Nucleon internal structure: a new set of quark, gluon momentum, angular momentum operators and parton distribution functions*

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Abstract It is unavoidable to deal with the quark and gluon momentum and angular momentum contributions to the nucleon momentum and spin in the study of nucleon internal structure. However we never have the quark and gluon momentum, orbital angular momentum and gluon spin operators which satisfy both the gauge invariance and the canonical momentum and angular momentum commutation relation. The conflicts between the gauge invariance and canonical quantization requirement of these operators are discussed. A new set of quark and gluon momentum, orbital angular momentum and spin operators, which satisfy both the gauge invariance and canonical momentum and angular momentum commutation relation, are proposed. The key point to achieve such a proper decomposition is to separate the gauge field into the pure gauge and the gauge covariant parts. The same conflicts also exist in QED and quantum mechanics and have been solved in the same manner. The impacts of this new decomposition to the nucleon internal structure are discussed.

Key words nucleon internal structure, quark-gluon momentum and angular momentum, canonical commutation relation, gauge invariance

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1 Introduction

In quantum physics, any observable is expressed as a Hermitian operator in Hilbert space. The fundamental operators, such as momentum, orbital angular momentum, spin, satisfy the canonical momentum and angular momentum commutation relation. These commutation relations or Lie algebras define the properties of these operators.

Gauge invariance has been recognized as the first principle through the development of the standard model. In classical gauge field theory, gauge invariance principle requires any observable must be expressed in terms of gauge invariant variable. In quantum gauge field theory, in general one only requires the matrix elements of an operator in between physical states to be gauge invariant. However one usually requires the operators themselves to be gauge invariant. This is called the strong gauge invariance in[1]. We will restrict our discussion in strong gauge invariance in this paper and leave the other possibility to the future study[1, 2]

In the study of nucleon (atom) internal structure, it is unavoidable to study the quark, gluon (electron, photon) momentum, orbital angular momentum and spin contributions to the nucleon (atom) momentum and spin. However we never have the quark, gluon (electron, photon) momentum, orbital angular momentum and spin operators which satisfy both the gauge invariance and canonical momen-

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tum and angular momentum commutation relation except the quark (electron) spin. Even it has been claimed in some textbooks that one can not define separately the photon spin and orbital angular momentum operators[3] and almost a proper gluon spin operator search has been given up in the nucleon spin structure study for the last ten years. This situation has left puzzles in quantum mechanics, quantum electrodynamics (QED) and quantum chromodynamics (QCD), for example, the expectation value of the Hamiltonian of hydrogen atom is gauge dependent under a time dependent gauge transformation[4], the meaning of the multipole radiation analysis from atom to hadron spectroscopy would be obscure if the photon spin and orbital angular momentum operators were not well defined especially the parity of these microscopic states determined from the multipole radiation analysis would be obscure, there will be no way to compare the measured gluon spin contribution to nucleon spin with the theoretical calculated one if one has not a proper gluon spin operator, etc.

In section 2 the conflict between gauge invariance and canonical quantization of the usual quark gluon (electron photon) momentum, orbital angular momentum and spin operators are discussed from the simple quantum mechanics of a charged particle moving in an electromagnetic field to those of quark and gluon in QCD. In the third section a new set of momentum, orbital angular momentum and spin operators, which satisfy both the gauge invariance and canonical momentum and angular momentum commutation relation, are given. The key point is to separate the gauge field into pure gauge and gauge covariant (invariant) parts. The possible impacts of these modification to the nucleon internal structure will be discussed in section 4. the last section is a summary and a prospect of further studies.

2 Conflict between gauge invariance and canonical quantization of the momentum and angular momentum operators of the fermion and gauge field parts

