Systematical law of β^+-decay half-lives of nuclei far from β-stable line*

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Abstract Experimental data of nuclear β^+-decay half-lives are systematically analyzed and investigated. We present an exponential law between the half-life of β^+-decay with the same forbiddenness and the nucleon number (Z,N) of parent nucleus far from the β-stable line. A formula with four parameters is proposed to describe the β^+-decay half-lives of nuclei far from stability. Experimental β^+-decay half-lives of the first and second forbidden transitions are well reproduced by this simple formula. The physics of the exponential law is related to the statistical properties of β^+-decay far from β-stable line.

Key words β^+-decay half-life, exponential law, formula

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1 Introduction

In the 1930s, Fermi proposed the theory of β-decay[1]. From then on, β-decay has been extensively studied both in theory and experiment[1—5]. However, nuclear β-decay is not only dominated by weak interaction but also affected by strong interaction. Because of the inherent complexity of strong interaction and nuclear many-body problem, at present it is still a difficult task to calculate nuclear β-decay half-life precisely in most cases. Therefore, a simple and reliable formula to calculate β-decay probability would be very valuable. In 1933, Sargent made an empirical study of β-decay half-lives and discovered a law which is consistent with Fermi theory of β-decay proposed one year later[1, 6]. Since then, a few parametric models were proposed based on some aspects of real physical behavior prescribed to the complex quantum many-body system, such as Kratz-Herrmann formula[7, 8] and the gross theory[9]. Along with the development of radioactive nuclear beams and the improvement of detector technology, there are many experimental data[10] available for the systematical analysis of new rule and properties of nuclear β^+-decay far from stability. This is helpful for the calculation and prediction of β^+-transition probability and for satisfying the data needs of nuclear astrophysics and experiments[11].

This paper is organized as follows. In Sect. 2, we study the systematical law of β^+-decay half-lives. A formula is proposed to calculate β^+-decay half-lives of nuclei far from the β-stable line. In Sect. 3, we discuss the physical meaning of the systematical law. A summary is given in the last section.

2 Systematical law of β^+-decay half-lives of nuclei far from β-stable line

There exists common behavior between β^+-decay half-lives and the neutron number of the isotopes.

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Dramatic variations exist in the β^+-decay half-lives of nuclei close to β-stable line. This is because β^+-decay energy of the nucleus close to β-stable line is very small, and the nuclear structure effects and the level distribution of daughter nucleus, causes relatively large fluctuation of β^+-decay energy. In low decay energy region, the β^+-decay half-life is sensitive to the β^+-decay Q value, which causes dramatic fluctuation of β^+-decay half-life of the nucleus close to β-stable line. For nuclei far from stability, the decay energy is much larger than that of nucleus close to β-stable line. As a result, the influence of certain nuclear structure effects can be smooth according to Fermi theory of β-decay. Therefore, there may exist systematical rule for the β^+-decay half-lives of nuclei far from the β-stable line.

In Fig. 1 we plot the variations of β^+-decay half-lives with the neutron numbers of Pb isotopes. It can be seen from Fig. 1 that there are large fluctuations of β^+-decay half-lives of nuclei close to β-stable line, whereas for nuclei far from β-stable line, there is a linear relationship between the logarithm of half-life of β^+-decay with the same forbiddenness and the neutron number of parent nucleus.

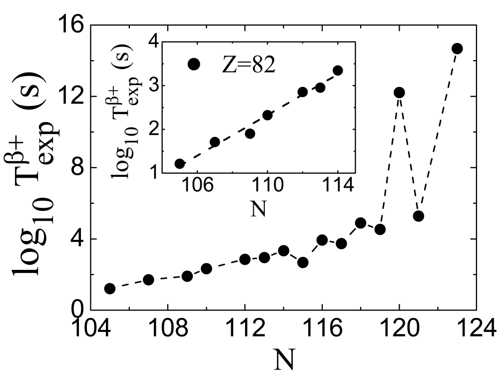

Fig. 1. The large figure plots the β^+-decay half-lives (in $\log_{10}$-scale) along Pb isotopic chain. The small figure in the inset shows the linear relationship between the logarithms of half-lives of β^+-decays with the same forbiddenness and the neutron number of Pb isotopes far from the β-stable line.

We now systematically investigate the above linear relationship. The experimental data of β^+-decays are taken from Nubase table of nuclear and decay properties by Audi et al.[10].

