

# Antikaon condensation in neutron star matter with strong magnetic fields<sup>\*</sup>

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**Abstract** We study the influence of strong magnetic fields on antikaon condensation in neutron star matter using the quark-meson coupling (QMC) model. The QMC model describes a nuclear many-body system as nonoverlapping MIT bags interacting through the self-consistent exchange of scalar and vector meson mean fields. It is found that the presence of strong magnetic fields alters the threshold density of antikaon condensation significantly. The results of the QMC model are compared with those obtained in a relativistic mean-field (RMF) model.

**Key words** quark-meson coupling model, antikaon condensation, magnetic fields

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## 1 Introduction

Observations of soft-gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) imply that the surface magnetic field of young neutron stars could be of order  $10^{14} - 10^{15}$  G<sup>[1]</sup>. Although there is no direct observational evidence for the internal magnetic field of the star, it may reach  $10^{18}$  G, as estimated in some theoretical works<sup>[2-4]</sup>. Motivated by a possible existence of strong magnetic fields in neutron stars, theoretical studies on the influence of extremely large fields on dense matter have been carried out by many authors<sup>[5-8]</sup>. It has been found that the composition of neutron star matter can be significantly affected by strong magnetic fields, while the maximum mass of neutron stars might be substantially increased<sup>[3, 4]</sup>. On the other hand, it is well known that the density in the interior of neutron stars is extremely high, and additional degrees of freedom such as hyperons, kaons, and even quarks may occur in the core of neutron stars. Recently, much attention has been paid

to the kaon/antikaon condensations<sup>[9-12]</sup>. In general, the presence of antikaon condensation tends to soften the equation of state (EOS) at high density and lower the maximum mass of neutron stars. It is interesting to investigate the influence of strong magnetic fields on antikaon condensation.

In this paper, we study the effects of strong magnetic fields on antikaon condensation in neutron star matter using the quark-meson coupling (QMC) model, in which the quark degrees of freedom are explicitly taken into account<sup>[13]</sup>. The QMC model has been applied to various problems of nuclear matter and finite nuclei with reasonable success<sup>[14-17]</sup>. Furthermore, the model has also been used to investigate the properties of neutron stars with the inclusion of hyperons, quarks, and  $K^-$  condensation<sup>[12, 18]</sup>. In the present work, both nucleons and antikaons are described as MIT bags which interact through the self-consistent exchange of isoscalar scalar and vector mesons ( $\sigma$  and  $\omega$ ) and isovector vector meson ( $\rho$ ) in the mean-field approximation. In contrast to the

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relativistic mean-field (RMF) model<sup>[19]</sup>, the quark structure plays a crucial role in the QMC model.

## 2 Model

We first study the properties of nucleons and antikaons under the influence of external meson and electromagnetic fields using the MIT bag model. For simplicity, we neglect the spatial variation of the fields over the small bag volume, and take the values at the center of the bag as their average quantities<sup>[14]</sup>.

For nucleons described as spherical MIT bags with external meson and electromagnetic fields, the up and down quarks inside the bag satisfy the Dirac equation

$$\left[ i\gamma_\mu \partial^\mu - (m_q + g_\sigma^q \sigma) - g_\omega^q \omega_\mu \gamma^\mu - g_\rho^q \tau_{3q} \rho_{3\mu} \gamma^\mu - \frac{e(1+3\tau_{3q})}{6} A_\mu \gamma^\mu \right] \psi_q = 0, \quad (1)$$

where  $g_\sigma^q$ ,  $g_\omega^q$ , and  $g_\rho^q$  are the quark-meson coupling constants, and  $m_q$  is the current quark mass.  $\tau_{3q}$  is the third component of the Pauli matrices.  $\sigma$ ,  $\omega_\mu$ ,  $\rho_{3\mu}$ , and  $A_\mu$  are the values of the meson and electromagnetic fields at the center of the bag. The normalized ground state for a quark in the bag is given by

$$\psi_q(\mathbf{r}, t) = \mathcal{N}_q e^{-i\epsilon_q t/R_N} \begin{pmatrix} j_0(x_q r/R_N) \\ i\beta_q \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} j_1(x_q r/R_N) \end{pmatrix} \frac{\chi_q}{\sqrt{4\pi}}, \quad (2)$$

where

$$\beta_q = \sqrt{\frac{\Omega_q - R_N m_q^*}{\Omega_q + R_N m_q^*}}, \quad (3)$$

$$\mathcal{N}_q^{-2} = 2R_N^3 j_0^2(x_q) [\Omega_q(\Omega_q - 1) + R_N m_q^*/2] / x_q^2, \quad (4)$$