The conflict between gauge invariance and canonical quantization of the momentum and orbital angular momentum operators of a charged particle moving in the electromagnetic field, a U(1) Abelian gauge field, has existed in quantum mechanics since the establishment of gauge invariance principle. Starting from the Lagrangian of a non-relativistic charged particle with mass m, velocity $\vec{v}$ and charge e moving in an electromagnetic field $A^{\mu}=(A^0,\vec{A})$,

$$\mathcal{L}(m,\vec{v},e,A_{\mu})=\frac{1}{2m}(m\vec{v})^2-e(A_0-\vec{v}\cdot\vec{A}), \qquad (1)$$

one obtains the canonical momentum,

$$\vec{p}=m\vec{v}+e\vec{A}, \qquad (2)$$

the orbital angular momentum,

$$\vec{L}=\vec{r}\times\vec{p}, \qquad (3)$$

and the Hamiltonian,

$$H=\frac{1}{2m}(\vec{p}-e\vec{A})^2+eA_0. \qquad (4)$$

All of these three classical dynamical variables are gauge dependent and so not observables. In the coordinate representation, the momentum operator $\vec{p}$ is quantized as

$$\vec{p}=\frac{\vec{\nabla}}{\mathrm{i}}. \qquad (5)$$

no matter what kind gauge is fixed on even though the classical canonical momentum operator, Eq. (2), is gauge dependent. The orbital angular momentum and Hamiltonian operators are quantized by replacing the $\vec{p}$ by $\frac{\vec{\nabla}}{\mathrm{i}}$. These quantized momentum and angular momentum operators satisfy the canonical commutation relation or the Lie algebra:

$$\begin{aligned}
[p_l,p_m] &= 0,\\
[L_l,L_m] &= \mathrm{i}\epsilon_{lmn}L_n,\\
[p_l,L_m] &= \mathrm{i}\epsilon_{lmn}p_n,\\
l,m,n &= 1,2,3,
\end{aligned} \qquad (6)$$

where $\epsilon_{l,m,n}$ is the rank three totally antisymmetric tensor and $\epsilon_{1,2,3}=1$. In general, the $[p_l,H]\neq 0$ which is different from the Poincaré algebra of the total momentum $P_l,(l=1,2,3)$ and total H of the whole system where $[P_l,H]=0$.

However, after a gauge transformation,

$$\psi' = \mathrm{e}^{-\mathrm{i}e\omega(x)}\psi, \qquad (7)$$

the matrix elements of the above operators transform as follows,

$$\begin{aligned}
\langle\psi'|\vec{p}|\psi'\rangle &= \langle\psi|\vec{p}|\psi\rangle-e\langle\psi|\vec{\nabla}\omega(x)|\psi\rangle,\\
\langle\psi'|\vec{L}|\psi'\rangle &= \langle\psi|\vec{L}|\psi\rangle-e\langle\psi|\vec{r}\times\vec{\nabla}\omega(x)|\psi\rangle,\\
\langle\psi'|H'|\psi'\rangle &= \langle\psi|H|\psi\rangle+e\langle\psi|\partial_t\omega(x)|\psi\rangle.
\end{aligned} \qquad (8)$$

It is obvious that the matrix elements of these three operators are all gauge dependent. Therefore they are not measurable and so these operators do not correspond to observables. This problem has left in quantum mechanics since the gauge principle was proposed.

The relativistic version of the quantum mechanics has the same problem. The gauge dependence of the expectation value of the Hamiltonian of the charged particle moving in electromagnetic field under a time dependent gauge transformation had been discussed by T. Goldman[4].

This conflict had been carried over to QED. Starting from a QED Lagrangian,

$$\mathcal{L} = \bar{\psi}[\gamma^\mu(\partial_\mu - \mathrm{i}eA_\mu) + \mathrm{i}m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{9}$$

By means of the Noether theorem one obtains the momentum and angular momentum operators as follows:

$$\vec{P} = \int \mathrm{d}^3x\psi^\dagger \frac{\vec{\nabla}}{\mathrm{i}}\psi + \int \mathrm{d}^3x E^i \vec{\nabla} A^i = P_\mathrm{e} + P_\mathrm{ph}, \tag{10}$$

$$\vec{J} = \vec{S}_\mathrm{e} + \vec{L}_\mathrm{e} + \vec{S}_\mathrm{ph} + \vec{L}_\mathrm{ph} = \int \mathrm{d}^3x\psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int \mathrm{d}^3x\vec{x}\times\psi^\dagger\frac{\vec{\nabla}}{\mathrm{i}}\psi + \int \mathrm{d}^3x\vec{E}\times\vec{A} + \int\mathrm{d}^3x\vec{x}\times E^i\vec{\nabla}A^i. \tag{11}$$

Here the $\Sigma^\mathrm{j} = \frac{\mathrm{i}}{2}\epsilon_{jkl}\gamma^k\gamma^l$. These electron and photon momentums, orbital angular momentums and spin, after quantization, satisfy momentum and angular momentum Lie algebra, however they are not gauge invariant except the electron spin.