The variations of the logarithms of first forbidden β^+-decay half-lives along different isotopic chains (Z=69—71) are plotted in Fig. 2(a). It is seen from Fig. 2(a) that the logarithms of β^+-decay half-lives on an isotopic chain approximately form a straight line. This can be written as

$$\log_{10} T_{1/2} = aN + b, \tag{1}$$

where a and b are two constants to be determined and N is the neutron number of the nucleus on the isotopic chain. This equation shows an exponential law between half-lives of β^+-transitions with the same forbiddenness and the neutron number of parent nuclei.

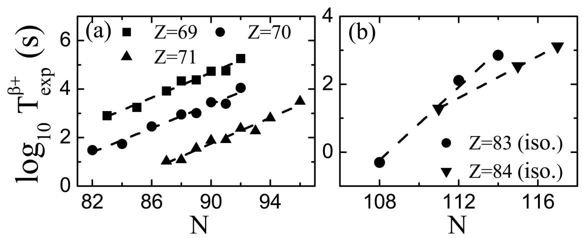

Fig. 2. The variations of half-lives of first forbidden β^+-transitions with the neutron number of parent nuclei: (a) Z=69—71 isotopic chains; (b) Z=83—84 isomeric states.

The above exponential law is also valid for β^+-decay half-lives of isomeric states. For example, we can see from Fig. 2(b) that there is a linear relationship between the logarithms of half-lives of the first forbidden β^+-transitions of isomeric states (Z=83—84) and the neutron number of parent nuclei. This shows there is the same exponential law for the β^+-decay half-lives of nuclei in ground state and in isomeric state, i.e., the relationship between the half-life of β^+-decay with the same forbiddenness and the neutron number.

Based on the above linear relationship between the logarithm of half-life of β^+-decay with the same forbiddenness and neutron number of parent nucleus, we propose the following formula

$$\log_{10} T_{1/2} = (c_1 Z + c_2)N + c_3 Z + c_4 \tag{2}$$

to calculate the β^+-decay half-lives of nuclei far from stability. In Eq. (2), Z denotes the proton number of parent nucleus, and c_1, c_2, c_3 and c_4 are four fitting parameters. This formula shows an exponential relationship between the β^+-decay half-life and the nucleon number (Z, N).

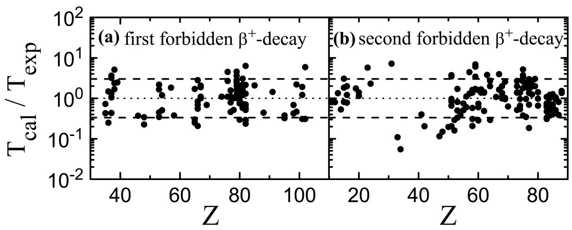

Fig. 3. Comparisons between calculated β^+-decay half-lives from the formula [Eq. (2)] and experimental ones: (a) first forbidden β^+-transition; (b) second forbidden β^+-transition.

We use the formula [Eq. (2)] to calculate the half-lives of first and second forbidden β^+-transitions and compare with the present experimental data. For the first forbidden transition, we get a set of parameters $c_1 = -0.00125$; $c_2 = 0.3923$; $c_3 = -0.4105$ and $c_4 = 3.8172$ according to a least-square fit of 102 nuclei. The standard deviation is 0.36 and the average deviation is 0.30. This shows the average ratio between theory and experiment is a factor of 2. The ratios between calculated data and experimental ones are plotted in Fig. 3(a). It is seen from Fig. 3(a) that the ratios for most nuclei are within a factor of 3.

According to a similar least-square fit of 144 nuclei with the second forbidden β^+-transitions, we get a set of parameters: $c_1 = -0.00161$; $c_2 = 0.3925$; $c_3 = -0.3214$ and $c_4 = -0.2701$. The standard deviation is 0.39 and the average deviation is 0.31. This shows the average ratio between calculated β^+-decay half-lives and experimental ones is a factor of 2. It is also seen from Fig. 3(b) that the ratios ($T_{\rm cal}/T_{\rm exp}$) are within a factor of 3 for most of the nuclei.

In a word, the formula [Eq. (2)] works well in describing the first and second forbidden β^+-transitions for the large range of nuclei ($A=9-237$) and for the large variations of β^+-decay half-lives (10^{-3}—10^6 s). The average ratio between calculated results and experimental ones is a factor of 2. It is comparable to the calculations of microscopic models[2, 3]. This shows that the formula [Eq. (2)] is reliable to describe the β^+-decay half-lives of nuclei far from β-stable line.

