Computer simulations for intense continuous beam transport in electrostatic lens systems*

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Abstract A code LEADS based on the Lie algebraic analysis for the continuous beam dynamics with space charge effect in beam transport has been developed. The program is used for the simulations of axial-symmetric and unsymmetrical intense continuous beam in the channels including drift spaces, electrostatic lenses and DC electrostatic accelerating tubes. In order to get the accuracy required, all elements are divided into many small segments, and the electric field in the segments is regarded as uniform field, and the dividing points are treated as thin lenses. Iteration procedures are adopted in the program to obtain self-consistent solutions. The code can be used in the designs of low energy beam transport systems, electrostatic accelerators and ion implantation machines.

Key words intense continuous beam, axial-symmetric electrostatic field Lie, algebraic analysis, space charge effect

PACS 52.59.Sa

1 Introduction

Intense continuous beam dynamic study plays an important role in the design of electrostatic accelerators of intense charged-particle beam, high-current ion implanters as well as some low energy beam transport systems. The analysis of pulsed beam transport in the electrostatic lenses was presented in Ref. [1], and accordingly the analytical results were coded[2]. The axial-symmetric continuous beam transport was worked out based on the Bessel equation[3], and the simulation code for this problem was presented[4]. However, the transport of the unsymmetrical continuous beams in the axial-symmetric electrostatic lenses has been a problem to be solved. Using the Lie algebraic method, we analyzed the non symmetric continuous beam transport in the axial-symmetric electrostatic lenses and the results have been incorporated in the program LEADS[5]. The code can be used in the designs and simulations of the axial-symmetric and unsymmetrical intense continuous beam transport in the ion-optic systems consisting of electrostatic lenses and electrostatic accelerating tubes.

2 Beam dynamics

2.1 Drift space

In the code, each drift space is divided into several small segments. The transport matrix of each segment is

$$\begin{bmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{bmatrix} = \begin{bmatrix} \cosh(LK_x) & \dfrac{\sinh(LK_x)}{K_x} & 0 & 0 \\ K_x\sinh(LK_x) & \cosh(LK_x) & 0 & 0 \\ 0 & 0 & \cosh(LK_y) & \dfrac{\sinh(LK_y)}{K_y} \\ 0 & 0 & K_y\sinh(LK_y) & \cosh(LK_y) \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{bmatrix}, \tag{1}$$

Received 19 October 2007

* Supported by National Natural Science Foundation of China (1057009)

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where L is the segment length, and x_i, x'_i, y_i, y'_i ($i=0,1$) are the coordinate in phase space of charged particle at the entrance and exit of segment respectively. If the particle distribution in phase space is KV type,

$$K_x^2=\frac{qI}{X(X+Y)c^3m\pi\beta^3\gamma\varepsilon},\quad K_y^2=\frac{qI}{Y(X+Y)c^3m\pi\beta^3\gamma\varepsilon}, \tag{2}$$

where q is the particle charge, m is the rest mass of reference particle respectively, I is the beam current, $\gamma=1/\sqrt{1-\beta^2}$, $\beta=v/c$, v is the velocity of the reference particle, c is the light velocity in the vacuum, ε is the vacuum permittivity, X and Y are the semi-axes of phase ellipse respectively. If the particle distribution type is Gaussian,

$$K_x^2=\frac{qI\mu_x}{2c^3m\pi\beta^3\gamma\varepsilon},\quad K_y^2=\frac{qI\mu_y}{2c^3m\pi\beta^3\gamma\varepsilon}, \tag{3}$$

where

$$\mu_x=\int_0^\infty\frac{\mathrm{d}\xi}{(2X^2+\xi)\sqrt{(2X^2+\xi)(2Y^2+\xi)}},$$

$$\mu_y=\int_0^\infty\frac{\mathrm{d}\xi}{(2Y^2+\xi)\sqrt{(2X^2+\xi)(2Y^2+\xi)}}.$$

2.2 Electrostatic lens

Electrostatic lenses are generally used to control the charged particle beams in low energy ion optical systems. When the particle beam moves in the paraxial region, linear approximation can be made for the particle trajectory solutions. As we do in the drift space mentioned above, the electrostatic lens is also divided into several small segments in the analysis. The electrical field in every segment is considered as uniform approximately, and each dividing point is treated as a thin lens. The transfer matrices of each segment and dividing point are expressed as follows.

