\mu\mu'}(p_a) = \sum_m \phi_\mu(p_a, m)\phi^*_{\mu'}(p_a, m) = -g_{\mu\mu'} + \frac{p_{a\mu}p_{a\mu'}}{p^2_a} \equiv -\tilde{g}_{\mu\mu'}(p_a), \qquad (19)$$

$$P^{(2)}_{\mu\nu\mu'\nu'}(p_a) = \sum_m \phi_{\mu\nu}(p_a, m)\phi^*_{\mu'\nu'}(p_a, m) = \frac{1}{2}(\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'} + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\mu'}) - \frac{1}{3}\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}, \qquad (20)$$

$$P^{(3)}_{\mu\nu\lambda\mu'\nu'\lambda'}(p_a) = \sum_m \phi_{\mu\nu\lambda}(p_a, m)\phi^*_{\mu'\nu'\lambda'}(p_a, m) = -\frac{1}{6}(\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'}\tilde{g}_{\lambda\lambda'} + \tilde{g}_{\mu\mu'}\tilde{g}_{\nu\lambda'}\tilde{g}_{\lambda\nu'} + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\mu'}\tilde{g}_{\lambda\lambda'} + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\lambda'}\tilde{g}_{\lambda\mu'} + \tilde{g}_{\mu\lambda'}\tilde{g}_{\nu\nu'}\tilde{g}_{\lambda\mu'} + \tilde{g}_{\mu\lambda'}\tilde{g}_{\nu\mu'}\tilde{g}_{\lambda\nu'}) + \frac{1}{15}(\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}\tilde{g}_{\lambda\lambda'} + \tilde{g}_{\mu\nu}\tilde{g}_{\nu'\lambda'}\tilde{g}_{\lambda\mu'} + \tilde{g}_{\mu\nu}\tilde{g}_{\mu'\lambda'}\tilde{g}_{\lambda\nu'} + \tilde{g}_{\mu\lambda}\tilde{g}_{\mu'\lambda'}\tilde{g}_{\nu\nu'} + \tilde{g}_{\mu\lambda}\tilde{g}_{\mu'\nu'}\tilde{g}_{\nu\lambda'} + \tilde{g}_{\mu\lambda}\tilde{g}_{\nu'\lambda'}\tilde{g}_{\nu\mu'} + \tilde{g}_{\nu\lambda}\tilde{g}_{\nu'\lambda'}\tilde{g}_{\mu\mu'} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu'\nu'}\tilde{g}_{\mu\lambda'} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu'\lambda'}\tilde{g}_{\mu\nu'}), \qquad (21)$$

$$P^{(4)}_{\mu\nu\lambda\sigma\mu'\nu'\lambda'\sigma'}(p_a) = \sum_m \phi_{\mu\nu\lambda\sigma}(p_a, m)\phi^*_{\mu'\nu'\lambda'\sigma'}(p_a, m) = \frac{1}{24}[\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'}\tilde{g}_{\lambda\lambda'}\tilde{g}_{\sigma\sigma'} + \cdots(\mu', \nu', \lambda', \sigma'\text{permutation, 24 terms})] - \frac{1}{84}[\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}\tilde{g}_{\lambda\lambda'}\tilde{g}_{\sigma\sigma'} + \cdots(\mu, \nu, \lambda, \sigma\text{permutation}, \mu', \nu', \lambda', \sigma'\text{ permutation, 72 terms})] + \frac{1}{105}(\tilde{g}_{\mu\nu}\tilde{g}_{\lambda\sigma} + \tilde{g}_{\mu\lambda}\tilde{g}_{\nu\sigma} + \tilde{g}_{\mu\sigma}\tilde{g}_{\nu\lambda})(\tilde{g}_{\mu'\nu'}\tilde{g}_{\lambda'\sigma'} + \tilde{g}_{\mu'\lambda'}\tilde{g}_{\nu'\sigma'} + \tilde{g}_{\mu'\sigma'}\tilde{g}_{\nu'\lambda'}). \qquad (22)$$

Note that

$$\tilde{t}^{(L)}_{\mu_1\cdots\mu_L} = (-1)^L P^{(L)}_{\mu_1\cdots\mu_L\mu'_1\cdots\mu'_L} r^{\mu'_1}\cdots r^{\mu'_L} B_L(Q_{abc}). \qquad (23)$$

3 Covariant tensor amplitudes and corresponding angular distributions for ψ radiative decays

For the decay $\psi \to \gamma X_J$, there are two independent momenta which we choose to be p of ψ and the momentum of the photon q. We use these two momenta and spin wave functions of the three particles to construct the covariant tensor amplitudes.

For $\psi \to \gamma f_0$, the e_μ can only contract with ψ^μ since $e_\mu p^\mu = e_\mu q^\mu = 0$; hence there is only one independent amplitude for $\psi \to \gamma 0^{++}$,

$$A = \psi_\mu(m_J)e^*_\nu(m_\gamma)\Lambda_0 U^{\mu\nu}_{\gamma f_0} = \Lambda_0\psi_\mu(m_J)e^*_\nu(m_\gamma)g^{\mu\nu}. \qquad (24)$$

By use of Eqs. (24), (3) and (4) we have

$$|A|^2 = |\Lambda|^2(1 + \cos^2\theta_\gamma). \qquad (25)$$

For $\psi \to \gamma 0^{-+}$, there is only one independent amplitude,

$$A = \psi_\mu(m_J)e^*_\nu(m_\gamma)\Lambda U^{\mu\nu}_{(\gamma 0^{-+})} = \Lambda\psi_\mu e^*_\nu \varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta. \qquad (26)$$

By using Eqs. (26), (3) and (4) and $\varepsilon^{\mu\nu\alpha\beta}p_\alpha p_\beta = 0$ relations we have

$$|A|^2 = |\Lambda|^2(1 + \cos^2\theta_\gamma)E^2_\gamma M^2. \qquad (27)$$

For $\psi \to \gamma 1^{++}$ vertex, there are two independent amplitudes,

$$A_1 = \Lambda_1 \psi_\mu(m_J) e_\nu^*(m_\gamma) \chi_\alpha^*(m_f) \varepsilon^{\mu\nu\alpha\beta} p_\beta, \tag{28}$$

$$A_2 = \Lambda_2 \psi^\mu(m_J) e_\alpha^*(m_\gamma) \times \chi^{*\beta}(m_f) q_\mu q_\nu \varepsilon^{\nu\alpha\beta\sigma} p_\sigma. \tag{29}$$