The multipole radiation analysis is the basis of atomic, molecular, nuclear and hadron spectroscopy. The multipole field is based on the decomposition of the electromagnetic field into field with definite orbital angular momentum and spin quantum numbers. If the photon spin and orbital angular momentum operators were gauge dependent, then the physical meaning of the multipole field would be obscure especially the parity of these microscopic states determined by the measurement of the orbital angular momentum quantum number of the multipole radiation field would be obscure.

QCD has the same problem as QED. The quark gluon momentum, orbital angular momentum and spin operators derived from QCD Lagrangian by Noether theorem have the same form as those of electron and photon if one omits the color indices. They satisfy the momentum and angular momentum Lie algebra but they are not gauge invariant except the quark spin.

Because of the lack of gauge invariant quark, gluon momentum operators, the present operator product expansion (OPE) used the following two operators as quark gluon momentum operators,

$$\vec{P} = \vec{\mathcal{P}}_q + \vec{\mathcal{P}}_g = \int \mathrm{d}^3x\psi^\dagger\frac{\vec{D}}{\mathrm{i}}\psi + \int\mathrm{d}^3x\vec{E}\times\vec{B},$$
$$\vec{D} = \vec{\nabla} - \mathrm{i}g\vec{A}. \tag{12}$$

Both the quark and gluon "momentum" operators $\vec{\mathcal{P}}_q$ and $\vec{\mathcal{P}}_g$ defined in Eq. (12) are gauge invariant but neither the quark "momentum" $\vec{\mathcal{P}}_q$ nor the gluon "momentum" $\vec{\mathcal{P}}_g$ satisfies the momentum algebra, for example,

$$[D_l, D_m] = -\mathrm{i}g(\partial_l A_m - \partial_m A_l) - \mathrm{i}g^2 C_{abc}A_l^a A_m^b T^c, \tag{13}$$

where C_{abc} is the $SU(3)$ group structure constant. The $\vec{\mathcal{P}}_g$ does not satisfy the momentum algebra either in the interacting quark-gluon field, i.e. QCD case. Therefore neither the $\vec{\mathcal{P}}_q$ nor the $\vec{\mathcal{P}}_g$ used in the OPE is the real momentum operator.

The gluon spin contribution is under intensive study, PHENIX, STAR, COMPASS, HERMES, and others are measuring the gluon spin contribution to nucleon spin. However there is no gluon spin operator which satisfies both gauge invariance and angular momentum algebra. There is also no quark, gluon orbital angular momentum operator which satisfies the gauge invariance and orbital angular momentum algebra. These situations hindered the study of the nucleon spin structure.

3 A new set of momentum, orbital angular momentum and spin operators for the fermion and gauge field parts

3.1 Decomposing the gauge field A_μ into pure gauge part A_pure and gauge invariant (covariant) part A_phys

Let us start from the simpler QED case. It is well known that to use gauge potential A_μ to describe the electromagnetic field the A_μ is not unique, i.e., there is gauge freedom. Under a gauge transformation,

$$A'^\mu = A^\mu + \partial^\mu\omega(x), \tag{14}$$

one obtains a new gauge potential A'^μ from A^μ. A^μ and A'^μ describe the same electromagnetic field,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_\mu A'_\nu - \partial_\nu A'_\mu. \tag{15}$$

Such a gauge freedom is necessary because the gauge potential A^μ plays two role in gauge field theory: the first is to provide a pure gauge field A_pure to compensate the induced field due to the phase change in a local gauge transformation of the Fermion field $\psi'(x) = e^{-ie\omega(x)}\psi(x)$ which must be varied with the arbitrary changed phase parameter $\omega(x)$; the second is to provide a physical field $A_{\rm phys}$ for the physical interaction between Fermion field and gauge field which should be gauge invariant under gauge transformation. The pure gauge potential $A_{\rm pure}$ should not contribute to electromagnetic field,

$$F^{\mu\nu}_{\rm pure}=\partial^\mu A^\nu_{\rm pure}-\partial^\nu A^\mu_{\rm pure}=0. \tag{16}$$