$\Omega_q = \sqrt{x_q^2 + (R_N m_q^*)^2}$  and  $m_q^* = m_q + g_\sigma^q \sigma$ .  $R_N$  is the bag radius of the nucleon, and  $\chi_q$  is the quark spinor. The energy of a static nucleon bag consisting of three ground-state quarks is then given by

$$E_N^{\text{bag}} = 3 \frac{\Omega_q}{R_N} - \frac{Z_N}{R_N} + \frac{4}{3} \pi R_N^3 B_N, \quad (5)$$

where the parameter  $Z_N$  accounts for various corrections including the zero-point motion, and  $B_N$  is the bag constant. The effective nucleon mass is then taken to be

$$M_N^* = E_N^{\text{bag}}. \quad (6)$$

In the present calculation, we take the current quark mass  $m_q = 5.5$  MeV. The parameter  $B_N^{1/4} = 210.854$  MeV and  $Z_N = 4.00506$ , as given in Ref. [18],

are determined by reproducing the nucleon mass  $M_N = 939$  MeV and the bag radius  $R_N = 0.6$  fm in free space.

For antikaons, negatively charged  $K^-$  and neutral  $\bar{K}^0$ , we assume that they are described as MIT bags in the same way as nucleons<sup>[12, 20]</sup>. The exchanged  $\sigma$ ,  $\omega$ , and  $\rho$  mesons are assumed to couple exclusively to the up and down quarks (and antiquarks), not to the strange quark<sup>[12]</sup>. Similarly, the effective mass of antikaons and kaons is given by

$$m_K^* = \frac{\Omega_q + \Omega_s}{R_K} - \frac{Z_K}{R_K} + \frac{4}{3} \pi R_K^3 B_K. \quad (7)$$

where  $\Omega_q = \sqrt{x_q^2 + (R_K m_q^*)^2}$  and  $\Omega_s = \sqrt{x_s^2 + (R_K m_s)^2}$ . We take the strange quark mass  $m_s = 150$  MeV and the bag constant  $B_K = B_N$ . The parameters  $Z_K = 3.362$  and  $R_K = 0.457$  fm, as given in Ref. [12], are determined from the kaon mass and the stability condition in free space.

To describe neutron star matter with antikaon condensation in the presence of strong magnetic fields, we adopt the total Lagrangian density written at the hadron level as the sum of nucleonic, kaonic and leptonic parts,

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_K + \mathcal{L}_l, \quad (8)$$

$$\begin{aligned} \mathcal{L}_N = & \sum_{b=n,p} \bar{\psi}_b \left[ i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - M_N^* - g_{\omega N} \gamma_\mu \omega^\mu - \right. \\ & g_{\rho N} \gamma_\mu \tau_{iN} \rho_i^\mu - \frac{1}{2} \kappa_b \sigma_{\mu\nu} F^{\mu\nu} \left. \right] \psi_b + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \\ & \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \\ & \frac{1}{4} R_{i\mu\nu} R_i^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_{i\mu} \rho_i^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \end{aligned} \quad (9)$$

$$\mathcal{L}_K = D_\mu^* \bar{K} D^\mu K - m_K^{*2} \bar{K} K, \quad (10)$$

$$\mathcal{L}_l = \sum_{l=e,\mu} \bar{\psi}_l [i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l] \psi_l. \quad (11)$$

Here  $A^\mu = (0, 0, Bx, 0)$  refers to a constant external magnetic field  $B$  along the  $z$ -axis. The effective masses  $M_N^*$  and  $m_K^*$  given in Eqs. (6) and (7) are obtained at the quark level. The covariant derivative is defined as  $D_\mu = \partial_\mu + iq_K A_\mu + ig_{\omega K} \omega_\mu + ig_{\rho K} \tau_{iK} \rho_i^\mu$ . The anomalous magnetic moments of nucleons are included with  $\kappa_p = \mu_N (g_p/2 - 1) = 1.7928 \mu_N$  and  $\kappa_n = \mu_N g_n/2 = -1.9130 \mu_N$ , where  $\mu_N$  is the nuclear magneton. In the QMC model, the coupling constants at the hadron level are related to the quark-meson coupling constants as  $g_{\omega N} = 3g_\omega^q$ ,  $g_{\omega K} = g_\omega^q$ ,

and  $g_{\rho N} = g_{\rho K} = g_{\rho}^q$ <sup>[12, 20]</sup>. The quark-meson coupling constants  $g_{\sigma}^q = 5.957$ ,  $g_{\omega}^q = 2.994$ , and  $g_{\rho}^q = 4.325$  are determined by fitting the saturation properties of nuclear matter<sup>[18]</sup>.

