Model with strong γ_4 T-violation*

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Abstract We extend the T violating model of the paper on "Hidden symmetry of the CKM and neutrino-mapping matrices" by assuming its T-violating phases $\chi_\uparrow$ and $\chi_\downarrow$ to be large and the same, with $\chi=\chi_\uparrow=\chi_\downarrow$. In this case, the model has 9 real parameters: $\alpha_\uparrow$, $\beta_\uparrow$, $\xi_\uparrow$, $\eta_\uparrow$ for the $\uparrow$-quark sector, $\alpha_\downarrow$, $\beta_\downarrow$, $\xi_\downarrow$, $\eta_\downarrow$ for the $\downarrow$ sector and a common χ. We examine whether these nine parameters are compatible with ten observables: the six quark masses and the four real parameters that characterize the CKM matrix (i.e., the Jarlskog invariant $\mathcal{J}$ and three Eulerian angles). We find that this is possible only if the T violating phase χ is large, between $-120°$ to $-135°$. In this strong T violating model, the smallness of the Jarlskog invariant $\mathcal{J}\cong 3\times 10^{-5}$ is mainly accounted for by the large heavy quark masses, with $\frac{m_c}{m_t}<\frac{m_s}{m_b}\approx 0.02$, as well as the near complete overlap of t and b quark, with $(c|b)$=-0.04.

Key words Jarlskog invariant, CKM matrix, strong γ_4 T-violation

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1 Introduction

In a previous paper on the "Hidden symmetry of the CKM and neutrino-mapping matrices" [1], we have posited a mass-generating Hamiltonian $H_\uparrow+H_\downarrow$ where

$$H_\uparrow=\alpha_\uparrow|q_3^\uparrow-\xi_\uparrow q_2^\uparrow|^2+\beta_\uparrow|q_2^\uparrow-\eta_\uparrow q_1^\uparrow|^2+\beta_\uparrow|q_3^\uparrow-\xi_\uparrow\eta_\uparrow q_1^\uparrow|^2$$
$$H_\downarrow=\alpha_\downarrow|q_3^\downarrow-\xi_\downarrow q_2^\downarrow|^2+\beta_\downarrow|q_2^\downarrow-\eta_\downarrow q_1^\downarrow|^2+\beta_\downarrow|q_3^\downarrow-\xi_\downarrow\eta_\downarrow q_1^\downarrow|^2 \quad (1.1)$$

with α, β, ξ, η real. This conserves T and leads to zero masses for the light quarks u and d. We then modified (1.1) by replacing $\xi_\uparrow$, $\xi_\downarrow$ with the corresponding T violating factors $\xi_\uparrow e^{i\chi_\uparrow}$ and $\xi_\downarrow e^{i\chi_\downarrow}$. To first order in $\chi_\uparrow$ and $\chi_\downarrow$ we found a relation of proportionality between $\mathcal{J}$, the Jarlskog invariant measuring T-violation, and a linear combination of square roots of the light masses. The ratio agreed roughly with known values. We shall call this the "weak γ_4-model" because to make the calculation we assumed $\chi_\uparrow$, $\chi_\downarrow$ to be small.

There were two reasons for dissatisfaction with this model. First, why not introduce the phase factor into η or $\xi\eta$, yielding different physics? And second, when we estimated not only $\mathcal{J}$ but the individual matrix elements of $U_{\rm CKM}$, we found that the data required $\chi_\uparrow$ and $\chi_\downarrow$ to be large angles, not small.

We now present a new model, the "strong γ_4-model". In this model we introduce phase factors independently into all three terms, but require them to have the same values in $H_\uparrow$ and $H_\downarrow$. Thus we take the mass-generating Hamiltonian to be $H_\uparrow+H_\downarrow$ where

$$\begin{aligned}H_\uparrow=&\ \alpha_\uparrow|q_3^\uparrow-\xi_\uparrow e^{i\rho}q_2^\uparrow|^2+\beta_\uparrow|q_2^\uparrow-\eta_\uparrow e^{i\omega}q_1^\uparrow|^2+\\&\beta_\uparrow|q_3^\uparrow-\xi_\uparrow\eta_\uparrow e^{-i\tau}q_1^\uparrow|^2\\H_\downarrow=&\ \alpha_\downarrow|q_3^\downarrow-\xi_\downarrow e^{i\rho}q_2^\downarrow|^2+\beta_\downarrow|q_2^\downarrow-\eta_\downarrow e^{i\omega}q_1^\downarrow|^2+\\&\beta_\downarrow|q_3^\downarrow-\xi_\downarrow\eta_\downarrow e^{-i\tau}q_1^\downarrow|^2\end{aligned} \quad . \quad (1.2)$$