This equation can not fix the $A_{\rm pure}$. One has to find additional condition to fix it. The spatial part of Eq. (16) is

$$\nabla\times\vec{A}_{\rm pure}=0, \tag{17}$$

which means $\vec{A}_{\rm pure}$ does not contribute to magnetic field. This equation can be expressed in another form,

$$\nabla\times\vec{A}_{\rm phys}=\nabla\times\vec{A}. \tag{18}$$

A natural choice of the additional condition in QED case is

$$\nabla\cdot\vec{A}_{\rm phys}=0, \tag{19}$$

which is the transverse wave condition and we know that this part is the physical one from the Coulomb gauge quantization. Combining these two conditions, Eqs. (18) and (19), under the natural boundary condition,

$$\vec{A}_{\rm phys}(|x|\to\infty)=0, \tag{20}$$

for any given set of gauge field $\vec{A}$, one can decompose it uniquely as follows,

$$\vec{A}=\vec{A}_{\rm pure}+\vec{A}_{\rm phys}, \tag{21}$$

where

$$\begin{aligned}\vec{A}_{\rm phys}(x) &= \vec{\nabla}\times\frac{1}{4\pi}\int {\rm d}^3x'\frac{\vec{\nabla}'\times\vec{A}(x')}{|\vec{x}-\vec{x}'|},\\ \vec{A}_{\rm pure}(x) &= \vec{A}-\vec{A}_{\rm phys}(x).\end{aligned} \tag{22}$$

We have to emphasize that for fixed $\vec{A}(x)$, the integration can be done and the obtained $\vec{A}_{\rm phys}$ is a local function. It is easy to prove that these two parts transform as follows in a gauge transformation Eq. (14),

$$\begin{aligned}\vec{A}'_{\rm phy} &= \vec{A}_{\rm phy},\\ \vec{A}'_{\rm pure} &= \vec{A}_{\rm pure}-\vec{\nabla}\omega(x).\end{aligned} \tag{23}$$

The time component A^0 can be decomposed in the same manner, from the condition $F^{i0}_{\rm pure}=0$, one obtains

$$\begin{aligned}\partial_i A^0_{\rm phys} &= \partial_i A^0+\partial_t(A^i-A^i_{\rm phys}),\\ A^0_{\rm phys} &= \int_{-\infty}^{x}{\rm d}x^i(\partial_i A^0+\partial_t A^i-\partial_t A^i_{\rm phys}).\end{aligned} \tag{24}$$

The $A^\mu_{\rm pure}=A^\mu-A^\mu_{\rm phys}$ can also be obtained from Eq. (17),(19) and (24) directly,

$$\begin{aligned}\vec{A}_{\rm pure} &= \vec{\nabla}\phi(x),\\ \phi(x) &= -\frac{1}{4\pi}\int{\rm d}^3x'\frac{\nabla'\cdot\vec{A}(x')}{|\vec{x}-\vec{x}'|}+\phi_0(x),\\ A^0_{\rm pure} &= -\partial_t\phi(x),\end{aligned} \tag{25}$$

where $\phi_0(x)$ satisfies the condition,

$$\nabla^2\phi_0(x)=0, \tag{26}$$

and is determined by the boundary condition. From Eq. (25) one can see that $A^\mu_{\rm pure}$, and so $A^\mu_{\rm phys}$, is Lorentz 4-vector.

To decompose the gauge potential $A_\mu=A^a_\mu T^a$ for the gluon field is more complicated than QED case. We first define the pure gauge potential $A^\mu_{\rm pure}$ (hereafter we omit the color indices if not necessary) by the same condition, i.e., it does not contribute to color electromagnetic field,

$$F^{\mu\nu}_{\rm pure}=\partial^\mu A^\nu_{\rm pure}-\partial^\nu A^\mu_{\rm pure}+{\rm i}g[A^\mu_{\rm pure},A^\nu_{\rm pure}]=0. \tag{27}$$

In order to make this definition condition looks similar to Eq. (17), we introduce a notation,

$$\begin{aligned}\vec{D}_{\rm pure} &= \vec{\nabla}-{\rm i}g\vec{A}_{\rm pure},\\ \vec{D}_{\rm pure}\times\vec{A}_{\rm pure} &= \vec{\nabla}\times\vec{A}_{\rm pure}-{\rm i}g\vec{A}_{\rm pure}\times\vec{A}_{\rm pure}=0.\end{aligned} \tag{28}$$