2.2.1 In the interval (z_i, z_{i+1})

In the interval $(z_i,\ z_{i+1})$, the transport matrix for intense continuous beam is

$$\begin{bmatrix} x_1\\ x'_1\\ y_1\\ y'_1\end{bmatrix}=\begin{bmatrix}\cosh(Le\cdot K_x) & \dfrac{\sinh(Le\cdot K_x)}{K_x} & 0 & 0\\ \dfrac{K_x}{\eta}\sinh(Le\cdot K_x) & \dfrac{\cosh(Le\cdot K_x)}{\eta} & 0 & 0\\ 0 & 0 & \cosh(Le\cdot K_y) & \dfrac{\sinh(Le\cdot K_y)}{K_y}\\ 0 & 0 & \dfrac{K_y}{\eta}\sinh(Le\cdot K_y) & \dfrac{\cosh(Le\cdot K_y)}{\eta}\end{bmatrix}\begin{bmatrix} x_0\\ x'_0\\ y_0\\ y'_0\end{bmatrix}, \tag{4}$$

where $Le=\dfrac{2(z_{i+1}-z_i)}{1+\eta}$, $\eta=\sqrt{\dfrac{V_{i+1}}{V_i}}$. V_i and V_{i+1} are energy gauged potentials at the points z_i and z_{i+1} of this segment respectively. In our studies, three kinds of electrostatic lenses are considered: two-tube lens, three-tube Einzel lens and three-aperture Einzel lens. The axial potential distribution of them is introduced in Ref. [6].

If the particle distribution in the phase space is KV type,

$$K_x^2=\frac{qI\log\eta}{X(X+Y)c^3m\pi\overline{\beta}^3\gamma\varepsilon(\eta-1)},\quad K_y^2=\frac{qI\log\eta}{Y(X+Y)c^3m\pi\overline{\beta}^3\gamma\varepsilon(\eta-1)}. \tag{5}$$

If the particle distribution type is Gaussian,

$$K_x^2=\frac{qI\mu_x\log\eta}{2c^3m\pi\overline{\beta}^3\gamma\varepsilon(\eta-1)},\quad K_y^2=\frac{qI\mu_y\log\eta}{2c^3m\pi\overline{\beta}^3\gamma\varepsilon(\eta-1)}. \tag{6}$$

where $\overline{\beta}$ is the average relative velocity of reference particle in this segment.

2.2.2 At dividing point z_i $(i=1,2,3,\cdots,n)$

The dividing point z_i $(i=1,2,3,\cdots,n)$ is regarded as a thin lens approximately. In the case of linear approximation, the transport matrix has been presented in Ref. [2].

2.3 DC electrostatic accelerating tube

The DC electrostatic accelerating tube can be considered as three parts, the entrance thin lens, the accelerating field, and the exit thin lens. The transfer matrices of the entrance thin lens and the exit thin lens are listed in Ref. [7]. The electrical field in accelerating tube is considered as uniform, and the uniform accelerating field is divided into many small segments also. The transfer matrices of these segments are the same as in Eq. (4).

3 Development of the code

The frame of the simulations for intense continuous beam transport in each segment is shown in Fig. 1, where "err" stands for the tolerance of the beam spot calculations. At the end of each segment, the number of particles lost on the channel boundary is recorded, and then the beam current carried by the lost particles will be taken off from the total current. If all of the particles are lost, process will stop.

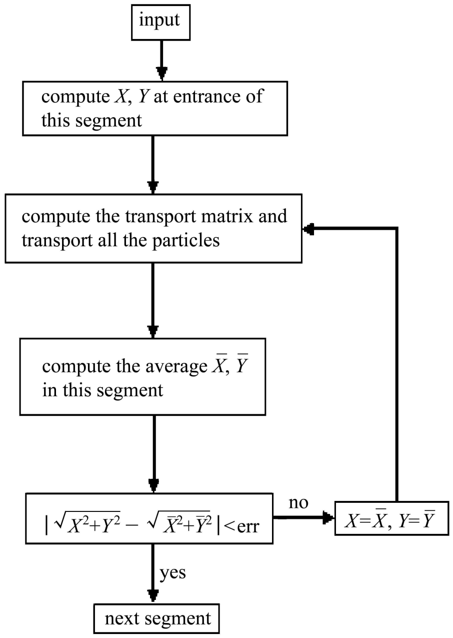

Fig. 1. Code frame.

4 Simulation example

As an example, we calculated a beam transport system, which consists of some drift spaces, a three-tube Einzel lens, a two-tube lens and a DC accelerating tube as shown in Fig. 2. At the first step of the simulations, we ignore the space charge forces, and the system is optimized. The particle distribution type in the phase space is Gaussian. The evolution of the particle distributions in the horizontal and vertical projections of the phase space are shown in Fig. 3 upper and Fig. 3 lower respectively, and the beam envelopes in Fig. 5(a). At the second step, the space charge effects are taken into account; and the element parameters of the beam line are taken from optimized results in first step; the particle distribution type in the phase space is also Gaussian, and beam current is 2 mA. The evolution of the particle distributions in the horizontal and vertical projections of the phase space is shown in Fig. 4 upper and Fig. 4 lower respectively, and the beam envelopes in Fig. 5(b). From these figures we can clearly see the effects of the space charge on the phase space diagrams and beam envelopes.