Here the spin-1 wave function $\chi_\beta(m_f)$ for the particle X(1^{++}) satisfies

$$\sum_{m_f=1}^{3} \chi^\mu(m_f) \chi^{*\nu}(m_f) = -g^{\mu\nu} + \frac{K^\mu K^\nu}{K^2} \equiv -\tilde{g}^{\mu\nu}(K), \tag{30}$$

with K the 4-momentum of the X and $\chi_\mu K^\mu = 0$. By using Eqs. (28)—(30), (3), and (4), we obtain

$$|A_1|^2 = |\Lambda_1|^2 \frac{3E_f^2 - 2E_\gamma^2}{m_f^2} \times \left(1 - \frac{E_f^2 - 2E_\gamma^2}{3E_f^2 - 2E_\gamma^2}\cos^2\theta_\gamma\right) M^2, \tag{31}$$

$$|A_2|^2 = |\Lambda_2|^2 (1 - \cos^2\theta_\gamma) 2E_\gamma^4 M^2. \tag{32}$$

For $\psi \to \gamma 1^{-+}$, we have two amplitudes here,

$$A_1 = \Lambda_1 \psi_\mu(m_J) e_\nu^*(m_\gamma) g^{\mu\nu} \chi_\alpha^*(m_f) p^\alpha, \tag{33}$$

$$A_2 = \Lambda_2 \psi_\mu(m_J) e_\nu^*(m_\gamma) \chi^{*\nu}(m_f) q^\mu. \tag{34}$$

By use of Eqs. (33), (34), (3), (4), and (30) we obtain

$$|A_1|^2 = |\Lambda_1|^2 (1 + \cos^2\theta_\gamma) \frac{E_\gamma^2 M^2}{m_f^2}, \tag{35}$$

$$|A_2|^2 = |\Lambda_2|^2 (1 - \cos^2\theta_\gamma) 2E_\gamma^2. \tag{36}$$

For $\psi \to \gamma f_2(2^{++})$, there are three independent covariant tensor amplitudes,

$$A_1 = \Lambda_1 \psi_\mu(m_J) e_\nu^*(m_\gamma) F^{*\mu\nu}(m_f), \tag{37}$$

$$A_2 = \Lambda_2 \psi_\mu(m_J) e_\nu^*(m_\gamma) g^{\mu\nu} F^{*\alpha\beta}(m_f) p_\alpha p_\beta, \tag{38}$$

$$A_3 = \Lambda_3 \psi_\mu(m_J) e_\nu^*(m_\gamma) q^\mu F^{*\alpha\nu}(m_f) p_\alpha, \tag{39}$$

where spin-2 tensors $F^{\mu\nu}$ are orthogonal to the momentum, symmetric, and traceless. Namely

$$K^\mu F_{\mu\nu} = 0,\ F_{\mu\nu} = F_{\nu\mu},\ g^{\mu\nu} F_{\mu\nu} = 0. \tag{40}$$

In addition, these spin-2 tensors satisfy the relation Eq. (20). By using Eqs. (37)—(39), (3), (4), and (20), we get

$$|A_1|^2 = |\Lambda_1|^2 \frac{13E_f^2 - 7E_\gamma^2}{6m_f^2} \times \left(1 + \frac{E_f^2 - 7E_\gamma^2}{13E_f^2 - 7E_\gamma^2}\cos^2\theta_\gamma\right), \tag{41}$$

$$|A_2|^2 = |\Lambda_2|^2 (1 + \cos^2\theta_\gamma) \frac{2E_\gamma^4 M^4}{3m_f^4}, \tag{42}$$

$$|A_3|^2 = |\Lambda_3|^2 (1 - \cos^2\theta_\gamma) \frac{E_\gamma^4 M^2}{m_f^2}. \tag{43}$$

For $\psi \to \gamma 2^{-+}$, we have

$$A_1 = \Lambda_1 \psi_\mu(m_J) e_\nu^*(m_J) \times \varepsilon^{\mu\nu\alpha\beta} p_\alpha q^\lambda F_{\beta\lambda}^*(m_f), \tag{44}$$

$$A_2 = \Lambda_2 \psi_\mu(m_J) e_\nu^*(m_\gamma) \times \varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta p^\gamma p^\delta F_{\gamma\delta}^*(m_f), \tag{45}$$

$$A_3 = \Lambda_3 \psi_\mu(m_J) e_\nu^*(m_\gamma) \times \varepsilon^{\nu\gamma\alpha\beta} p_\alpha q_\beta q^\mu p^\delta F_{\gamma\delta}^*(m_f). \tag{46}$$

By using Eqs. (44)—(46), (3), (4), and (20), we get

$$|A_1|^2 = |\Lambda_1|^2 \frac{E_\gamma^2 M^4 (2E_\gamma^2 + 5m_f^2)}{3m_f^4} \left(1 + \frac{2E_\gamma^2 - m_f^2}{2E_\gamma^2 + 5m_f^2}\cos^2\theta_\gamma\right),$$

$$|A_2|^2 = |\Lambda_2|^2 (1 + \cos^2\theta_\gamma) \frac{2E_\gamma^6 M^6}{3m_f^4},$$

$$|A_3|^2 = |\Lambda_3|^2 (1 - \cos^2\theta_\gamma) \frac{E_\gamma^6 M^4}{m_f^2}. \tag{47}$$