The additional condition is even more complicated, i.e., one does not have a natural choice as Eq. (19) in QED. We make the following choice[5],

$$\begin{aligned}\vec{\mathcal{D}}_{\rm pure} &= \vec{\nabla}-{\rm i}g[\vec{A}_{\rm pure},\ ]\\ \vec{\mathcal{D}}_{\rm pure}\cdot\vec{A}_{\rm phys} &= \vec{\nabla}\cdot\vec{A}_{\rm phys}-{\rm i}g[\vec{A}^i_{\rm pure},\vec{A}^i_{\rm phys}]=0.\end{aligned} \tag{29}$$

The summation over the vector component i has been assumed in the above equation and following ones. Please note that in the above adjoint representation of the new covariant derivative operator $\vec{\mathcal{D}}$, the bracket $[A^i_{\rm pure},A^i_{\rm phys}]$ is not the quantum bracket but a color $SU^{\rm c}(3)$ group one,

$$[A^i_{\rm pure},A^i_{\rm phys}]={\rm i}C_{\rm abc}A^{ib}_{\rm pure}A^{ic}_{\rm phys}T^a.$$

These equations can be rewritten as follows,

$$\begin{aligned}\vec{\nabla}\cdot\vec{A}_{\rm phys} &= {\rm i}g[\vec{A}^i-\vec{A}^i_{\rm phys},\vec{A}^i_{\rm phys}]={\rm i}g[\vec{A}^i,\vec{A}^i_{\rm phys}],\\ \vec{\nabla}\times\vec{A}_{\rm phys} &= \vec{\nabla}\times\vec{A}-{\rm i}g(\vec{A}-\vec{A}_{\rm phys})\times(\vec{A}-\vec{A}_{\rm phys}),\\ \partial_i A^0_{\rm phys} &= \partial_i A^0+\partial_t(A^i-A^i_{\rm phys})-{\rm i}g[A^i-A^i_{\rm phys},\\ &\quad A^0-A^0_{\rm phys}].\end{aligned} \tag{30}$$

These equations can be solved perturbatively: in the zeroth order, i.e., assuming $g=0$, these equations are the same as those of QED, one can obtain the zeroth order solution; then taking into account the nonlinear coupling through iteration one obtains a perturbative solution as a power expansion in g.

If one assumes a trivial boundary condition for the pure gauge field $A_{\rm pure}$, then one can use the following equations to obtain a perturbative solution too,

$$\begin{aligned}\vec{\nabla}\times\vec{A}_{\rm pure} &= {\rm i}g\vec{A}_{\rm pure}\times\vec{A}_{\rm pure},\\ \vec{\nabla}\cdot\vec{A}_{\rm pure} &= \vec{\nabla}\cdot\vec{A}-{\rm i}g[A^i_{\rm pure},A^i],\\ \partial_i A^0_{\rm pure} &= -\partial_t A^i_{\rm pure}+{\rm i}g[A^i_{\rm pure},A^0_{\rm pure}].\end{aligned}\tag{31}$$

Under a gauge transformation,

$$\begin{aligned}\psi' &= U\psi,\\ A'_\mu &= UA_\mu U^\dagger-\frac{\rm i}{g}U\partial_\mu U^\dagger,\end{aligned}\tag{32}$$

where $U=e^{-{\rm i}g\omega}$. The $\vec{A}_{\rm pure}$ and $\vec{A}_{\rm phys}$ will be transformed as

$$\begin{aligned}\vec{A'}_{\rm phys} &= U\vec{A}_{\rm phys}U^\dagger,\\ \vec{A'}_{\rm pure} &= U\vec{A}_{\rm pure}U^\dagger-\frac{\rm i}{g}U\nabla U^\dagger.\end{aligned}\tag{33}$$

3.2 Quantum mechanics

We have mentioned in the introduction part that even in quantum mechanics, there are already puzzles related to the fundamental operators, the matrix elements of canonical momentum, orbital angular momentum and Hamiltonian of a charged particle moving in an electromagnetic field are all not gauge invariant. In order to get rid of these puzzles, gauge invariant operators have been introduced,

$$\begin{aligned}\vec{p}\,' &= \vec{p}-e\vec{A},\\ \vec{L}' &= \vec{x}\times\vec{p}\,'.\end{aligned}\tag{34}$$

It is easy to check that the matrix elements of these operators are gauge invariant. However as we have pointed out in Eq.(13), that the gauge invariant "momentum" $\vec{p}\,'$ does not satisfy the canonical momentum Lie algebra, so they are not the real momentum. The gauge invariant "orbital angular momentum" $\vec{L}\,'$ does not satisfy the angular momentum Lie algebra either.