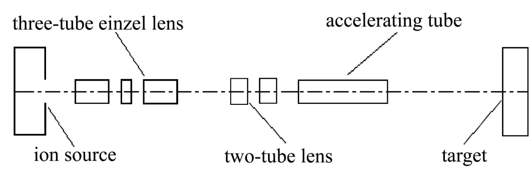

Fig. 2. Beam transport line.

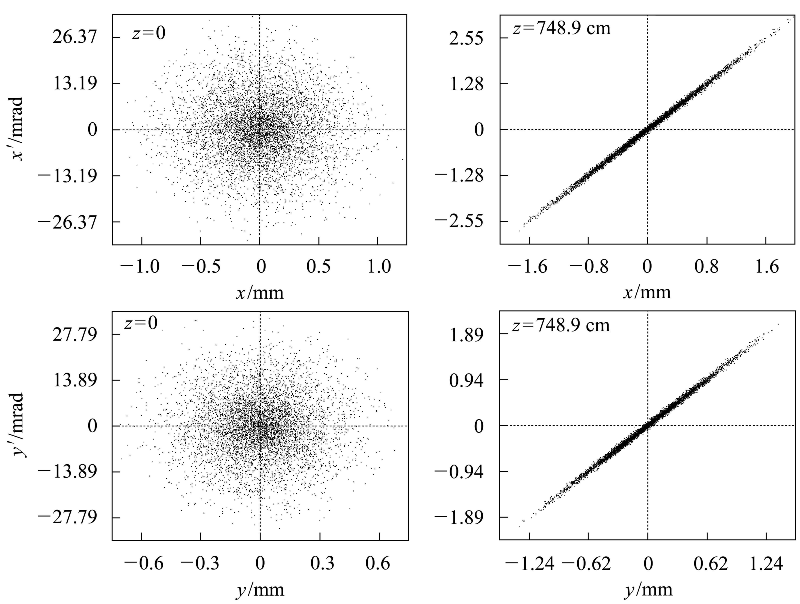

Fig. 3. Particle distribution at horizontal (upper) and vertical (lower) projection in phase space $I=0$.

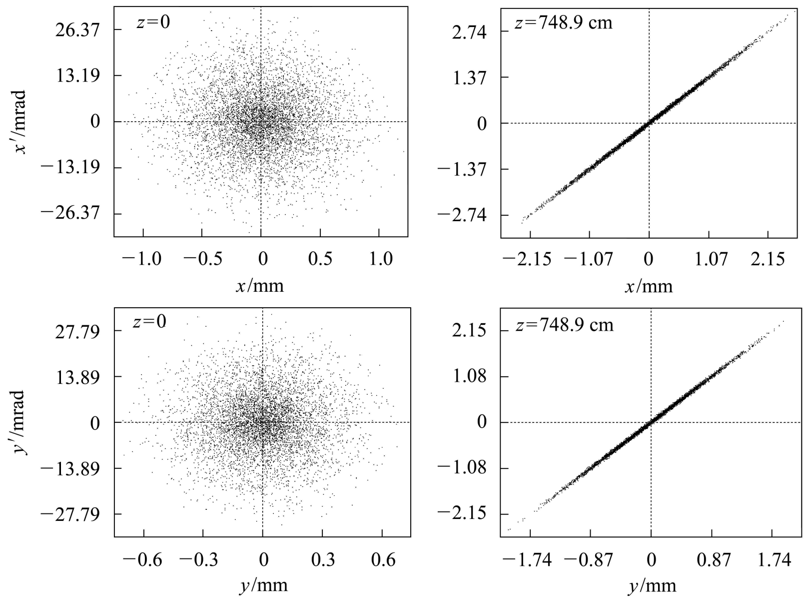

Fig. 4. Particle distribution at horizontal (upper) and vertical (lower) projection in the phase space $I=2$ mA.

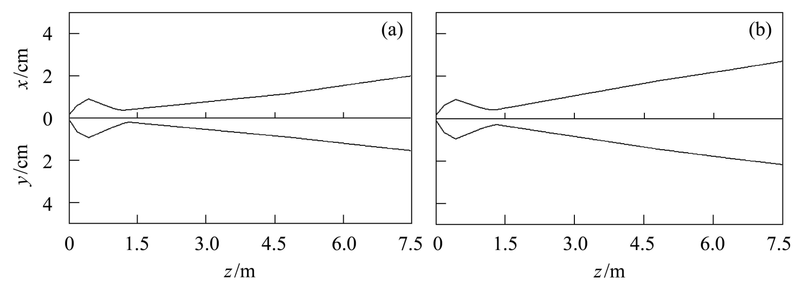

Fig. 5. Envelopes $I=0$ (a) and $I=2$ mA (b).

5 Conclusion

The code LEADS based on Lie algebraic analysis has been developed, which has been tested in simulation example mentioned above. It can be used to simulate the transport of either axial-symmetric or unsymmetrical intense continuous beam in ion-optic system, taking space charge effect into account.

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