The chemical potentials of nucleons and leptons are given by

$$\mu_p = E_f^p + g_{\omega N}\omega_0 + g_{\rho N}\rho_{30}, \quad (12)$$

$$\mu_n = E_f^n + g_{\omega N}\omega_0 - g_{\rho N}\rho_{30}, \quad (13)$$

$$\mu_l = E_f^l = \sqrt{k_{f,\nu,s}^2 + m_l^2} + 2\nu|q_l|B. \quad (14)$$

The Fermi energies  $E_f^p$  and  $E_f^n$  are related to the Fermi momenta  $k_{f,\nu,s}^p$  and  $k_{f,\nu,s}^n$  as

$$E_f^{p2} = k_{f,\nu,s}^{p2} + \left( \sqrt{M_N^{*2} + 2\nu q_p B} - s\kappa_p B \right)^2, \quad (15)$$

$$E_f^{n2} = k_{f,\nu,s}^{n2} + (M_N^* - s\kappa_n B)^2, \quad (16)$$

where  $\nu$  enumerates the Landau levels of electrical fermions. The quantum number  $s$  is  $+1$  for spin up and  $-1$  for spin down cases. For  $s$ -wave condensation of negatively charged  $K^-$  and neutral  $\bar{K}^0$ , we have the chemical potentials

$$\mu_{K^-} = \sqrt{m_{K^-}^{*2} + |q_{K^-}|B} - g_{\omega K}\omega_0 - g_{\rho K}\rho_{30}, \quad (17)$$

$$\mu_{\bar{K}^0} = m_{\bar{K}^0}^* - g_{\omega K}\omega_0 + g_{\rho K}\rho_{30}. \quad (18)$$

For neutron star matter with uniform distributions, the composition of matter is determined by the requirements of charge neutrality and  $\beta$ -equilibrium conditions. In the present calculation with antikaon condensation, the  $\beta$ -equilibrium conditions are expressed by

$$\mu_n - \mu_p = \mu_{K^-} = \mu_e = \mu_{\mu}, \quad (19)$$

$$\mu_{\bar{K}^0} = 0. \quad (20)$$

The charge neutrality condition is given by

$$\rho_p = \rho_{K^-} + \rho_e + \rho_{\mu}. \quad (21)$$

### 3 Results and discussion

In order to show how the results depend on the models based on different degrees of freedom, we make a systematic comparison between the QMC model and the RMF model with the TM1 parameter set<sup>[21]</sup>. The TM1 model includes nonlinear terms for both  $\sigma$  and  $\omega$  mesons, and has been widely used in many studies of nuclear physics<sup>[22–26]</sup>. It has been pointed in Refs. [9, 10] that antikaon condensation in the RMF models is quite sensitive to the antikaon

optical potential,  $U_{\bar{K}}(\rho_0) = g_{\sigma K}\sigma - g_{\omega K}\omega_0$ . In the QMC model, the antikaon optical potential is given by  $U_{\bar{K}}(\rho_0) = m_{\bar{K}}^* - m_{\bar{K}} - g_{\omega K}\omega_0$ <sup>[12]</sup>, and we obtain  $U_{\bar{K}}(\rho_0) = -123$  MeV with the parameters used in the present study. Here we take  $g_{\omega K} = g_{\omega N}/3$  and  $g_{\sigma K} = 0.926$  in the TM1 model, which is determined by fitting  $U_{\bar{K}}(\rho_0) = -123$  MeV obtained in the QMC model, so that the comparison between the QMC and TM1 models is more meaningful. It is found that the onset of  $K^-$  condensation shifts to higher density in the presence of strong magnetic fields, and it even shifts to densities beyond the threshold of  $\bar{K}^0$  condensation for strong enough magnetic fields. Although  $\bar{K}^0$  is a neutral particle, its chemical potential given by Eq. (18) is indirectly influenced by the magnetic field through the dependence of meson mean fields on the magnetic field. We conclude that  $K^-$  condensation depends more strongly on the magnetic field than  $\bar{K}^0$  condensation. By comparing the results of the QMC model with those of the TM1 model, one can see that the onset of antikaon condensation in the presence of strong magnetic fields occurs at lower densities within the QMC model. Although there are quantitative differences between the QMC and TM1 models, we find that the qualitative trends of magnetic field effects are quite similar in the two models.