It is now easily seen that the masses and CKM matrix depend on the phases only through the sum $\chi=\rho+\omega+\tau$. Accordingly, without loss of generality, we set $\rho=\omega=0$, $\tau=\chi$. The mass-generating Hamiltonian can then be written as

$$\left(\bar q_1^\uparrow,\ \bar q_2^\uparrow,\ \bar q_3^\uparrow\right)M_\uparrow\begin{pmatrix}q_1^\uparrow\\q_2^\uparrow\\q_3^\uparrow\end{pmatrix}+\left(\bar q_1^\downarrow,\ \bar q_2^\downarrow,\ \bar q_3^\downarrow\right)M_\downarrow\begin{pmatrix}q_1^\downarrow\\q_2^\downarrow\\q_3^\downarrow\end{pmatrix}$$

where $q_i^\uparrow$, $q_i^\downarrow$ and $\bar q_i^\uparrow$, $\bar q_i^\downarrow$ are related to the corresponding Dirac field operators $\psi(q_i(\uparrow))$, $\psi(q_i(\downarrow))$ and their hermitian conjugate $\psi^\dagger(q_i(\uparrow))$, $\psi^\dagger(q_i(\downarrow))$ by

$$q_i^{\uparrow/\downarrow}=\psi(q_i(\uparrow/\downarrow)) \quad\text{and}\quad \bar q_i^{\uparrow/\downarrow}=\psi^\dagger(q_i(\uparrow/\downarrow))\gamma_4, \quad (1.3)$$

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$$M_{\uparrow/\downarrow}=\begin{pmatrix}\beta\eta^2(1+\xi^2) & -\beta\eta & -\beta\xi\eta e^{i\chi}\\ -\beta\eta & \beta+\alpha\xi^2 & -\alpha\xi\\ -\beta\xi\eta e^{-i\chi} & -\alpha\xi & \alpha+\beta\end{pmatrix}_{\uparrow/\downarrow}, \quad (1.4)$$

with the arrow-subscripts $\uparrow$, $\downarrow$ referring to α, β, ξ, η, but not to χ.

In diagonalizing (1.4) we do not assume, as in the weak γ_4-model, that χ is small. We find that the smallness of $\mathcal{J}$ is mainly accounted for by the large heavy masses with

$$\frac{m_{\rm c}}{m_{\rm t}}<\frac{m_{\rm s}}{m_{\rm b}}\approx 0.02 \quad (1.5)$$

and by the nearly complete overlap of the statevectors for t and b since

$$|(u|b)|<|(c|b)|\cong 0.04. \quad (1.6)$$

We have been able to carry out complete calculations in which the only approximations are based on the smallness of $\frac{m_{\rm s}}{m_{\rm b}}$, $\frac{m_{\rm c}}{m_{\rm t}}$ and $(c|b)$. These calculations are described in Sections 2 and 3; we give here a brief outline.

We diagonalize $M_\uparrow$ and $M_\downarrow$ with the aid of parameters $r_{\uparrow,\downarrow}$, $B_{\uparrow,\downarrow}$, $\varPhi_{\uparrow,\downarrow}$, $\mathcal{S}$, $\mathcal{L}$ to be defined in the next two sections. They are shown there to satisfy the following ten equations (to first order in small quantities):

$$\frac{1-r_\uparrow^2}{r_\uparrow^2}\sin^2 B_\uparrow=\frac{4m_{\rm u}m_{\rm c}}{(m_{\rm c}-m_{\rm u})^2}, \quad (1.7)$$

$$\frac{1-r_\downarrow^2}{r_\downarrow^2}\sin^2 B_\downarrow=\frac{4m_{\rm d}m_{\rm s}}{(m_{\rm s}-m_{\rm d})^2}, \quad (1.8)$$

$$\sin^2\frac{1}{2}\chi=\frac{1-r_\uparrow^2}{\sin^2 2\varPhi_\uparrow}=\frac{1-r_\downarrow^2}{\sin^2 2\varPhi_\downarrow}, \quad (1.9)$$

$$\mathcal{L}=\frac{\sqrt{m_{\rm s}m_{\rm d}}}{m_{\rm b}}-\frac{\sqrt{m_{\rm c}m_{\rm u}}}{m_{\rm t}}, \quad (1.10)$$

$$\mathcal{S}=\sin(\varPhi_\uparrow-\varPhi_\downarrow)=(c|b), \quad (1.11)$$

$$|(u|b)+\mathcal{S}\sin\frac{1}{2}B_\uparrow|^2=\mathcal{L}^2\cos^2\frac{1}{2}B_\uparrow, \quad (1.12)$$

$$\mathrm{Im}(u|b)=-\mathcal{L}\frac{\cos\frac{1}{2}B_\uparrow\cos\frac{1}{2}\chi}{r_\uparrow} \quad (1.13)$$

and

$$(u|s)=\sin\frac{1}{2}(B_\downarrow-B_\uparrow). \quad (1.14)$$

Our strategy of solution is as follows. We take $m_{\rm s}$, $m_{\rm c}$, $m_{\rm b}$, $m_{\rm t}$, as well as $(u|s)$, $(u|b)$ and $(c|b)$, to be given from data (see Table 1). Then we have eleven unknowns $r_{\uparrow,\downarrow}$, $B_{\uparrow,\downarrow}$, $\varPhi_{\uparrow,\downarrow}$, $\mathcal{S}$, $\mathcal{L}$, χ, $m_{\rm d}$, $m_{\rm u}$ constrained by ten independent equations given above (with (1.9) and (1.11), each counted as two equations). Taking a trial value of $\sin\frac{1}{2}B_\uparrow$, we are able to solve numerically for the other ten unknowns by a self-correcting double iteration that converges to 4 decimal stability after $36=6\times6$ passes. We find that $m_{\rm u}$ is particularly sensitive to variations in $\sin\frac{1}{2}B_\uparrow$; a variation of 30% in the latter carries $m_{\rm u}$ through the whole of its experimental range from 1.5 to 3.0 MeV/c^2. Meanwhile $m_{\rm d}$ varies by only 25%, from 5.2 to 6.5 MeV/c^2, well within the experimental range, 3.0 to 8.0 MeV/c^2. The value of χ must be taken as negative and is in the neighborhood of $-125°$, between $-120°$ and $-135°$. We have also tried deviations in $m_{\rm s}$, $m_{\rm b}$, $(c|b)$, Re$(u|b)$ and Im$(u|b)$. Only in the case of $m_{\rm s}$ does it appear that a maximal deviation (-25%) from the "best value" might push $m_{\rm d}$ outside the range given by data. (See Tables 1 and 2, and Fig. 1).