For $\psi \to \gamma f_4(4^{++})$, there are three independent covariant tensor amplitudes,

$$A_1 = \psi_\mu(m_J) e_\nu^*(m_\gamma) \Lambda_1 U^{\mu\nu}_{(\gamma f_4)1} = \psi_\mu e_\nu^* \Lambda_1 F^{*\alpha\beta\mu\nu} p_\alpha p_\beta, \tag{48}$$

$$A_2 = \psi_\mu(m_J) e_\nu^*(m_\gamma) \Lambda_2 U^{\mu\nu}_{(\gamma f_4)2} = \psi_\mu e_\nu^* \Lambda_2 g^{\mu\nu} F^{*\alpha\beta\gamma\delta} p_\alpha p_\beta p_\gamma p_\delta, \tag{49}$$

$$A_3 = \psi_\mu(m_J) e_\nu^*(m_\gamma) \Lambda_3 U^{\mu\nu}_{(\gamma f_4)3} = \psi_\mu e_\nu^* \Lambda_3 r^\mu F^{*\alpha\beta\gamma\nu}(m_f) p_\alpha p_\beta p_\gamma. \tag{50}$$

By using Eqs. (48)—(50), (3), (4), and (22), we get

$$|A_1|^2 = |\Lambda_1|^2 \frac{E_\gamma^4 (17E_f^2 - 7E_\gamma^2)}{35m_f^6} \times \left(1 - \frac{3E_f^2 + 7E_\gamma^2}{17E_f^2 - 7E_\gamma^2}\cos^2\theta_\gamma\right) M^4, \tag{51}$$

$$|A_2|^2 = |\Lambda_2|^2 (1 + \cos^2\theta_\gamma) \frac{8E_\gamma^8 M^8}{35m_f^8}, \tag{52}$$

$$|A_3|^2 = |A_3|^2(1-\cos^2\theta_\gamma)\frac{8E_\gamma^8 M^6}{7m_f^6}. \tag{53}$$

4 Amplitudes for $\psi\to\gamma\pi^+\pi^-$ in helicity formalism and covariant tensor formalism

To illustrate the relation between the covariant tensor formalism and the helicity formalism, we take $\psi\to\gamma\pi^+\pi^-$ as an example. The same formulae work for ψ radiative decay to any two pseudoscalars. For simplicity, in the following deduction, we drop the well-known Breit-Wigner propagators for the intermediate resonances and centrifugal barrier form factors for each decay vertex.

The general formulation of helicity amplitudes can be found in Ref. [12]. The total helicity amplitude for the process

$$\psi\to\gamma f_J(0^{++},2^{++},4^{++}),\quad f_J\to\pi^+\pi^- \tag{54}$$

may be written as

$$\begin{aligned}M^1_{\lambda_J\lambda_\gamma}(\Omega_\gamma,\Omega_\pi) &= \sum_{J_f,\lambda_f} M^1_{\lambda_\gamma\lambda_f}(\Omega_\gamma;\lambda_J)M^{J_f}_{\lambda_\pi\lambda_\pi}\times \\ &(\Omega_\pi;\lambda_{J_f}) = \sum_{J_f,\lambda_f} A^1_{\lambda_\gamma\lambda_f}D^{1*}_{\lambda_J,\lambda_\gamma-\lambda_f}\times \\ &(\Omega_\gamma)A^{J_f}_{\lambda_\pi\lambda_\pi}D^{J_f*}_{\lambda_f,\lambda_\pi-\lambda_\pi}(\Omega_\pi) = \\ &\sum_{\lambda_f}A^1_{\lambda_\gamma\lambda_f}D^{1*}_{\lambda_J,\lambda_\gamma-\lambda_f}(\Omega_\gamma)D^{2*}_{\lambda_f,0}(\Omega_\pi) + \\ &\sum_{\lambda_f}B^1_{\lambda_\gamma\lambda_f}D^{1*}_{\lambda_J,\lambda_\gamma-\lambda_f}(\Omega_\gamma)D^{4*}_{\lambda_f,0}(\Omega_\pi) + \\ &C^1_{\lambda_\gamma,0}D^{1*}_{\lambda_J,\lambda_\gamma},\end{aligned} \tag{55}$$

where λ_J, λ_γ, and λ_f are helicities of ψ, γ, and f_J respectively; $A^1_{\lambda_\gamma\lambda_f}$, $B^1_{\lambda_\gamma\lambda_f}$, and $C^1_{\lambda_\gamma 0}$ are helicity coupling amplitudes for 2^{++}, 4^{++} and 0^{++}, respectively; $\Omega_\gamma=(\theta_\gamma,\phi_\gamma)$ describes the direction of the momentum $\boldsymbol{q}$ of the photon in the rest frame of vector particle ψ. $\Omega_\pi=(\theta_\pi,\phi_\pi)$ describes the direction of the 3-momentum $\boldsymbol{Q}_\pi$ of the π^+ in the rest frame of the particle f_J. The function D^J is the $(2J+1)$-dimensional representation of the rotation group, and relevant information about it are presented in the Appendix. Parity conservation of the helicity-coupling amplitude A^J leads to the following relationship,

$$A^J_{\lambda_\gamma\lambda_f} = P_\psi P_\gamma P_f(-1)^{J_\psi-J_\gamma-J_f}A^J_{-\lambda_\gamma,-\lambda_f}, \tag{56}$$

where P_ψ, P_γ, and P_f are the intrinsic parities of the particles J/ψ, γ, and f_J respectively. From Eq. (55) we may write

$$\begin{aligned}M^1_{1,1}(\Omega_\gamma,\Omega_\pi) =\ & A^1_{1,0}D^{1*}_{1,1}(\Omega_\gamma)D^{2*}_{0,0}(\Omega_\pi) + \\ & A^1_{1,1}D^{1*}_{1,0}(\Omega_\gamma)D^{2*}_{1,0}(\Omega_\pi) + \\ & A^1_{1,2}D^{1*}_{1,-1}(\Omega_\gamma)D^{2*}_{2,0}(\Omega_\pi) + \\ & B^1_{1,0}D^{1*}_{1,1}(\Omega_\gamma)D^{4*}_{0,0}(\Omega_\pi) + \\ & B^1_{1,1}D^{1*}_{1,0}(\Omega_\gamma)D^{4*}_{1,0}(\Omega_\pi) + \\ & B^1_{1,2}D^{1*}_{1,-1}(\Omega_\gamma)D^{4*}_{2,0}(\Omega_\pi) + \\ & C^0_{1,0}D^{*1}_{1,-1}(\Omega_\gamma) = \\ & \Big(A_0\frac{1+\cos\theta_\gamma}{2}\Big(\frac{3}{2}\cos^2\theta_\pi-\frac{1}{2}\Big) + \\ & A_1\frac{\sin\theta_\gamma}{\sqrt{2}}\sqrt{\frac{3}{2}}\sin\theta_\pi\cos\theta_\pi e^{i\phi_\pi} + \\ & A_2\frac{1-\cos\theta_\gamma}{2}\frac{\sqrt{6}}{4}\sin^2\theta_\pi e^{2i\phi_\pi} + \\ & C\frac{1+\cos\theta_\gamma}{2} + B_0\frac{1+\cos\theta_\gamma}{2}\times \\ & \frac{1}{8}(35\cos^4\theta_\pi-30\cos^2\theta_\pi+3) + \\ & B_1\frac{\sin\theta_\gamma}{\sqrt{2}}\frac{\sqrt{5}}{8}\sin2\theta_\pi(7\cos^2\theta_\pi-3)e^{i\phi_\pi} + \\ & B_2\frac{1-\cos\theta_\gamma}{2}\frac{1}{4}\sqrt{\frac{5}{2}}\sin^2\theta_\pi\times \\ & (7\cos^2\theta_\pi-1)e^{2i\phi_\pi}\Big)e^{i\phi_\gamma},\end{aligned} \tag{57}$$

