

# Effects of the Little Higgs Models on Single Top Production at the $e^- \gamma$ Colliders<sup>\*</sup>

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**Abstract** In the framework of the littlest Higgs(LH) model and the littlest Higgs model with  $T$ -parity(LHT), we investigate the single top production process  $e^- \gamma \rightarrow \nu_e b \bar{t}$ , and calculate the corrections of these two models to the cross section of this process. We find that in the reasonable parameter space, the correction terms for the tree-level  $Wtb$  couplings coming from the LHT model can generate significantly corrections to the cross section of this process, which might be detected in the future high energy linear  $e^+e^-$  collider(ILC) experiments. However, the contributions of the new gauge boson  $W_H^\pm$  predicted by the LH model to this process are very small.

**Key words** littlest Higgs model,  $T$ -parity, cross section

Recently, the little Higgs model offers a very promising solution to the hierarchy problem in which the Higgs boson is naturally light as a result of nonlinearly realized symmetry<sup>[1]</sup>. The key feature of this model is that the Higgs boson is a pseudo-Goldstone boson of an approximate global symmetry which is spontaneously broken by a vacuum expectation value(VEV) at a scale of a few TeV and thus is naturally light. The most economical little Higgs model is the so-called littlest Higgs model, which is based on a  $SU(5)/SO(5)$  nonlinear sigma model<sup>[2]</sup>. It consists of a  $SU(5)$  global symmetry, which is spontaneously broken down to  $SO(5)$  by a vacuum condensate  $f$ . In this model, a set of new heavy gauge bosons ( $B_H, Z_H, W_H$ ) and a new heavy-vector-like quark( $T$ ) are introduced which just cancel the quadratic divergence induced by the SM gauge boson loops and the top quark loop, respectively. Furthermore, these new particles might produce characteristic signatures at

the present and future collider experiments<sup>[3, 4]</sup>.

It has been shown that the LH model suffers from severe constraints from the precision electroweak measurement, which would require raising the mass of new particles to be much higher than 1TeV<sup>[5]</sup>. To avoid this problem,  $T$ -parity is introduced into the LH model, which is called LHT model<sup>[6]</sup>. Under  $T$ -parity, the SM particles are  $T$ -even and most of the new heavy particles are  $T$ -odd. Thus, the SM gauge bosons can-not mix with the new gauge bosons, and the electroweak precision observables are not modified at tree level. In the top-quark sector, the LHT model contains a  $T$ -odd and  $T$ -even partner of the top quark. The  $T$ -even partner of the top quark cancels the quadratic divergence contribution of top quark to Higgs boson mass and mixes with top quark. It has been shown that the loop corrections to precision electroweak observables are much small and the scale parameter  $f$  can be decreased to 500GeV<sup>[6, 7]</sup>.

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Thus, this model can produce rich phenomenology in the present and future experiments.

The top quark is by far the heaviest known fermion with a mass of the order of the electroweak scale  $m_t=172.7 \pm 2.9\text{GeV}^{[8]}$ . Assuming this is not a coincidence, it is hoped that a detailed study of top quark couplings to other particles will be of utility in clarifying whether the SM provides the correct mechanism for electroweak symmetry-breaking, or whether new physics is responsible. It is therefore of interest to provide a general description of the top quark couplings, which might be modified due to the presence of new interactions or particles.

Future linear colliders are expected to be designed to function also as  $\gamma\gamma$  or  $e\gamma$  colliders with the photon beams generated by laser-scattering method, in these modes the flexibility in polarizing both lepton and photon beams will allow unique opportunities to analyze the top quark properties and interactions. The aim of this paper is to consider the process  $e^- \gamma \rightarrow \nu_e b \bar{t}$  in the context of the LH model and the LHT model, respectively, and see whether the effects of these two models on this process can be detected in the future ILC experiments.