Table 1*.

Parameter	"Best" value
$m_{\rm s}$	95 MeV
$m_{\rm b}$	4.5 GeV
$(c\|b)$	0.04
Re$(u\|b)$	0.002
Im$(u\|b)$	-0.003

* These values are used to obtain the top two rows in Table 2.

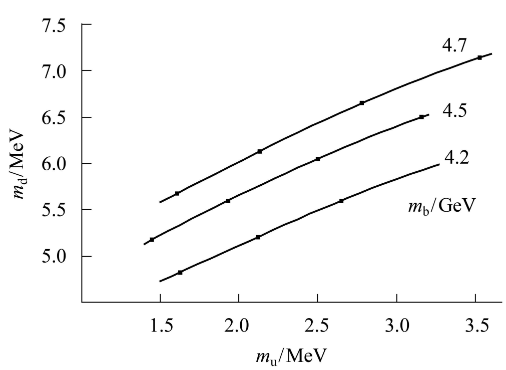

Fig. 1. $m_{\rm d}$ versus $m_{\rm u}$ for $m_{\rm s}=95$ MeV, $(c|b)=0.04$, $(u|b)=0.002-0.003i$ and $m_{\rm b}=4.2$ GeV, 4.5 GeV and 4.7 GeV.

The next two sections are devoted to defining the parameters that appear in (1.7)—(1.14) and proving that these equations are satisfied. In Section 2, we discuss the separate diagonalization of $M_\uparrow$ and $M_\downarrow$, and in Section 3, we examine the CKM matrix.

In Section 4, we discuss briefly a third model[2], which we may call a $i\gamma_5$ model, because its Hamiltonian contains a term in $i\gamma_4\gamma_5$ as well as the usual one in γ_4.

Table 2. Values of m_u, m_d and χ calculated from the strong γ_4-model*.

Input parameters		m_u/MeV	m_d/MeV	$\cos\frac{1}{2}\chi$
As in Table 1		1.45	5.18	0.487
		3.16	6.50	0.428
Table 1 except	$m_\mathrm{s}=85$ MeV	1.39	5.43	0.479
		3.29	6.86	0.418
Table 1 except	$m_\mathrm{s}=105$ MeV	1.52	5.00	0.490
		3.09	6.22	0.433
Table 1 except	$m_\mathrm{b}=4.2$ GeV	1.63	4.83	0.483
		3.33	6.02	0.427
Table 1 except	$m_\mathrm{b}=4.7$ GeV	1.61	5.68	0.476
		3.53	7.14	0.417
Table 1 except	$(c\|b)=0.035$	1.40	4.86	0.507
		2.98	5.96	0.454
Table 1 except	$(c\|b)=0.045$	1.51	5.52	0.468
		3.36	7.07	0.405
Table 1 except	$\mathrm{Re}(u\|b)=0.0015$	1.63	4.74	0.525
		3.33	5.96	0.463
Table 1 except	$\mathrm{Re}(u\|b)=0.0025$	1.72	6.09	0.432
		2.96	7.06	0.397
Table 1 except	$\mathrm{Im}(u\|b)=-0.0025$	1.64	4.93	0.428
		2.75	5.81	0.389
Table 1 except	$\mathrm{Im}(u\|b)=-0.0035$	1.73	5.96	0.510
		2.93	6.83	0.473

*The values of five input parameters are taken as in Table 1, except for single departures as shown in the left-hand column here. For each setting of the input parameters, there is a one-parameter family of solutions of Eqs. (1.7)—(1.14). We show two members of each family, chosen roughly to span the experimental range of m_u from 1.5 to 3.0 MeV. The corresponding values of m_d stay within its experimental range from 3 to 8 MeV, and χ remains large from $-120°$ to $-135°$.