$$\begin{aligned}M^1_{1,-1}(\Omega_\gamma,\Omega_\pi) =\ & A^1_{-1,0}D^{1*}_{1,-1}(\Omega_\gamma)D^{2*}_{0,0}(\Omega_\pi) + \\ & A^1_{-1,-1}D^{1*}_{1,0}(\Omega_\gamma)D^{2*}_{-1,0}(\Omega_\pi) + \\ & A^1_{-1,-2}D^{1*}_{1,1}(\Omega_\gamma)D^{2*}_{-2,0}(\Omega_\pi) + \\ & B^1_{-1,0}D^{1*}_{1,-1}(\Omega_\gamma)D^{4*}_{0,0}(\Omega_\pi) + \\ & B^1_{-1,-1}D^{1*}_{1,0}(\Omega_\gamma)D^{4*}_{-1,0}(\Omega_\pi) + \\ & B^1_{-1,-2}D^{1*}_{1,1}(\Omega_\gamma)D^{4*}_{-2,0}(\Omega_\pi) + \\ & C^0_{-1,0}D^{*1}_{1,-1}(\Omega_\gamma) = \\ & \Big(A_0\frac{1-\cos\theta_\gamma}{2}\Big(\frac{3}{2}\cos^2\theta_\pi-\frac{1}{2}\Big) - \\ & A_1\frac{\sin\theta_\gamma}{\sqrt{2}}\sqrt{\frac{3}{2}}\sin\theta_\pi\cos\theta_\pi e^{-i\phi_\pi} + \\ & A_2\frac{1+\cos\theta_\gamma}{2}\frac{\sqrt{6}}{4}\sin^2\theta_\pi e^{-2i\phi_\pi} + \\ & C\frac{1-\cos\theta_\gamma}{2} + B_0\frac{1-\cos\theta_\gamma}{2}\times \\ & \frac{1}{8}(35\cos^4\theta_\pi-30\cos^2\theta_\pi+3) - \\ & B_1\frac{\sin\theta_\gamma}{\sqrt{2}}\frac{\sqrt{5}}{8}\sin2\theta_\pi(7\cos^2\theta_\pi-3)e^{-i\phi_\pi} +\end{aligned}$$

$$B_2 \frac{1+\cos\theta_\gamma}{2} \frac{1}{4}\sqrt{\frac{5}{2}}\sin^2\theta_\pi \times (7\cos^2\theta_\pi - 1)e^{-2i\phi_\pi}\Big) e^{i\phi_\gamma}, \quad (58)$$

and

$$\begin{aligned} M^1_{0,1}(\Omega_\gamma,\Omega_\pi) = {} & A^1_{1,0} D^{1*}_{0,1}(\Omega_\gamma) D^{2*}_{0,0}(\Omega_\pi) + A^1_{1,1} D^{1*}_{0,0}(\Omega_\gamma) D^{2*}_{1,0}(\Omega_\pi) + \\ & A^1_{1,2} D^{1*}_{0,-1}(\Omega_\gamma) D^{2*}_{2,0}(\Omega_\pi) + B^1_{1,0} D^{1*}_{0,1}(\Omega_\gamma) D^{4*}_{0,0}(\Omega_\pi) + \\ & B^1_{1,1} D^{1*}_{0,0}(\Omega_\gamma) D^{4*}_{1,0}(\Omega_\pi) + B^1_{1,2} D^{1*}_{0,-1}(\Omega_\gamma) D^{4*}_{1,0}(\Omega_\pi) + \\ & C^1_{1,0} D^{1*}_{0,1}(\Omega_\gamma) = \\ & \Big(A_0 \frac{\sin\theta_\gamma}{\sqrt{2}}\Big(\frac{3}{2}\cos^2\theta_\pi - \frac{1}{2}\Big) - A_1\cos\theta_\gamma\sqrt{\frac{3}{2}}\sin\theta_\pi\cos\theta_\pi e^{i\phi_\pi} - \\ & A_2\frac{\sin\theta_\gamma}{\sqrt{2}}\frac{\sqrt{6}}{4}\sin^2\theta_\pi e^{2i\phi_\pi} + C\frac{\sin\theta_\gamma}{\sqrt{2}} + B_0\frac{\sin\theta_\gamma}{\sqrt{2}} \times \\ & \frac{1}{8}(35\cos^4\theta_\pi - 30\cos^2\theta_\pi + 3) - B_1\cos\theta_\gamma\frac{\sqrt{5}}{8}\sin 2\theta_\pi(7\cos^2\theta_\pi - 3)e^{i\phi_\pi} - \\ & B_2\frac{\sin\theta_\gamma}{\sqrt{2}}\frac{1}{4}\sqrt{\frac{5}{2}}\sin^2\theta_\pi \times (7\cos^2\theta_\pi - 1)e^{2i\phi_\pi}\Big) e^{i\phi_\gamma}. \end{aligned} \quad (59)$$