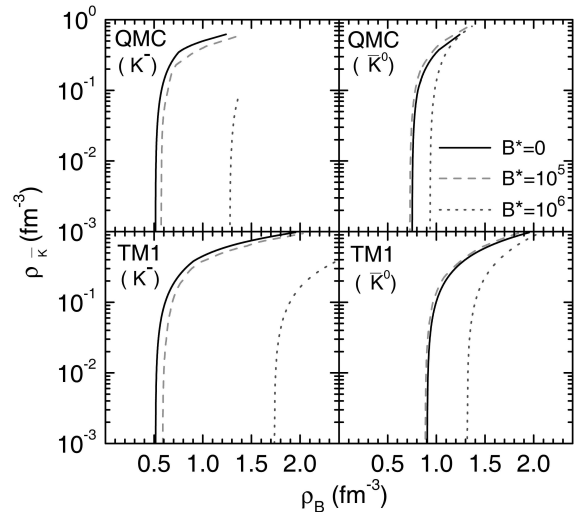


Fig. 1. The densities of antikaons,  $\rho_{K^-}$  (left panels) and  $\rho_{\bar{K}^0}$  (right panels), as a function of the baryon density,  $\rho_B$ , for  $B^* = B/B_c^e = 0, 10^5$ , and  $10^6$  ( $B_c^e = 4.414 \times 10^{13}$  G is the electron critical field). The results of the QMC and TM1 models are shown in the upper and lower panels, respectively.

## 4 Summary

We have studied the effects of strong magnetic fields on antikaon condensation in neutron star matter using the QMC model. We found that the threshold of antikaon condensation shifts to higher density in the presence of strong magnetic fields. The thresh-

old density of  $K^-$  condensation strongly depends on the magnetic field strength, and it even shifts beyond the threshold of  $\bar{K}^0$  condensation for sufficiently strong magnetic fields. There exist quantitative differences between the QMC and TM1 models, but qualitative trends of magnetic field effects on antikaon condensation and EOS are quite similar.

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## References

- 1 Thompson C, Duncan R C. *Astrophys. J.*, 1996, **473**: 322—342
- 2 Broderick A, Prakash M, Lattimer J M. *Astrophys. J.*, 2000, **537**: 351—367
- 3 Cardall C Y, Prakash M, Lattimer J M. *Astrophys. J.*, 2001, **554**: 322—339
- 4 MAO G J, Iwamoto A, LI Z X. *Chin. J. Astron. Astrophys.*, 2003, **3**: 359—374
- 5 Chakrabarty S, Bandyopadhyay D, Pal S. *Phys. Rev. Lett.*, 1997, **78**: 2898—2901
- 6 WEI F X, MAO G J, Ko C M et al. *J. Phys. G*, 2006, **32**: 47—61
- 7 YUE P, SHEN H. *Phys. Rev. C*, 2006, **74**: 045807
- 8 Harding A K, LAI D. *Rep. Prog. Phys.*, 2006, **69**: 2631—2708
- 9 Pal S, Bandyopadhyay D, Greiner W. *Nucl. Phys. A*, 2000, **674**: 553—577
- 10 Banik S, Bandyopadhyay D. *Phys. Rev. C*, 2001, **63**: 035802; *Phys. Rev. C*, 2001, **64**: 055805; *Phys. Rev. C*, 2002, **66**: 065801
- 11 GUO H, LIU B, ZHANG J W. *Phys. Rev. C*, 2003, **67**: 024902
- 12 Menezes D P, Panda P K, Providencia C. *Phys. Rev. C*, 2005, **72**: 035802
- 13 Guichon P. *Phys. Lett. B*, 1988, **200**: 235—240
- 14 Saito K, Tsushima K, Thomas A W. *Nucl. Phys. A*, 1996, **609**: 339—363
- 15 Muller H, Jennings B K. *Nucl. Phys. A*, 1998, **640**: 55—76
- 16 Tsushima K, Saito K, Haidenbauer J et al. *Nucl. Phys. A*, 1998, **630**: 691—718
- 17 Saito K, Tsushima K, Thomas A W. *Prog. Part. Nucl. Phys.*, 2007, **58**: 1—167
- 18 Panda P K, Menezes D P, Providencia C. *Phys. Rev. C*, 2004, **69**: 025207
- 19 Serot B D, Walecka J D. *Adv. Nucl. Phys.*, 1986, **16**: 1—327
- 20 Tsushima K, Saito K, Thomas A W et al. *Phys. Lett. B*, 1998, **429**: 239—246
- 21 Sugahara Y, Toki H. *Nucl. Phys. A*, 1994, **579**: 557—572
- 22 SHEN H, Toki H, Oyamatsu K et al. *Nucl. Phys. A*, 1998, **637**: 435—450
- 23 Hirata D, Sumiyoshi K, Carlson B V et al. *Nucl. Phys. A*, 1996, **609**: 131—146
- 24 SHEN H. *Phys. Rev. C*, 2002, **65**: 035802
- 25 Dey P, Bhattacharyya A, Bandyopadhyay D. *J. Phys. G*, 2002, **28**: 2179—2186
- 26 SHEN H, YANG F, Toki H. *Prog. Theor. Phys.*, 2006, **115**: 325—335