2 Diagonalization of $M_\uparrow$ and $M_\downarrow$

In this section, we shall drop the arrow-subscripts and write (1.4) as

$$M=\begin{pmatrix} T^2\beta & -T\beta\cos\Phi & -T\beta\sin\Phi e^{i\chi} \\ -T\beta\cos\Phi & \alpha\tan^2\Phi+\beta & -\alpha\tan\Phi \\ -T\beta\sin\Phi e^{-i\chi} & -\alpha\tan\Phi & \alpha+\beta \end{pmatrix}, \tag{2.1}$$

where

$$\Phi=\tan^{-1}\xi\ , \tag{2.2}$$

$$T=\eta\sqrt{1+\xi^2}\ , \tag{2.3}$$

so that $T^2\beta=\beta\eta^2(1+\xi^2)$, $\sin\Phi=\xi/\sqrt{1+\xi^2}$, $\cos\Phi=1/\sqrt{1+\xi^2}$ and (2.1)=(1.4). We denote the eigenvalues of M by m_l, m_m, m_h (light, medium, heavy), and seek a unitary matrix $\boldsymbol{W}$ (with $\boldsymbol{W}\boldsymbol{W}^\dagger=1$) such that

$$M=\boldsymbol{W}\begin{pmatrix} m_\mathrm{l} & 0 & 0 \\ 0 & m_\mathrm{m} & 0 \\ 0 & 0 & m_\mathrm{h}\end{pmatrix}\boldsymbol{W}^\dagger. \tag{2.4}$$

The W matrix will be built up in stages, as we shall discuss. First we isolate the heavy mass by writing

$$M=\Omega\left(\begin{array}{c:c} (\boldsymbol{n}) & \begin{matrix} L \\ 0 \end{matrix} \\ \hdashline \begin{matrix} L^* & 0 \end{matrix} & \mu_\mathrm{h} \end{array}\right)\Omega^\dagger\ , \tag{2.5}$$

where

$$\Omega^\dagger=\left(\begin{array}{c:cc} 1 & 0 & 0 \\ \hdashline 0 & \multicolumn{2}{c}{\multirow{2}{*}{$e^{\mathrm{i}\Phi\tau_y}$}} \\ 0 & & \end{array}\right), \tag{2.6}$$

$$\mu_\mathrm{h}=\alpha\sec^2\Phi+\beta\ , \tag{2.7}$$

$$L=T\beta\cos\Phi\sin\Phi(1-e^{i\chi}) \tag{2.8}$$

and

$$(\boldsymbol{n})=\beta\begin{pmatrix} T^2 & -T(\cos^2\Phi+\sin^2\Phi e^{\mathrm{i}\chi}) \\ -T(\cos^2\Phi+\sin^2\Phi e^{-\mathrm{i}\chi}) & 1 \end{pmatrix}. \tag{2.9}$$

Thus, (2.1) can be obtained by a simple substitution of (2.6)—(2.9) into (2.5).

Next, we diagonalize the 2×2 matrix $(\boldsymbol{n})$ of (2.9) by setting

$$\cos^2\Phi+\sin^2\Phi e^{\mathrm{i}\chi}=re^{\mathrm{i}A}\ , \tag{2.10}$$

with r, A both real. Then

$$(\boldsymbol{n})=\beta\begin{pmatrix} T^2 & -Tre^{\mathrm{i}A} \\ -Tre^{-\mathrm{i}A} & 1 \end{pmatrix}= e^{\frac{1}{2}\mathrm{i}\tau_z A}e^{-\frac{1}{2}\mathrm{i}\tau_y B}\begin{pmatrix} \mu_l & 0 \\ & \mu_m \end{pmatrix}e^{\frac{1}{2}\mathrm{i}\tau_y B}e^{-\frac{1}{2}\mathrm{i}\tau_z A}\ , \tag{2.11}$$

provided that

$$\begin{aligned} \mu_\mathrm{m}+\mu_\mathrm{l}&=\beta(1+T^2), \\ (\mu_\mathrm{m}-\mu_\mathrm{l})\cos B&=\beta(1-T^2), \\ (\mu_\mathrm{m}-\mu_\mathrm{l})\sin B&=2\beta Tr. \end{aligned} \tag{2.12}$$

By quadratic combination of (2.12) we obtain

$$\mu_{\rm m}\mu_{\rm l}=\beta^2T^2(1-r^2); \tag{2.13}$$

then, by dividing the above equation by the square of the last line of (2.12), we have

$$\frac{4\mu_{\rm m}\mu_{\rm l}}{(\mu_{\rm m}-\mu_{\rm l})^2}=\frac{1-r^2}{r^2}\sin^2 B\ , \tag{2.14}$$

which leads to (1.7) and (1.8).

Also, by applying the Law of Sines to the complex triangle described by (2.10), followed by trigonometric identities, we find

$$\cos\left(\frac{1}{2}\chi-A\right)=\frac{\cos\frac{1}{2}\chi}{r}, \tag{2.15}$$

a relation that will be useful later.

Applying (2.11) to (2.5), we now have

$$M=\Omega V\times\begin{pmatrix}\mu_{\rm l} & 0 & L\Delta^*\cos\frac{1}{2}B\\ 0 & \mu_{\rm m} & -L\Delta^*\sin\frac{1}{2}B\\ L^*\Delta\cos\frac{1}{2}B & -L^*\Delta\sin\frac{1}{2}B & \mu_{\rm h}\end{pmatrix}V^\dagger\Omega^\dagger, \tag{2.16}$$

where

$$\Delta={\rm e}^{\frac{1}{2}{\rm i}A} \tag{2.17}$$

and

$$V^\dagger=\left(\begin{array}{cc|c}\multicolumn{2}{c|}{\left({\rm e}^{\frac{1}{2}{\rm i}\tau_yB}{\rm e}^{-\frac{1}{2}{\rm i}\tau_zA}\right)} & \begin{matrix}0\\0\end{matrix}\\ \hline 0 & 0 & 1\end{array}\right). \tag{2.18}$$