Above we have used $A_0 \equiv A^1_{1,0} = A^1_{-1,0}$, $A_1 \equiv A^1_{1,1} = A^1_{-1,-1}$, $A_2 \equiv A^1_{1,2} = A^1_{-1,-2}$; $B_0 \equiv B^1_{1,0} = B^1_{-1,0}$, $B_1 \equiv B^1_{1,1} = B^1_{-1,-1}$, $B_2 \equiv B^1_{1,2} = B^1_{-1,-2}$; $C \equiv C^1_{1,0} = C^1_{-1,0}$. When $f_J = 0^{++}$, there is only one independent helicity amplitude C. When $f_J = 2^{++}$, there are three independent helicity coupling amplitudes A_0, A_1, and A_2. When $f_J = 4^{++}$, there are also three independent helicity coupling amplitudes B_0, B_1, and B_2. The angular distribution of the decay process Eq. (54) can be written as

$$\begin{aligned} W(\Omega_\gamma, \Omega_\pi) \propto {} & |M^1_{1,1}(\Omega_\gamma,\Omega_\pi)|^2 + |M^1_{1,-1}(\Omega_\gamma,\Omega_\pi)|^2 + |M^1_{-1,1}(\Omega_\gamma,\Omega_\pi)|^2 + |M^1_{-1,-1}(\Omega_\gamma,\Omega_\pi)|^2 = \\ & 2(|M^1_{1,1}(\Omega_\gamma,\Omega_\pi)|^2 + |M^1_{1,-1}(\Omega_\gamma,\Omega_\pi)|^2), \end{aligned} \quad (60)$$

where $|M^1_{1,1}(\Omega_\gamma,\Omega_\pi)|^2 = |M^1_{-1,-1}(\Omega_\gamma,\Omega_\pi)|^2$ and $|M^1_{1,-1}(\Omega_\gamma,\Omega_\pi)|^2 = |M^1_{-1,1}(\Omega_\gamma,\Omega_\pi)|^2$. One can write down the explicit expression of the total angular distributions for the process Eq. (54) by inserting Eqs. (57) and (58) into Eq. (60). The angular distribution of the photon has the following form,

$$\begin{aligned} W(\cos\theta_\gamma) = {} & \int_{-1}^{+} d\cos\theta_\pi \int_0^{2\pi} d\phi_\pi W(\Omega_\gamma,\Omega_\pi) = \\ & \frac{4\pi}{5}(|A_0|^2 + 2|A_1|^2 + |A_2|^2 + 5|C|^2) \times \left(1 + \frac{|A_0|^2 - 2|A_1|^2 + |A_2|^2 + 5|C|^2}{|A_0|^2 + 2|A_1|^2 + |A_2|^2 + 5|C|^2}\cos^2\theta_\gamma\right) + \\ & \frac{4\pi}{9}(|B_0|^2 + 2|B_1|^2 + |B_2|^2) \times \left(1 + \frac{|B_0|^2 - 2|B_1|^2 + |B_2|^2}{|B_0|^2 + 2|B_1|^2 + |B_2|^2}\cos^2\theta_\gamma\right), \end{aligned} \quad (61)$$

and the angular distribution of the pion is

$$\begin{aligned} W(\cos\theta_\pi) = {} & \int_{-1}^{+} d\cos\theta_\gamma \int_0^{2\pi} d\phi_\pi W(\Omega_\gamma,\Omega_\pi) = \\ & \frac{4\pi}{3}|A_0|^2(3\cos^2\theta_\pi - 1)^2 + 8\pi|A_1|^2(1-\cos^2\theta_\pi)\cos^2\theta_\pi + \\ & 2\pi|A_2|^2(1-\cos^2\theta_\pi)^2 + \frac{16\pi}{3}|C|^2 + \frac{8\pi}{3}(A_0C^* + CA_0^*)(3\cos^2\theta_\pi - 1) + \\ & \frac{\pi}{12}|B_0|^2(35\cos^4\theta_\pi - 30\cos^2\theta_\pi + 3)^2 + \frac{5\pi}{3}|B_1|^2(1-\cos^2\theta_\pi)\cos^2\theta_\pi(7\cos^2\theta_\pi - 3)^2 + \\ & \frac{5\pi}{6}|B_2|^2(1-\cos^2\theta_\pi)^2(7\cos^2\theta_\pi - 1)^2 + \frac{2\pi}{3}(CB_0^* + B_0C^*)(35\cos^4\theta_\pi - 30\cos^2\theta_\pi + 3) + \\ & \frac{\pi}{3}(A_0B_0^* + B_0A_0^*)(3\cos^2\theta_\pi - 1) \times (35\cos^4\theta_\pi - 30\cos^2\theta_\pi + 3) + \\ & \frac{2\pi\sqrt{30}}{3}(A_1B_1^* + B_1A_1^*)(1-\cos^2\theta_\pi) \times \cos^2\theta_\pi(7\cos^2\theta_\pi - 3) + \frac{\pi\sqrt{15}}{3}(A_2B_2^* + \\ & B_2A_2^*)(1-\cos^2\theta_\pi)^2(7\cos^2\theta_\pi - 1). \end{aligned} \quad (62)$$

On the other hand, in the covariant tensor formalism, by using the covariant tensor amplitudes Eqs. (37—39), (24), (48), (49) and (50) for the decay process

Eq. (54) we have

$$A(m_J, m_\gamma) = \psi^*_\mu(m_J)e_\nu(m_\gamma)[\Lambda_1\tilde{T}^{\mu\nu}(K_{f_2}) + \Lambda_2 g^{\mu\nu}\tilde{T}^{00}(K_{f_2})M^2 + \Lambda_3(K^\mu_{f_2} - q^\mu)\tilde{T}^{0\nu}(K_{f_2})M + \Lambda_0 g^{\mu\nu} + \Lambda'_1\tilde{T}^{\mu\nu00}(K_{f_4})M^2 + \Lambda'_2 g^{\mu\nu}\tilde{T}^{0000}(K_{f_4})M^4 + \Lambda'_3(K^\mu_{f_4} - q^\mu)\tilde{T}^{\nu000}(K_{f_4})M], \quad (63)$$

where Λ_0 represents the single independent amplitude for 0^{++} process; Λ_i with $i = 1,2,3$ correspond to three independent amplitudes for 2^{++}; and Λ'_i with $i = 1,2,3$ for 4^{++}.