In the LH model, the couplings constants of the SM gauge boson  $W$  and the new heavy gauge boson  $W_H$  to ordinary particles, which are related to our calculation, can be written as<sup>[3]</sup>:

$$g_V^{\text{Wev}} = -g_A^{\text{Wev}} = \frac{ie}{2\sqrt{2}s_W} \left[ 1 - \frac{v^2}{2f^2} c^2 (c^2 - s^2) \right], \quad (1)$$

$$g_V^{\text{WHev}} = -g_A^{\text{WHev}} = -\frac{ie}{2\sqrt{2}s_W} \frac{c}{s}, \quad (2)$$

$$g_V^{\text{Wtb}} = -g_A^{\text{Wtb}} = \frac{ie}{2\sqrt{2}s_W} \left[ 1 - \frac{v^2}{2f^2} (x_L^2 + c^2 (c^2 - s^2)) \right], \quad (3)$$

$$g_V^{\text{W_Htb}} = -g_A^{\text{W_Htb}} = -\frac{ie}{2\sqrt{2}s_W} \frac{c}{s}, \quad (4)$$

where  $f$  is the scalar parameter,  $v=246\text{GeV}$  is the electroweak scale,  $s_W$  represents the sine of the weak mixing angle, and  $c$  is the mixing parameter between  $SU(2)_1$  and  $SU(2)_2$  gauge bosons with  $s = \sqrt{1 - c^2}$ .  $x_L$  is the mixing parameter between the SM top quark  $t$  and the vector-like top quark  $T$ , which is defined as  $x_L = \lambda_1^2 / (\lambda_1^2 + \lambda_2^2)$ ,  $\lambda_1$  and  $\lambda_2$  are the Yukawa couplings

parameters. We write the gauge boson-fermion couplings in the form of  $i\gamma^\mu (g_V + g_A \gamma^5)$ .

Compared with the process  $e^- \gamma \rightarrow \nu_e b \bar{t}$  in the SM, this process in the LH model receives additional contributions from the heavy boson  $W_H^\pm$  proceed through the Feynman diagrams depicted in Fig. 1. Furthermore, the modification of the relations among the SM parameters, the precision electroweak input parameters, the correction terms to the SM  $W e \nu_e$  and  $W b t$  coupling can also produce corrections to this process.

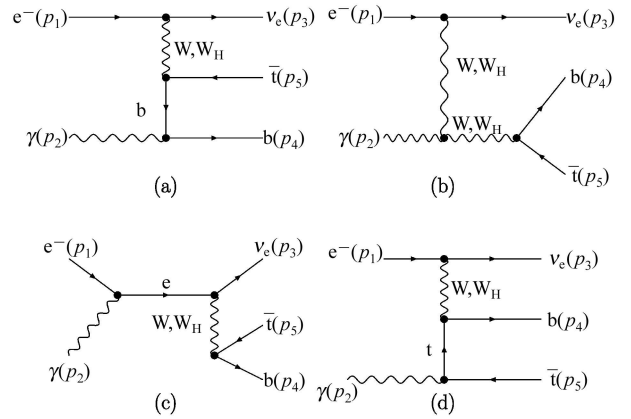


Fig. 1. Feynman diagrams of the process  $e^- \gamma \rightarrow \nu_e b \bar{t}$  in the LH model.

In order to write a compact expression for the amplitudes, it is necessary to define the triple-boson couplings coefficient as:

$$\Gamma^{\alpha\beta\gamma}(p_1, p_2, p_3) = g^{\alpha\beta}(p_1 - p_2)^\gamma + g^{\beta\gamma}(p_2 - p_3)^\alpha + g^{\gamma\alpha}(p_3 - p_1)^\beta, \quad (5)$$

with all momenta out-going.

The invariant production amplitudes of the process in the LH model can be written as:

$$M = M_a + M_b + M_c + M_d, \quad (6)$$

with

$$\begin{aligned} M_a &= \bar{u}(p_3) g_V^{\text{Wev}} \gamma_\mu (1 - \gamma_5) u(p_1) \{ G(p_3 - p_1, M_W) + \\ &\quad \frac{c^2}{s^2} G(p_3 - p_1, M_{W_H}) \} \times \\ &\quad g^{\mu\nu} \bar{u}(p_4) g^{\gamma b \bar{b}} \gamma_\rho G(p_4 - p_2, m_b) \times \\ &\quad g_V^{\text{Wtb}} \gamma_\nu (1 - \gamma_5) v(p_5) \varepsilon^\rho(p_2), \end{aligned} \quad (7)$$