Thus M is almost diagonalized. Let us study the magnitude of L. From (2.13) and (2.10) we find

$$\mu_{\rm m}\mu_{\rm l}=\beta^2T^2(1-r^2)=2\beta^2T^2(1-\cos\chi)\cos^2\Phi\sin^2\Phi \tag{2.19}$$

and comparing this with (2.8) we have

$$|L|=2|T\beta\cos\Phi\sin\Phi\sin\frac{1}{2}\chi|=\sqrt{\mu_{\rm m}\mu_{\rm l}}. \tag{2.20}$$

Hence, if we write

$$\begin{pmatrix}\mu_{\rm l} & 0 & L\Delta^*\cos\frac{1}{2}B\\ 0 & \mu_{\rm m} & -L\Delta^*\sin\frac{1}{2}B\\ L^*\Delta\cos\frac{1}{2}B & -L^*\Delta\sin\frac{1}{2}B & \mu_{\rm h}\end{pmatrix}= P\begin{pmatrix}m_{\rm l} & 0 & 0\\ 0 & m_{\rm m} & 0\\ 0 & 0 & m_{\rm h}\end{pmatrix}P^\dagger \tag{2.21}$$

the elements of P will differ from those of the unit matrix by $O\left[\frac{\sqrt{m_{\rm l}m_{\rm m}}}{m_{\rm h}}\right]\ll 1$. A careful examination shows that all the m's may be approximated by μ's; in particular, we also have $\left|\frac{\mu_{\rm l}}{m_{\rm l}}-1\right|\sim O\left[\frac{m_{\rm m}}{m_{\rm h}}\right]$. Therefore (2.14) becomes

$$\frac{4m_{\rm m}m_{\rm l}}{(m_{\rm m}-m_{\rm l})^2}=\frac{1-r^2}{r^2}\sin^2 B \tag{2.22}$$

and (1.7) and (1.8) are established.

Also, (1.9) is a direct consequence of (2.13) and (2.20). We may take (1.10) as the definition of $\mathcal{L}$, and from (2.20) we may write it as

$$\mathcal{L}=\frac{|L_\downarrow|}{m_{\rm b}}-\frac{|L_\uparrow|}{m_{\rm t}}. \tag{2.23}$$

The first equality of (1.11) is the definition of $\mathcal{S}$. Thus what remains is to establish the second part of (1.11), and (1.12)—(1.14). This requires studying the CKM matrix which relates "up" to "down" eigenstates, as we shall see.

3 The CKM matrix

In this section we restore the arrow subscripts $\uparrow$, $\downarrow$. On account of (2.16) and (2.21), the matrix $\boldsymbol{W}$ defined in (2.4) is given by

$$\boldsymbol{W}^\dagger_{\uparrow,\downarrow}=P^\dagger_{\uparrow,\downarrow}V^\dagger_{\uparrow,\downarrow}\Omega^\dagger_{\uparrow,\downarrow}. \tag{3.1}$$

If we define

$$U=\boldsymbol{W}^\dagger_\uparrow\boldsymbol{W}_\downarrow=P^\dagger_\uparrow U_0P_\downarrow\ , \tag{3.2}$$

where

$$U_0=V^\dagger_\uparrow\Omega^\dagger_\uparrow\Omega_\downarrow V_\downarrow=\left(\begin{array}{cc|c}\multicolumn{2}{c|}{\left({\rm e}^{\frac{1}{2}{\rm i}\tau_yB_\uparrow}{\rm e}^{-\frac{1}{2}{\rm i}\tau_zA_\uparrow}\right)} & \begin{matrix}0\\0\end{matrix}\\ \hline 0 & 0 & 1\end{array}\right)\times \left(\begin{array}{c|cc}1 & 0 & 0\\ \hline \begin{matrix}0\\0\end{matrix} & \multicolumn{2}{c}{{\rm e}^{{\rm i}(\Phi_\uparrow-\Phi_\downarrow)\tau_y}}\end{array}\right)\left(\begin{array}{cc|c}\multicolumn{2}{c|}{\left({\rm e}^{\frac{1}{2}{\rm i}\tau_zA_\downarrow}{\rm e}^{-\frac{1}{2}{\rm i}\tau_yB_\downarrow}\right)} & \begin{matrix}0\\0\end{matrix}\\ \hline 0 & 0 & 1\end{array}\right), \tag{3.3}$$

then U transforms eigenstates of $M_\downarrow$ into eigenstates of $M_\uparrow$, provided that the phases of the eigenstates are suitably chosen. To obtain the CKM matrix $U_{\rm CKM}$, which relates eigenstates whose phases follow a standard convention, we shall need an additional transformation

$$U_{\rm CKM}=Q^\dagger_\uparrow UQ_\downarrow\ , \tag{3.4}$$

where $Q_{\uparrow,\downarrow}$ are diagonal unitary matrices to be chosen presently.