In the following we derive the relation between helicity coupling amplitudes ($A^1_{\lambda_\gamma,\lambda_{f_2}}$, $B^1_{\lambda_\gamma,\lambda_{f_4}}$, $C^1_{\lambda_\gamma,0}$) and covariant tensor coupling amplitudes (Λ_i, Λ'_i, Λ_0). By use of Eqs. (57)—(59), and (63) one can write

$$M^1_{1,1}(\Omega_\gamma = \Omega_\pi = 0) = A(\Omega_\gamma = \Omega_\pi = 0,1,1) = A_0 + B_0 + C = \psi^*_\mu(+)e_\nu(+)[\Lambda_1\tilde{T}^{\mu\nu}(K_{f_2}) + \Lambda_2 g^{\mu\nu}\tilde{T}^{00}(K_{f_2})M^2 + \Lambda_3(K^\mu_{f_2} - q^\mu)\tilde{T}^{0\nu}(K_{f_2})M + \Lambda_0 g^{\mu\nu} + \Lambda'_1\tilde{T}^{\mu\nu00}(K_{f_4})M^2 + \Lambda'_2 g^{\mu\nu}\tilde{T}^{0000}(K_{f_4})M^4 + \Lambda'_3(K^\mu_{f_4} - q^\mu)\tilde{T}^{\nu000}(K_{f_4})M],$$

$$M^1_{0,1}\left(\Omega_\gamma = 0, \theta_\pi = \frac{\pi}{4}, \phi_\pi = 0\right) = A\left(\Omega_\gamma = 0, \theta_\pi = \frac{\pi}{4}, \phi_\pi = 0; 0, 1,\right) = -\frac{\sqrt{6}}{4}A_1 - \frac{\sqrt{5}}{16}B_1 = \psi^*_\mu(0)e_\nu(+)[\Lambda_1\tilde{T}^{\mu\nu}(K_{f_2}) + \Lambda_2 g^{\mu\nu}\tilde{T}^{00}(K_{f_2})M^2 + \Lambda_3(K^\mu_{f_2} - q^\mu)\tilde{T}^{0\nu}(K_{f_2})M + \Lambda_0 g^{\mu\nu} + \Lambda'_1\tilde{T}^{\mu\nu00}(K_{f_4})M^2 + \Lambda'_2 g^{\mu\nu}\tilde{T}^{0000}(K_{f_4})M^4 + \Lambda'_3(K^\mu_{f_4} - q^\mu)\tilde{T}^{\nu000}(K_{f_4})M], \quad (64)$$

$$M^1_{1,-1}\left(\Omega_\gamma = 0, \theta_\pi = \frac{\pi}{2}, \phi_\pi = 0\right) = A\left(\Omega_\gamma = 0, \theta_\pi = \frac{\pi}{2}, \phi_\pi = 0; -1, 1,\right) = \frac{\sqrt{6}}{4}A_2 - \frac{\sqrt{10}}{8}B_2 = \psi^*_\mu(-)e_\nu(+)[\Lambda_1\tilde{T}^{\mu\nu}(K_{f_2}) + \Lambda_2 g^{\mu\nu}\tilde{T}^{00}(K_{f_2})M^2 + \Lambda_3(K^\mu_{f_2} - q^\mu)\tilde{T}^{0\nu}(K_{f_2})M + \Lambda_0 g^{\mu\nu} + \Lambda'_1\tilde{T}^{\mu\nu00}(K_{f_4})M^2 + \Lambda'_2 g^{\mu\nu}\tilde{T}^{0000}(K_{f_4})M^4 + \Lambda'_3(K^\mu_{f_4} - q^\mu)\tilde{T}^{\nu000}(K_{f_4})M].$$

To calculate the helicity coupling amplitudes A_0, A_1, A_2, C, B_0, B_1, and B_2, it is necessary to write down all the relevant momenta and the spin-1 polarization four vectors along the z-axis in the rest frame of the ψ,

$$p^\alpha = (M;0,0,0),$$
$$q^\alpha = (E_\gamma;0,0,E_\gamma), \quad (65)$$
$$K^\alpha_{f_J} = (E_{f_J};0,0,-E_\gamma),$$

where $M = E_\gamma + E_{f_J}$, $E^2_\gamma = |\boldsymbol{q}|^2$, $E^2_{f_J} = m^2_{f_J} + E^2_\gamma$. The relevant polarization four vectors for ψ and γ are given by

$$\psi^\alpha(\pm) = \mp\frac{1}{\sqrt{2}}(0;1,\pm i,0),$$
$$\psi^\alpha(0) = (0;0,0,1), \quad (66)$$
$$e^\alpha(\pm) = \mp\frac{1}{\sqrt{2}}(0;1,\pm i,0).$$

We determine the helicity coupling amplitudes in the rest frame of ψ. To do that we first write down $\tilde{r}^\mu$ in the rest frame of f_J, and then by making Lorentz transformation we obtain $\tilde{r}'^\mu$ in the rest frame of the parent particle J/ψ. Namely

$$\tilde{r}^\mu = \tilde{g}^{\mu\nu}(K_{f_J})r_\nu = (0,r^1,r^2,r^3) = (0;0,0,Q_\pi), \quad (67)$$

where $r^\mu = Q^\mu_{\pi^+} - Q^\mu_{\pi^-}$. By Lorentz transformation the relative momentum can be written as follows in the rest frame of the ψ,

$$\tilde{r}'^\mu_{A_0,B_0} = (\tilde{r}'^0,\tilde{r}'^1,\tilde{r}'^2,\tilde{r}'^3) = 2Q_\pi\left(-\frac{E_\gamma}{m_{f_J}},0,0,\frac{E_{f_J}}{m_{f_J}}\right),$$
$$\tilde{r}'^\mu_{A_1,B_1} = (\tilde{r}'^0,\tilde{r}'^1,\tilde{r}'^2,\tilde{r}'^3) = \sqrt{2}Q_\pi\left(-\frac{E_\gamma}{m_{f_J}},1,0,\frac{E_{f_J}}{m_{f_J}}\right),$$
$$\tilde{r}'^\mu_{A_2,B_2} = (\tilde{r}'^0,\tilde{r}'^1,\tilde{r}'^2,\tilde{r}'^3) = 2Q_\pi(0,1,0,0), \quad (68)$$