$$\begin{aligned}
M_b = & -\bar{u}(p_3)g_V^{\text{W}^{\text{ev}}}\gamma_\mu(1-\gamma_5)u(p_1)G^{\mu\nu\rho} \times \\
& (p_3-p_1, -p_2, p_4+p_5)\{G(p_3-p_1, M_W) \times \\
& G(p_4+p_5, M_W) + \\
& \frac{c^2}{s^2}G(p_3-p_1, M_{\text{WH}})G(p_4+p_5, M_{\text{WH}})\} \\
& \bar{u}(p_4)g_V^{\text{W}^{\text{tb}}}\gamma_\nu(1-\gamma_5)v(p_5)\varepsilon^\rho(p_2), \quad (8)
\end{aligned}$$

$$\begin{aligned}
M_c = & \bar{u}(p_4)g_V^{\text{W}^{\text{tb}}}\gamma_\mu(1-\gamma_5)v(p_5)\{G(p_4+p_5, M_W) + \\
& \frac{c^2}{s^2}G(p_4+p_5, M_{\text{WH}})\}g^{\mu\nu}\bar{u}(p_3)g_V^{\text{W}^{\text{ev}}}\gamma_\nu(1-\gamma_5) \times \\
& G(p_1+p_2, 0)g^{\gamma\text{e}\bar{\text{e}}}\gamma_\rho u(p_1)\varepsilon^\rho(p_2), \quad (9)
\end{aligned}$$

$$\begin{aligned}
M_d = & \bar{u}(p_3)g_V^{\text{W}^{\text{ev}}}\gamma_\mu(1-\gamma_5)u(p_1)\{G(p_3-p_1, M_W) + \\
& \frac{c^2}{s^2}G(p_3-p_1, M_{\text{WH}})\}g^{\mu\nu}\bar{u}(p_4)g_V^{\text{W}^{\text{tb}}}\gamma_\nu(1-\gamma_5) \times \\
& G(p_2-p_5, m_t)g^{\gamma\text{t}\bar{\text{t}}}\gamma_\rho v(p_5)\varepsilon^\rho(p_2), \quad (10)
\end{aligned}$$

where  $G(p, m) = 1/(p^2 - m^2)$  denotes the propagator of the particle.

The hard photon beam of the  $e\gamma$  collider can be obtained from laser backscattering at the  $e^+e^-$  linear collider. Let  $\hat{s}$  and  $s$  be the center-of-mass energies of the  $e\gamma$  and  $e^+e^-$  systems, respectively. After calculating the cross section  $\sigma(\hat{s})$  for the subprocess  $e^-\gamma \rightarrow \nu_e b \bar{t}$ , the total cross section at the  $e^+e^-$  linear collider can be obtained by folding  $\sigma(\hat{s})$  with the photon distribution function that is given in Ref. [9]:

$$\sigma(\text{tot}) = \int_{(M_t+M_b)^2/s}^{x_{\text{max}}} dx \sigma(\hat{s}) f_\gamma(x), \quad (11)$$

where

$$f_\gamma(x) = \frac{1}{D(\xi)} \left[ 1-x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2} \right], \quad (12)$$

with

$$D(\xi) = \left( 1 - \frac{4}{\xi} - \frac{8}{\xi^2} \right) \ln(1+\xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1+\xi)^2}. \quad (13)$$

In the above equation,  $\xi = 4E_e\omega_0/m_e^2$  in which  $m_e$  and  $E_e$  stand, respectively, for the incident electron mass and energy,  $\omega_0$  stands for the laser photon energy, and  $x = \omega/E_e$  stands for the fraction of energy of the incident electron carried by the backscattered photon.  $f_\gamma$  vanishes for  $x > x_{\text{max}} = \omega_{\text{max}}/E_e = \xi/(1+\xi)$ . In order to avoid the creation of  $e^+e^-$  pairs by the interaction of the incident and backscattered photons, we require  $\omega_0 x_{\text{max}} \leq m_e^2/E_e$ , which implies that  $\xi \leq 2+2\sqrt{2} \simeq 4.8$ .

For the choice of  $\xi = 4.8$ , we obtain

$$x_{\text{max}} \approx 0.83, \quad D(\xi_{\text{max}}) \approx 1.8. \quad (14)$$

For simplicity, we have ignored the possible polarization for the electron and photon beams.