In evaluating (3.3) it is convenient to introduce new symbols:

$$\delta=\Delta_\uparrow\Delta^*_\downarrow={\rm e}^{\frac{1}{2}{\rm i}(A_\uparrow-A_\downarrow)}, \tag{3.5}$$

$$\Gamma=\cos\frac{1}{2}B_\uparrow,\quad \gamma=\cos\frac{1}{2}B_\downarrow, \tag{3.6}$$

$$\Sigma=\sin\frac{1}{2}B_{\uparrow},\quad \sigma=\sin\frac{1}{2}B_{\downarrow}, \tag{3.7}$$

$$\mathcal{S}=\sin(\Phi_{\uparrow}-\Phi_{\downarrow}) \text{ and } C=\cos(\Phi_{\uparrow}-\Phi_{\downarrow}). \tag{3.8}$$

We note that the first equation in (3.8) is the same in (1.11). By using (3.5)—(3.8), we find U_0 of (3.3) can be written as

$$U_0=\begin{pmatrix} \delta^*\Gamma\gamma+C\delta\Sigma\sigma & -\delta^*\Gamma\sigma+C\delta\Sigma\gamma & \mathcal{S}\Delta_{\uparrow}\Sigma \\ -\delta^*\Sigma\gamma+C\delta\Gamma\sigma & \delta^*\Sigma\sigma+C\delta\Gamma\gamma & \mathcal{S}\Delta_{\uparrow}\Gamma \\ -\mathcal{S}\Delta_{\downarrow}^*\sigma & -\mathcal{S}\Delta_{\downarrow}^*\gamma & C \end{pmatrix}. \tag{3.9}$$

The next step is to prepare for a perturbative treatment of (3.2) by writing

$$P_{\uparrow,\downarrow}\cong I+p_{\uparrow,\downarrow}\ , \tag{3.10}$$

where (in arrowless notation)

$$p^{\dagger}=\frac{1}{m_{\rm h}}\begin{pmatrix} 0 & 0 & -\Delta^*L\cos\frac{1}{2}B \\ 0 & 0 & \Delta^*L\sin\frac{1}{2}B \\ \Delta L^*\cos\frac{1}{2}B & -\Delta L^*\sin\frac{1}{2}B & 0 \end{pmatrix}. \tag{3.11}$$

We note that by putting (3.11) into (3.10), we can satisfy (2.21) to first order in L.

Thus we have

$$U\cong U_0+U'\ , \tag{3.12}$$

where

$$U'=p_{\uparrow}^{\dagger}U_0+U_0p_{\downarrow}. \tag{3.13}$$

Let us carefully evaluate the lower left element of $p_{\uparrow}^{\dagger}U_0$:

$$\begin{aligned}(p_{\uparrow}^{\dagger}U_0)_{31}=&\ \frac{1}{m_{\rm t}}(L_{\uparrow}^*\Delta_{\uparrow}\cos\tfrac{1}{2}B_{\uparrow})(\delta^*\Gamma\gamma+C\delta\Sigma\sigma)+\\ &\frac{1}{m_{\rm t}}\left(-L_{\uparrow}^*\Delta_{\uparrow}\sin\frac{1}{2}B_{\uparrow}\right)(-\delta^*\Sigma\gamma+C\delta\Gamma\sigma)=\\ &\frac{L_{\uparrow}^*\Delta_{\uparrow}}{m_{\rm t}}\big[\Gamma(\delta^*\Gamma\gamma+C\delta\Sigma\sigma)+\\ &\Sigma(\delta^*\Sigma\gamma-C\delta\Gamma\sigma)\big]=\\ &\frac{L_{\uparrow}^*\Delta_{\uparrow}}{m_{\rm t}}\delta^*(\Gamma^2+\Sigma^2)\gamma=\frac{L_{\uparrow}^*}{m_{\rm t}}\Delta_{\downarrow}\gamma.\end{aligned} \tag{3.14}$$

(Note how the calculation converts $\Delta_{\uparrow}$ to $\Delta_{\downarrow}$ and Γ to γ.) The corresponding element of $U_0p_{\downarrow}$ is trivial:

$$(U_0p_{\downarrow})_{31}=C\left(\frac{1}{m_{\rm b}}\Delta_{\downarrow}^*L_{\downarrow}\cos\frac{1}{2}B_{\downarrow}\right)^*=-\frac{L_{\downarrow}^*}{m_{\rm b}}\Delta_{\downarrow}\gamma C. \tag{3.15}$$

Anticipating that $B_{\uparrow}$ will turn out fairly small, ~ 0.2, we now observe that the matrix element U_{23} is going to be dominated by $(U_0)_{23}=\mathcal{S}\Delta_{\uparrow}\Gamma\sim\mathcal{S}\Delta_{\uparrow}$. Therefore, $\mathcal{S}$ must have magnitude $\sim .04$. It follows that $C\sim 1-\frac{1}{2}\mathcal{S}^2$ can be replaced by 1, and that all elements of U' other than $(U')_{13,23,31,32}$ being of order $\mathcal{S}\cdot\frac{\sqrt{m_{\rm d}m_{\rm s}}}{m_{\rm b}}$, can be neglected.