where $Q_\pi = |\boldsymbol{Q}_\pi|$. By using Eqs. (10), (11), (13) and absorbing centrifugal barrier factor into Λ_i and Λ'_i, we get

$$\tilde{T}^{(2)\mu\nu}(K_{f_2}) = \tilde{r}'^\mu\tilde{r}'^\nu + \frac{\boldsymbol{r}^2}{3}\tilde{g}^{\mu\nu}(K_{f_2}),$$

$$\tilde{T}^{\mu\nu00}(K_{f_4}) = \tilde{r}'^\mu\tilde{r}'^\nu(\tilde{r}'^0)^2 + \frac{\boldsymbol{r}^2}{7}(\tilde{g}^{\mu\nu}(\tilde{r}'^0)^2 + 2\tilde{g}^{\mu0}\tilde{r}'^\nu\tilde{r}'^0 + 2\tilde{g}^{\nu0}\tilde{r}'^\mu\tilde{r}'^0 + \tilde{g}^{00}\tilde{r}'^\mu\tilde{r}'^\nu) + \frac{\boldsymbol{r}^4}{35}(\tilde{g}^{\mu\nu}\tilde{g}^{00} + 2\tilde{g}^{\mu0}\tilde{g}^{\nu0}),$$

$$\tilde{T}^{0000}(K_{f_4}) = (\tilde{r}'^0)^4 + \frac{6\boldsymbol{r}^2}{7}\tilde{g}^{00}(\tilde{r}'^0)^2 + \frac{3\boldsymbol{r}^4}{35}(\tilde{g}^{00})^2 = \frac{128}{35}\frac{E^4_\gamma}{m^4_{f_4}}Q^4_\pi, \quad (69)$$

$$\tilde{T}^{\nu000}(K_{f_4}) = \tilde{r}'^{\nu}(\tilde{r}'^{0})^3 + \frac{3\boldsymbol{r}^2}{7}(\tilde{g}^{\nu 0}(\tilde{r}'^{0})^2 + \tilde{g}^{00}\tilde{r}'^{\nu}\tilde{r}'^{0}) + \frac{3\boldsymbol{r}^4}{35}\tilde{g}^{\nu 0}\tilde{g}^{00},$$

with

$$\tilde{g}^{00}(K_{f_J}) = g^{00} - \frac{K^0_{f_J}K^0_{f_J}}{m^2_{f_J}} = -\frac{E^2_\gamma}{m^2_{f_J}};\ \boldsymbol{r}^2 = 4Q^2_\pi. \quad (70)$$

By using Eqs. (64)—(66), (68)—(70) we obtain the relation between helicity coupling amplitudes and tensor amplitudes for 0^{++}, 2^{++}, and 4^{++} as follows,

$$A_0 + C + B_0 = \left(-\Lambda_1 - 2\Lambda_2\frac{E^2_\gamma}{m^2_{f_2}}M^2\right)\frac{4}{3}Q^2_\pi - \Lambda_0 + \left(-\Lambda'_1 - 2\Lambda'_2\frac{E^2_\gamma}{m^2_{f_4}}M^2\right)\frac{64}{35}\frac{E^2_\gamma}{m^2_{f_4}}M^2Q^4_\pi, \quad (71)$$

$$-\frac{\sqrt{6}}{4}A_1 - \frac{\sqrt{5}}{16}B_1 = \left(-\Lambda_1\frac{E_{f_2}}{m_{f_2}} - 2\Lambda_3\frac{E^2_\gamma}{m_{f_2}}M\right)\sqrt{2}Q^2_\pi + \left(-5\Lambda'_1\frac{E_{f_4}}{m_{f_4}} - 2\Lambda'_3\frac{E^2_\gamma}{m_{f_4}}M\right)\times \frac{2\sqrt{2}}{7}\frac{E^2_\gamma}{m^2_{f_4}}M^2Q^4_\pi, \quad (72)$$

$$\frac{\sqrt{6}}{4}A_2 - \frac{\sqrt{10}}{8}B_2 = -2\Lambda_1Q^2_\pi + \frac{8}{7}\Lambda'_1\frac{E^2_\gamma}{m^2_{f_4}}M^2Q^4_\pi. \quad (73)$$

From Eqs. (71)—(73) we may write

$$C = \Lambda_0,$$

$$A_0 = \left(-\Lambda_1 - 2\Lambda_2\frac{E^2_\gamma}{m^2_{f_2}}M^2\right)\frac{4}{3}Q^2_\pi,$$

$$A_1 = \left(\Lambda_1\frac{E_{f_2}}{m_{f_2}} + 2\Lambda_3\frac{E^2_\gamma}{m_{f_2}}M\right)\frac{4\sqrt{3}}{3}Q^2_\pi,$$

$$A_2 = -\Lambda_1\frac{4\sqrt{6}}{3}Q^2_\pi, \quad (74)$$

$$B_0 = \left(-\Lambda'_1 - 2\Lambda'_2\frac{E^2_\gamma}{m^2_{f_4}}M^2\right)\frac{64}{35}\frac{E^2_\gamma}{m^2_{f_4}}M^2Q^4_\pi,$$

$$B_1 = \left(5\Lambda'_1\frac{E_{f_4}}{m_{f_4}} + 2\Lambda'_3\frac{E^2_\gamma}{m_{f_4}}M\right)\frac{32\sqrt{10}}{35}\frac{E^2_\gamma}{m^2_{f_4}}M^2Q^4_\pi,$$

$$B_2 = -\Lambda'_1\frac{32\sqrt{10}}{35}\frac{E^2_\gamma}{m^2_{f_4}}M^2Q^4_\pi.$$

From these relations, after one gets the covariant tensor amplitudes, one can easily get the corresponding helicity amplitudes, and vice versa.