In Table 1 we use the formula [Eq. (2)] to predict the β^+-decay half-lives of nuclei with first forbidden transition (Z=74—79). The first and fifth columns, and the second and sixth columns are respectively the proton number and mass number of parent nucleus. The predicted β^+-decay half-lives are listed in the third and seventh columns. The calculated β^+-decay half-lives by Möller et al.[2] are listed in the fourth and eighth columns for comparisons. It is seen from Table 1 that the calculated results from the formula [Eq. (2)] are comparable to those from the microscopic model[2] within a few times.

Table 1. Predicted half-lives of nuclei with first forbidden β^+-transitions (Z=74—79). The results calculated by Möller et al.[2] are listed for comparisons.

Z	A	$T_{\rm cal.}$/s	$T_{\rm Möller}$/s	Z	A	$T_{\rm cal.}$/s	$T_{\rm Möller}$/s
74	158	0.042	0.3287	77	167	0.071	0.3235
74	160	0.17	0.4927	77	169	0.28	0.6105
74	163	1.33	1.8577	77	171	1.09	1.0064
74	167	20.97	14.3608	77	173	4.26	1.6296
74	168	41.82	19.1796	78	166	0.0055	0.1146
74	169	83.41	29.5178	78	169	0.042	0.2550
75	160	0.025	0.3330	78	171	0.16	0.7941
75	162	0.10	0.3694	78	173	0.64	1.8848
75	163	0.20	0.5922	79	169	0.0064	0.1753
75	165	0.79	1.1065	79	170	0.013	0.2725
75	166	1.58	0.6647	79	171	0.025	0.2531
76	162	0.015	0.1529	79	173	0.096	0.3360
76	166	0.24	0.8145	79	175	0.37	0.4979
77	164	0.0092	0.1423	79	177	1.43	0.9285
77	166	0.036	0.2434				

3 Discussions on physics behind the exponential law

In the above section we demonstrate that the exponential law [Eq. (1)] and the formula [Eq. (2)] describe the β^+-decay half-lives of nuclei far from the β-stable line well. We discuss the physical meaning of this exponential law in this section. According to the linear relationship between the maximum kinetic energy of the positron emitted in the β^+-decay and the neutron number of parent nucleus,

$$E_{\mathrm{m}} = d_1 N + d_2, \tag{3}$$

we obtain the linear relation between the logarithms of half-lives of β^+-decays with the same forbiddenness and the maximum kinetic energy of the emitted positron,

$$\log_{10} T_{1/2} = d_3 E_{\mathrm{m}} + d_4. \tag{4}$$

Here, d_1, d_2, d_3 and d_4 in Eqs. (3) and (4) are four parameters. The linear relation [Eq. (4)] is demonstrated in Fig. 4(a). It is seen from Eq. 4(a) that the experimental data lie on a straight line. This suggests that Eq. (4) can describe the variations of β^+-decay half-lives well.

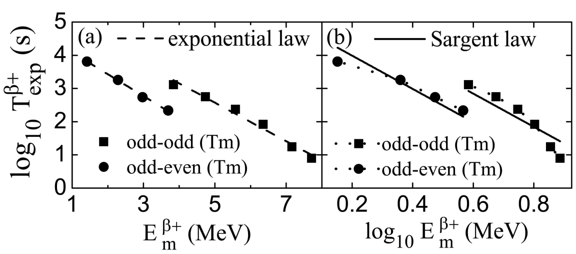

Fig. 4. Comparisons between the exponential law and Sargent law for describing the β^+-decay half-lives of nuclei far from stability: (a) exponential law; (b) Sargent law.

It is well known that there is Sargent law[4, 6] between β^+-decay half-life and the maximum kinetic energy E_{m} of the emitted positron, i.e., the power law $T_{1/2} \propto E_{\mathrm{m}}^{-5}$. This is consistent with the Fermi theory of β-decay when the decay energy is large. However, it seems that the exponential law [Eqs. (1) and (4)] contradicts Sargent law. In Fig. 4(b) we plot the logarithms of β^+-decay half-lives of Tm isotopes ($Z=69$) versus the logarithm of E_{m}. The solid line denotes the Sargent law. It is seen from Fig. 4(b) that experimental data slightly deviate from Sargent law. Comparing Fig. 4(a) with Fig. 4(b), we find that the exponential law describes β^+-decay half-lives of Tm isotopes better than Sargent law. This is probably caused by the statistical properties of β^+-decay of nucleus far from the β-stable line. For the nucleus far from stability, the decay energy is large, and the parent nucleus could decay to a series of levels of daughter nucleus.