Thus, by repeating for $(U')_{13,23,32}$ the calculations leading to (3.14) and (3.15), we have

$$U'\cong\begin{pmatrix} 0 & 0 & +\left(\frac{L_{\downarrow}}{m_{\rm b}}-\frac{L_{\uparrow}}{m_{\rm t}}\right)\Delta_{\uparrow}\Gamma \\ 0 & 0 & -\left(\frac{L_{\downarrow}}{m_{\rm b}}-\frac{L_{\uparrow}}{m_{\rm t}}\right)\Delta_{\uparrow}\Sigma \\ -\left(\frac{L_{\downarrow}^*}{m_{\rm b}}-\frac{L_{\uparrow}^*}{m_{\rm t}}\right)\Delta_{\downarrow}\gamma & +\left(\frac{L_{\downarrow}^*}{m_{\rm b}}-\frac{L_{\uparrow}^*}{m_{\rm t}}\right)\Delta_{\downarrow}\sigma & 0 \end{pmatrix}. \tag{3.16}$$

But from (2.8), taking T,β , $\cos\Phi$, $\sin\Phi$ positive, we find

$$\frac{L_{\downarrow}}{|L_{\downarrow}|}=\frac{L_{\uparrow}}{|L_{\uparrow}|}=\frac{1-{\rm e}^{{\rm i}\chi}}{|1-{\rm e}^{{\rm i}\chi}|} \tag{3.17}$$

and so

$$\frac{L_{\downarrow}}{m_{\rm b}}-\frac{L_{\uparrow}}{m_{\rm t}}=\frac{1-{\rm e}^{{\rm i}\chi}}{|1-{\rm e}^{{\rm i}\chi}|}\mathcal{L} \tag{3.18}$$

by (2.23). We now anticipate that χ will have to be negative in order to make everything come out right. Hence,

$$\frac{1-{\rm e}^{{\rm i}\chi}}{|1-{\rm e}^{{\rm i}\chi}|}=\frac{{\rm e}^{\frac{1}{2}{\rm i}\chi}\left(-2{\rm i}\sin\frac{1}{2}\chi\right)}{|2\sin\frac{1}{2}\chi|}=+{\rm i}{\rm e}^{\frac{1}{2}{\rm i}\chi} \tag{3.19}$$

and (3.16) leads to

$$U'\simeq\begin{pmatrix} 0 & 0 & +{\rm i}{\rm e}^{\frac{1}{2}{\rm i}\chi}\mathcal{L}\Delta_{\downarrow}^*\Gamma \\ 0 & 0 & -{\rm i}{\rm e}^{\frac{1}{2}{\rm i}\chi}\mathcal{L}\Delta_{\downarrow}^*\Sigma \\ +{\rm i}{\rm e}^{-\frac{1}{2}{\rm i}\chi}\mathcal{L}\Delta_{\downarrow}\gamma & -{\rm i}{\rm e}^{-\frac{1}{2}{\rm i}\chi}\mathcal{L}\Delta_{\downarrow}\sigma & 0 \end{pmatrix}. \tag{3.20}$$

For reasons shortly to be evident, let us now introduce the phase factors

$$\varepsilon_{\uparrow,\downarrow}=-{\rm i}{\rm e}^{\frac{1}{2}{\rm i}\chi}(\Delta_{\uparrow,\downarrow}^*)^2={\rm e}^{-\frac{{\rm i}\sigma}{2}}{\rm e}^{{\rm i}(\frac{1}{2}\chi-A_{\uparrow,\downarrow})}. \tag{3.21}$$

Then we have

$$U'=\begin{pmatrix} 0 & 0 & -\varepsilon_{\uparrow}\mathcal{L}\Delta_{\uparrow}\Gamma \\ 0 & 0 & +\varepsilon_{\uparrow}\mathcal{L}\Delta_{\uparrow}\Sigma \\ +\varepsilon_{\downarrow}^*\mathcal{L}\Delta_{\downarrow}^*\gamma & -\varepsilon_{\downarrow}^*\mathcal{L}\Delta_{\downarrow}^*\sigma & 0 \end{pmatrix}. \tag{3.22}$$

In treating (3.9), let us note that since $\Phi_\uparrow - \Phi_\downarrow \approx \sin^{-1}\mathcal{S}$ is small, $A_\uparrow - A_\downarrow$ is also small by (2.10). Hence $|\mathrm{Im}\delta|$ is small (see(3.5)) and $1-\mathrm{Re}\delta$ is second order. So $\mathrm{Re}\delta$ can be taken $=1$, and the imaginary parts of $(U_0)_{11,12,21,22}$ can be adjusted by small adjustments in $Q_\uparrow, Q_\downarrow$. We shall treat such adjustments imprecisely and simply neglect these imaginary parts. By taking $C\to 1$ and using (3.6)—(3.7), we find

$$\begin{pmatrix} (U_0)_{11} & (U_0)_{12} \\ (U_0)_{21} & (U_0)_{22} \end{pmatrix} = \begin{pmatrix} \Gamma\gamma+\Sigma\sigma & -\Gamma\sigma+\Sigma\gamma \\ -\Sigma\gamma+\Gamma\sigma & \Sigma\sigma+\Gamma\gamma \end{pmatrix} = \begin{pmatrix} \cos\frac{1}{2}(B_\downarrow - B_\uparrow) & -\sin\frac{1}{2}(B_\downarrow - B_\uparrow) \\ \sin\frac{1}{2}(B_\downarrow - B_\uparrow) & \cos\frac{1}{2}(B_\downarrow - B_\uparrow) \end{pmatrix}. \tag{3.23}$$