5 Conclusion

The covariant tensor amplitudes and the corresponding angular distributions have been calculated for the processes $\psi\to\gamma0^{++}$, $\gamma0^{-+}$, $\gamma1^{++}$, $\gamma1^{-+}$, $\gamma2^{++}$, $\gamma2^{-+}$, $\gamma4^{++}$. The full amplitude for the process $\psi\to\gamma\{0^{++}, 2^{++}, 4^{++}\}\to\gamma\pi\pi$ is given in both helicity formalism and covariant tensor formalism. The corresponding angular distributions of photon and pion are given in Eqs. (61) and (62), respectively. The relations between covariant tensor amplitudes and helicity amplitudes are given in Eqs. (74). Except some smooth energy dependent factors caused by the requirement of Lorentz covariance in the covariant tensor formalism, the covariant tensor formalism is equivalent to the helicity formalism. For the bin-by-bin fit[8] with energy interval small enough, the smooth energy dependence could be ignored and the two formalisms are equivalent.

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Appendix A

Rotation matrix

In this appendix we review some of the properties of rotation matrix[17] which are relevant to our studies. The elements of the rotation matrix are then given by

$$D^J_{m'm}(\alpha,\beta,\gamma) = \langle Jm' \mid R(\alpha,\beta,\gamma) \mid Jm\rangle = \langle Jm' \mid e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z} \mid Jm\rangle = e^{-i\alpha m'} d^J_{m'm}(\beta) e^{-i\gamma m}, \quad (A1)$$

where J is the spin of the resonance, R denotes the rotation through Euler angles α,β,γ, and d^J is defined to be the matrix representative of the rotation β about the y-axis. Namely,

$$d^J_{m'm}(\beta) = \langle Jm' \mid e^{-i\beta J_y} \mid Jm\rangle. \quad (A2)$$

The following general formula for the d is

$$d^J_{m'm}(\theta) = \sum_n \frac{(-1)^n \sqrt{(J+m)!(J-m)!(J+m')!(J-m')!}}{(J-m'-n)!(J+m-n)!(n+m'-m)!n!} = \left(\cos\frac{1}{2}\theta\right)^{2J+m-m'-2n}\left(-\sin\frac{1}{2}\theta\right)^{m'-m+2n}. \quad (A3)$$

The symmetry relation for the d function is

$$d^J_{m'm}(\theta) = (-1)^{m'-m} d^J_{-m',-m}(\theta) = (-1)^{m'-m} d^J_{mm'}(\theta). \quad (A4)$$

By using Eqs. (A3) and (A4) we have

$$\begin{aligned}
D^1_{1,1}(\Omega) &= e^{-i\phi} d^1_{1,1}(\theta) = \frac{1+\cos\theta}{2} e^{-i\phi},\\
D^1_{1,-1}(\Omega) &= e^{-i\phi} d^1_{1,-1}(\theta) = \frac{1-\cos\theta}{2} e^{-i\phi},\\
D^1_{1,0}(\Omega) &= e^{-i\phi} d^1_{1,0}(\theta) = -\frac{\sin\theta}{\sqrt{2}} e^{-i\phi},\\
D^2_{0,0}(\Omega) &= d^2_{0,0}(\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2},\\
D^2_{1,0}(\Omega) &= e^{-i\phi} d^2_{1,0}(\theta) = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta e^{-i\phi},\\
D^2_{-1,0}(\Omega) &= e^{i\phi} d^2_{-1,0}(\theta) = -e^{i\phi} d^2_{1,0}(\theta) = \sqrt{\frac{3}{2}}\sin\theta\cos\theta e^{i\phi},\\
D^1_{0,0}(\Omega) &= \cos\theta, \qquad (A5)\\
D^2_{2,0}(\Omega) &= e^{-2i\phi} d^2_{2,0}(\theta) = \frac{\sqrt{6}}{4}\sin^2\theta e^{-2i\phi},\\
D^2_{-2,0}(\Omega) &= e^{2i\phi} d^2_{-2,0}(\theta) = \frac{\sqrt{6}}{4}\sin^2\theta e^{2i\phi},\\
D^4_{0,0}(\Omega) &= \frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3),\\
D^4_{1,0}(\Omega) &= e^{-i\phi} d^4_{1,0}(\theta) = -\frac{\sqrt{5}}{8}\sin 2\theta(7\cos^2\theta - 3)e^{-i\phi},\\
D^4_{-1,0}(\Omega) &= e^{i\phi} d^4_{-1,0}(\theta) = \frac{\sqrt{5}}{8}\sin 2\theta(7\cos^2\theta - 3)e^{i\phi},\\
D^4_{2,0}(\Omega) &= e^{-2i\phi} d^4_{-1,0}(\theta) = \frac{1}{4}\sqrt{\frac{5}{2}}\sin^2\theta(7\cos^2\theta - 1)e^{-2i\phi},\\
D^4_{-2,0}(\Omega) &= e^{2i\phi} d^4_{-1,0}(\theta) = \frac{1}{4}\sqrt{\frac{5}{2}}\sin^2\theta(7\cos^2\theta - 1)e^{2i\phi},
\end{aligned}$$

here

$$D^J_{m'm}(\Omega) \equiv D^J_{m'm}(\phi,\theta,0) = e^{-im'\phi} d^J_{m'm}(\theta). \quad (A6)$$

ψ辐射衰变的角分布*

沙依甫加马力·达吾来提[2,3;1)] 邹冰松[1,2;2)] 吴济民[1,2;3)]

1(中国高等技术中心 北京 100080)
2(中国科学院高能物理研究所 北京 100039)
3(新疆大学物理系 乌鲁木齐 830046)

摘要 J/ψ和ψ′粒子辐射衰变到介子的过程是寻找胶球,混杂态,抽取gg-q$\bar{q}$耦合的极佳场所. 北京正负电子对撞机(BEPC)已经积累了大量的J/ψ和ψ′事例,即将改进升级的BEPC及美国的CLEO-c还将获取更多的事例. 这里首先利用协变张量方法提供了该类过程的角分布公式,然后利用螺旋度方法给出了ψ辐射衰变到两个赝标介子的角分布公式,以此为例讨论了螺旋度方法和协变张量方法的关系.

关键词 ψ辐射衰变 角分布 振幅

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1) E-mail: sayipjamal@mail.ihep.ac.cn; 2) E-mail: zoubs@mail.ihep.ac.cn; 3) E-mail: wujm@mail.ihep.ac.cn