Based on our proposed gauge field decomposition in the above section, we introduce another set of momentum and orbital angular momentum operators which satisfy both gauge invariance and the corresponding commutation relation,

$$\begin{aligned}\vec{p}_{\rm pure} &= \vec{p}-e\vec{A}_{\rm pure},\\ \vec{L}_{\rm pure} &= \vec{x}\times\vec{p}_{\rm pure}.\end{aligned}\tag{35}$$

The long standing puzzle, the gauge non-invariance of the expectation value of the Hamiltonian[4] can be solved in the same manner, for the non-relativistic quantum mechanics, the new Hamiltonian is

$$H=\frac{(\vec{p}-e\vec{A})^2}{2m}+eA^0-e\partial_t\phi(x).\tag{36}$$

The last term is a pure gauge term, it cancels the unphysical energy appeared in eA^0 induced by the pure gauge term and then guarantees the expectation value of this Hamiltonian gauge invariant. It is a direct extension of Eq. (35) to the fourth momentum component.

The Dirac Hamiltonian has the same unphysical energy part and has to be canceled in the same manner as that for the Schrödinger Hamiltonian. Here we have done a check: starting from a QED Lagrangian with both electron and proton, under the infinite proton mass approximation, we derived the Dirac equation of electron and the gauge invariant Hamiltonian of the electron part and verified the difference between the Dirac Hamiltonian obtained from the Dirac equation and the gauge invariant one.

Our study shows that the canonical momentum, orbital angular momentum and the Hamiltonian used in the quantum mechanics are not obserevables, one must subtract the pure gauge part, the unphysical one, from these operators as we did in Eqs. (35) and (36) to obtain the observable ones.

3.3 QED

We have explained that the momentum and angular momentum operators of the Fermion and gauge field part, Eqs. (10) and (11), derived from the QED Lagrangian by means of Noether theorem are not gauge invariant except the Fermion spin. One can obtain a gauge invariant decomposition by adding a surface term to Eqs. (10) and (11) or from the Belinfante symmetric energy-momentum tensor,

$$\begin{aligned}\vec{P} &= \vec{\mathcal{P}}_{\rm e}+\vec{\mathcal{P}}_{\rm ph}=\\ &\int{\rm d}^3x\psi^\dagger\frac{\vec{D}}{\rm i}\psi+\int{\rm d}^3x\vec{E}\times\vec{B}\end{aligned}\tag{37}$$

$$\begin{aligned}\vec{J} &= \vec{S}_{\rm e}+\vec{L}'_{\rm e}+\vec{J}'_{\rm ph}=\int{\rm d}^3x\psi^\dagger\frac{\vec{\Sigma}}{2}\psi+\\ &\int{\rm d}^3x\vec{x}\times\psi^\dagger\frac{\vec{D}}{\rm i}\psi+\int{\rm d}^3x\vec{x}\times(\vec{E}\times\vec{B}).\end{aligned}\tag{38}$$

There are two problems with this decomposition: (1), $\vec{L}'_{\rm e}$ and $\vec{J}'_{\rm ph}$ do not satisfy the angular momentum commutation relation even though in free electromagnetic field the photon total angular momentum $\vec{J}'_{\rm ph}$ does; (2), there is no separate photon spin and orbital angular momentum operators and this feature will ruin the multipole radiation analysis as we discussed in the second section.