Now $B_\downarrow - B_\uparrow$ must be positive to fit U_{13} and U_{31}, and so U_{12} is negative, whereas the standard presentation gives $(U_{\mathrm{CKM}})_{12}$ positive. Therefore, we shall use the Q-transformation to change the sign of the first row and column, and also to remove the factors $\Delta_\uparrow$, $\Delta_\downarrow^*$ now appearing in the third row and column. Thus

$$Q_\uparrow^\dagger = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \Delta_\downarrow \end{pmatrix}, \quad Q_\downarrow^\dagger = \begin{pmatrix} +1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \Delta_\uparrow^* \end{pmatrix} \tag{3.24}$$

and

$$U_{\mathrm{CKM}} = Q_\uparrow^\dagger U_0 Q_\downarrow + Q_\uparrow^\dagger U' Q_\downarrow = \begin{pmatrix} \cos\frac{1}{2}(B_\downarrow - B_\uparrow) & \sin\frac{1}{2}(B_\downarrow - B_\uparrow) & -\mathcal{S}\Sigma + \varepsilon_\uparrow\mathcal{L}\Gamma \\ -\sin\frac{1}{2}(B_\downarrow - B_\uparrow) & \cos\frac{1}{2}(B_\downarrow - B_\uparrow) & \mathcal{S}\Gamma + \varepsilon_\uparrow\mathcal{L}\Sigma \\ \mathcal{S}\sigma - \varepsilon_\downarrow^*\mathcal{L}\gamma & -\mathcal{S}\gamma - \varepsilon_\downarrow^*\mathcal{L}\sigma & 1 \end{pmatrix}, \tag{3.25}$$

where we have again allowed a slight imprecision of phase in the (3,3) element.

Comparing (3.25) with the array

$$U_{\mathrm{CKM}} = \begin{pmatrix} (u|d) & (u|s) & (u|b) \\ (c|d) & (c|s) & (c|b) \\ (t|d) & (t|s) & (t|b) \end{pmatrix}, \tag{3.26}$$

we obtain the second half of (1.11) and (1.12)—(1.14).

Note: there is an ambiguity, $\Phi_{\uparrow,\downarrow} >$ or $< \frac{\pi}{4}$. We take both Φ's$> \frac{\pi}{4}$, so that $|A| > |\chi - A|$ and hence $|A| > |\frac{1}{2}\chi|$. Since χ and A are negative, $\frac{1}{2}\chi - A > 0$ and hence $\mathrm{Re}\,\varepsilon_{\uparrow,\downarrow} > 0$, as required in $(u|b)$ and $(t|d)$. Because $\mathrm{Im}\,\varepsilon_\uparrow = -\cos\left(\frac{1}{2}\chi - A\right)$, we can then derive (1.13) by using (2.15).

4 The "Timeon" model

The merit of the "strong γ_4 T-violation model" examined in this paper suggests that there may be large T-violation somewhere in physics although its manifestation in the quark mass sector is small. In the "strong γ_4 T-violation model" the T-violating effects are produced by the phase χ which enters non-linearly into the Hamiltonian. This non-linear interaction makes it difficult to construct a renormalizable quantum field theory that can be extended beyond the mass matrix. For this and other reasons, we have considered a different model[3] in which the T-violating effect enters linearly; therefore, the model can lead to a renormalizable field theory, called "timeon".

In the timeon theory, the mass-generating Hamiltonian can be written by replacing $M_{\uparrow/\downarrow}$ in (1.4) by

$$G_{\uparrow/\downarrow} + \mathrm{i}\gamma_5 F_{\uparrow/\downarrow}, \tag{4.1}$$

where $G_{\uparrow/\downarrow}$ and $F_{\uparrow/\downarrow}$ are real symmetric matrices, and the $F_{\uparrow/\downarrow}$ term in $i\gamma_5$ arises from coupling to the vacuum expectation value of a new T-negative and P-negative field $\tau(x)$, the timeon field. Thus, the whole field theory conserves T, but T-violation arises from the spontaneous symmetry breaking that makes the vacuum expectation value

$$\tau_0 = \langle\tau(x)\rangle_{\mathrm{vac}} \neq 0. \tag{4.2}$$

The timeon field $\tau(x)$ is real, so that there is no Goldstone boson[4]. However, the oscillation of $\tau(x)$ around its vacuum expectation value τ_0 gives rise to a new particle, called "timeon", whose production can lead to large T-violating effects. In Ref. [3], it is shown that the parameters determining $G_{\uparrow/\downarrow}$ and $F_{\uparrow/\downarrow}$ can be adjusted to simulate an arbitrary complex γ_4 model, as far as the quark masses are concerned, but not the CKM matrix. Thus, for example, in the timeon γ_5-model the light quark masses in the small mass limit turn out to be proportional to $\mathcal{J}$, whereas in the γ_4-model, they are proportional to $\mathcal{J}^2$.

References

1 Friedberg R, Lee T D. Ann. Phys., 2008, **323**: 1087
2 Particle Data Group. J. Phys. G, 2006, **33**: 1
3 Friedberg R, Lee T D. arXiv:0809.3633
4 Goldstone J. Nuovo Cimento, 1961, **9**: 154