Therefore the decay constant of β^+-decay is the sum of different decay branches, i.e.,

$$\lambda = \sum_i p_i (E_{\mathrm{m}} - E_i)^5. \tag{5}$$

Here E_i denotes the excited energy of the ith level of daughter nucleus, p_i is the product of const term and matrix element of the transition from the ground state of parent nucleus to the ith level of daughter nucleus. After expanding Eq. (5), we get

$$\begin{aligned}\lambda = {} & \Big(\sum_i p_i\Big) E_{\mathrm{m}}^5 - 5\Big(\sum_i p_i E_i\Big) E_{\mathrm{m}}^4 + \\ & 10\Big(\sum_i p_i E_i^2\Big) E_{\mathrm{m}}^3 - 10\Big(\sum_i p_i E_i^3\Big) E_{\mathrm{m}}^2 + \\ & 5\Big(\sum_i p_i E_i^4\Big) E_{\mathrm{m}} - \sum_i p_i E_i^5. \end{aligned} \tag{6}$$

Eq. (6) is similar to the expanded form of the exponential relationship:

$$\begin{aligned}\lambda = {} & f_0 \Big[1 + f_1 E_{\mathrm{m}} + \frac{(f_1 E_{\mathrm{m}})^2}{2!} + \cdots + \frac{(f_1 E_{\mathrm{m}})^5}{5!} + \\ & \cdots + \frac{(f_1 E_{\mathrm{m}})^n}{n!} + \cdots \Big], \end{aligned} \tag{7}$$

where $f_0 = 0.693 \times 10^{-d_4}$ and $f_1 = -2.303 d_3$. And Sargent law can be seen as a special term E_{m}^5 in Eqs. (6) and (7). This shows that the exponential law is more suitable to describe β^+-decay half-lives of nuclei far from stability. It is shown that there is common behavior for the half-lives of α-decay, β-decay, and cluster radioactivity[12—14] although they are governed by different interactions such as strong, weak and electromagnetic interactions, that is, there is the exponential law between the decay half-life and the decay energy (or nucleon number) for nuclear α-decay, β-decay, and cluster radioactivity.

4 Summary and conclusions

The rule of experimental β^+-decay half-lives is systematically analyzed. We have discovered a new exponential law between half-lives of β^+-decays with the same fobiddenness and the neutron number of parent nuclei far from the β-stable line. This law is also valid for isomeric states. Based on the exponential law, a formula is proposed to calculate the β^+-decay half-lives of nuclei far from stability. The comparison between theory and experiment shows that this formula is reliable to calculate the β^+-decay half-lives of nuclei far from the β-stable line. Some predictions on β^+-decay half-lives are presented. We also discussed the physics of the exponential law. Because the contribution of the transitions from ground state of parent nucleus to excited levels of daughter nucleus to the total decay probability is included, this new exponential law is reliable to describe β^+-decay half-lives of nuclei far from the β-stable line. This is helpful for scientists to analyze the data of β^+-decay and for the calculation in nuclear astrophysics.

References

1 Fermi E. Z. Phys., 1934, **88**: 161
2 Möller P, Nix J R, Kratz K. At. Data Nucl. Data Tables, 1997, **66**: 131
3 Nabi Jameel-Un, Klapdor-Kleingrothaus H V. At. Data Nucl. Data Tables, 2004, **88**: 237
4 LU Xi-Ting, JIANG Dong-Xing, YE Yan-Lin. Nuclear Physics. Beijing: Atomic Energy Press, 2000. 151 (in Chinese)
5 ZHANG Xiao-Ping, REN Zhong-Zhou. Phys. Rev. C, 2006, **73**: 014305
6 Sargent B W. Proc. R. Soc. London Ser. A, 1933, **139**: 659
7 Kratz K L, Herrmann G. Z. Physik, 1973, **263**: 435
8 Pfeiffer B, Kratz K L, Möller P. Prog. Nucl. Energy, 2002, **41**: 39
9 Takahashi K, Yamada M. Prog. Theor. Phys., 1969, **41**: 1470
10 Audi G, Bersillon O, Blachot J et al. Nucl. Phys. A, 2003, **729**: 3
11 SHEN Wen-Qing, ZHAN Wen-Long, YE Yan-Lin et al. Nucl. Phys. Rev., 2001, **18**(4): 206 (in Chinese)
12 XU Chang, REN Zhong-Zhou. Nucl. Phys. Rev., 2006, **23**(4): 431 (in Chinese)
13 Viola V E, Seaborg G T. J. Inorg. Nucl. Chem., 1966, **28**: 741
14 REN Zhong-Zhou, XU Chang, WANG Zai-Jun. Phys. Rev. C, 2004, **70**: 034304