With the above production amplitudes, we can obtain the production cross section directly. In the calculation of the cross section, instead of calculating the square of the amplitudes analytically, we calculate the amplitudes numerically by using the method of the references<sup>[10]</sup> which can greatly simplify our calculation.

In our numerical results, we take the input parameters as  $M_t = 172.7\text{GeV}$ <sup>[8]</sup>,  $\alpha_e = 1/128.8$ ,  $M_Z = 91.187\text{GeV}$ ,  $s_W^2 = 0.2315$  and  $m_W = 80.45\text{GeV}$ <sup>[11]</sup>. The value of the relative correction parameter is insensitive to the degree of the electron and positron polarization and the c.m. energy  $\sqrt{s}$ . Therefore, we do not consider the polarization of the initial states and take  $\sqrt{s} = 500\text{GeV}$  in our numerical calculation. Except for these SM input parameters, the contributions of the LH model to single top quark production are dependent on the free parameters ( $f$ ,  $c$ ,  $x_L$ ). Considering the constraints of the electroweak precision data on these free parameters, we will assume  $f = 1 \sim 2\text{TeV}$ ,  $0.3 \leq x_L \leq 0.6$ ,  $0 < c \leq 0.5$  for the LH model<sup>[12]</sup>. The relative correction of the LH model to the cross section of single top production is in the expression of the relative correction parameter  $\delta\sigma/\sigma^{\text{SM}}$  with  $\delta\sigma = |\sigma^{\text{tot}} - \sigma^{\text{SM}}|$  and  $\sigma^{\text{SM}}$  is the tree-level cross section of  $e^-\gamma \rightarrow \nu_e b \bar{t}$  production predicted by the SM. The numerical results are summarized in Fig. 2.

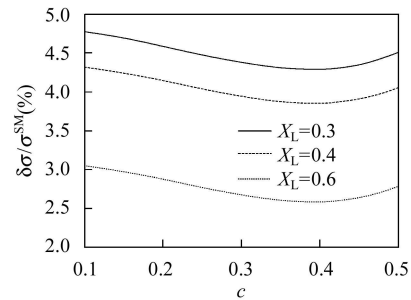


Fig. 2. The relative correction  $\delta\sigma/\sigma^{\text{SM}}$  as a function of the mixing parameter  $c$  for  $f=1\text{TeV}$  and three values of the mixing parameter  $x_L$  in LH model.

In the LH model, the extra contributions to the

process  $e^- \gamma \rightarrow \nu_e b \bar{t}$  come from the heavy gauge boson  $W_H^\pm$ , the modification of the relations among the SM parameters and the precision electroweak input parameters, and the correction terms of the SM  $W_{\text{tb}}$  and  $W_{\text{tb}}$  couplings. From Fig. 2, we can see that the absolute values of the relative correction  $\delta\sigma/\sigma^{\text{SM}}$  are smaller than 5% in most of the parameter space preferred by the electroweak precision data, which is difficult to be detected.

Under  $T$ -parity of the LH model, the couplings of the electroweak gauge boson to light fermions are not modified from their corresponding SM couplings at tree level. Nonetheless, the  $W_{\text{tb}}$  coupling is modified at tree level by the mixing of the top quark with its  $T$ -even partner<sup>[6, 7]</sup>. The expression of the coupling  $W_{\text{tb}}$  can be written as:

$$g_V^{\text{Wtb}} = -g_A^{\text{Wtb}} = \frac{ie}{2\sqrt{2}s_W} \left[ 1 - \frac{c_\lambda^4}{2} \frac{v^2}{f^2} \right], \quad (15)$$

where the mixing parameter  $c_L = \lambda_1 / \sqrt{(\lambda_1^2 + \lambda_2^2)}$ , in which  $\lambda_1$  and  $\lambda_2$  are the Yukawa couplings parameters in the LHT model.

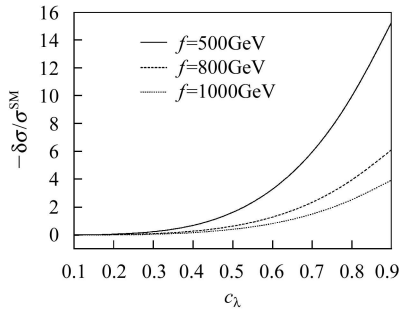


Fig. 3. The relative correction  $\delta\sigma/\sigma^{\text{SM}}$  as a function of the mixing parameters  $c_\lambda$  for three values of the scale parameter  $f$  in LHT model.

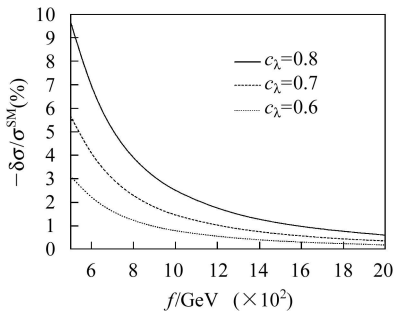


Fig. 4. The relative correction  $\delta\sigma/\sigma^{\text{SM}}$  as a function of the the scale parameter for three values of the mixing parameters  $c_\lambda$  in LHT model.

From the above discussions, we can see that the LHT model can also generate corrections to the production cross sections for the process  $e^- \gamma \rightarrow \nu_e b \bar{t}$  via the modification of the coupling  $W_{\text{tb}}$ . The value of the relative correction parameter  $\delta\sigma/\sigma^{\text{SM}}$  depends on two free parameters  $f$  and  $c_\lambda$  in the LHT model. Considering the parameter space of  $f$  and  $c_\lambda$  constrained by Ref. [7], we take  $0.1 \leq c_\lambda \leq 0.9$  and  $500\text{GeV} \leq f \leq 2000\text{GeV}$ . The relative correction parameters generated by the LHT model to the cross section of single top production at the  $e\gamma$  collider are shown in Fig. 3 and Fig. 4. In these figures, we have taken  $\delta\sigma = \sigma^{\text{LHT}} - \sigma^{\text{SM}}$ . From Fig. 3, we can see that the absolute value of the relative correction increases with an increase of the mixing parameter  $c_\lambda$ . As long as  $f \leq 800\text{GeV}$  and  $c_\lambda \geq 0.7$ , in sizable regions of the parameter space in the LHT model, the absolute value of the relative correction  $\delta\sigma/\sigma^{\text{SM}}$  is larger than 5%, which might be detected in the future ILC experiments. To see the effect of varying the scale parameter  $f$  on the relative correction  $\delta\sigma/\sigma^{\text{SM}}$ , we plot  $\delta\sigma/\sigma^{\text{SM}}$  as a function of  $f$  for three values of the mixing parameter  $c_\lambda$  in Fig. 4. One can see from Fig. 4 that the absolute value of the relative correction  $\delta\sigma/\sigma^{\text{SM}}$  decreases as  $f$  increase, which is consistent with the conclusions for the corrections of the LH model and LHT model to other observables<sup>[13]</sup>.

The little Higgs model, which can solve the hierarchy problem, is a promising alternative new physics model. The LH model is one of the simplest and phenomenologically viable models, which realizes the little Higgs idea. In order to provide a valuable theoretical instruction to test the little Higgs idea, people have done a lot of phenomenological work within the context of the little Higgs models. In this paper, we have considered single top production process  $e^- \gamma \rightarrow \nu_e b \bar{t}$  in the LH model and the LHT model. We find that the contribution of the LH model to this process is very small in most of the parameter space, which is difficult to be detected in future ILC experiments. However, in sizable regions of the parameter space in the LHT model, the absolute value of the relative correction  $\delta\sigma/\sigma^{\text{SM}}$  is larger than 5%, which might be detected in the future ILC experiments.

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## 小 Higgs 模型对顶夸克在 $e\gamma$ 对撞机上产生的影响\*

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**摘要** 在最小 Higgs 模型和  $T$  宇称下的最小 Higgs 模型中, 计算了单个顶夸克的产生过程  $e\gamma \rightarrow \nu_e b\bar{t}$  的产生截面, 并讨论了这两种模型对该过程产生截面的贡献. 发现在合适的参数范围内, 来自  $T$  宇称下的最小 Higgs 模型中对  $Wtb$  耦合的修正项对该过程产生截面的相对修正大于 5%, 这种影响能在将来的直线对撞机实验中探测到. 然而, 在最小 Higgs 模型中, 新的规范玻色子  $W_{\frac{1}{2}}^{\pm}$  对该过程的贡献非常小.

**关键词** 最小 Higgs 模型  $T$  宇称 产生截面

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