Based on the decomposition of the gauge field potential into pure gauge and the physical parts, Eq.(21), we obtain the following decomposition,

$$\vec{P} = \vec{P}_{\rm e}+\vec{P}_{\rm ph},=\int {\rm d}^3x\psi^\dagger\frac{\vec{D}_{\rm pure}}{\rm i}\psi+ \int {\rm d}^3xE^i\vec{\mathcal{D}}_{\rm pure}A^i_{\rm phys}. \quad (39)$$

$$\vec{J} = \vec{S}_{\rm e}+\vec{L}_{\rm e}+\vec{S}_{\rm ph}+\vec{L}_{\rm ph}=\int {\rm d}^3x\psi^\dagger\frac{\vec{\Sigma}}{2}\psi+ \int {\rm d}^3x\vec{x}\times\psi^\dagger\frac{\vec{D}_{\rm pure}}{\rm i}\psi+\int {\rm d}^3x\vec{E}\times\vec{A}_{\rm phys}+ \int {\rm d}^3x\vec{x}\times E^i\vec{\mathcal{D}}_{\rm pure}A^i_{\rm phys}. \quad (40)$$

Here the operator $\vec{D}_{\rm pure}$ and $\vec{\mathcal{D}}_{\rm pure}$ are the same as giving in Eqs. (28) and (29) but with g replaced by e. Because of the Abelian property of the U(1) gauge field, the adjoint representation of the operator $\vec{\mathcal{D}}$ is simplified to be a simple $\vec{\nabla}$. It is not hard to check that every operator in the above decomposition, Eqs. (39) and (40) is gauge invariant and satisfies the momentum, angular momentum commutation relation.

The photon spin and orbital angular momentum operators are well defined as shown in Eq. (40). The multipole radiation analysis is theoretically sound now as it should be.

3.4 QCD

One can copy the results for QED, the Eqs. (39,40), to QCD to obtain the quark, gluon momentum, orbital angular momentum and spin operators which satisfy both the gauge invariance and the canonical momentum and angular momentum commutation relations.

A decomposition of the form of Eqs. (37,38) has been used in the nucleon spin structure study for the last ten years[6]. Every operator in those decomposition is gauge invariant and so corresponding to observable, however because they do not satisfy the momentum, angular momentum Lie algebra so the measured ones are not the quark, gluon momentum and orbital angular momentum and can not be connected to those used in hadron spectroscopy.

The gluon spin operator had been searched for more than ten years in the nucleon spin structure study and no satisfied one was obtained. Now one can calculate the matrix element of the gluon spin operator $\vec{S}_g=\int {\rm d}^3x\vec{E}\times\vec{A}_{\rm phys}$ between the polarized nucleon state $|{\rm N(p,s)}\rangle$ to obtain the gluon spin contribution to nucleon spin and compared it with the measured ones.

4 Reexamination of the nucleon internal structure

The nucleon internal structure has been studied based on the gauge invariant but canonical momentum, angular momentum Lie algebra violated operators given in Eqs. (37, 38) for the past years. This led to a distorted picture of the nucleon internal structure. For example, the quark and gluon carry half of the nucleon momentum in the asymptotic limit has been a deeply rooted picture of nucleon internal momentum structure. Using the new quark, gluon momentum operator we recalculated their scale evolution and obtained the new mixing matrix,

$$\gamma^P = -\frac{\alpha_{\rm s}}{4\pi}\begin{pmatrix} -\frac{2}{9}n_g & \frac{4}{3}n_f \\ \frac{2}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}. \quad (41)$$

Which gives the new asymptotic limit for the renormalized gluon momentum,

$$\vec{P}^R_g=\frac{\frac{1}{2}n_g}{\frac{1}{2}n_g+3n_f}\vec{p}_{\rm total}. \quad (42)$$

For typical gluon number $n_g=8$ and quark flavor number $n_f=5$, the above equation gives $\vec{p}^R_g\simeq\frac{1}{5}\vec{P}_{\rm total}$. This is distinctly different from the renowned results $\vec{P}^R_g\simeq\frac{1}{2}\vec{P}_{\rm total}$. This latter result is obtained from the mixing matrix,

$$\gamma^P = -\frac{\alpha_{\rm s}}{4\pi}\begin{pmatrix} -\frac{8}{9}n_g & \frac{4}{3}n_f \\ \frac{8}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}, \quad (43)$$

which is obtained by means of the quark and gluon momentum operators given in Eq. (37). The mixing matrix element of Eq. (43) leads to the well known asymptotic limit,

$$\vec{\mathcal{P}}^R_g=\frac{2n_g}{2n_g+3n_f}\vec{P}_{\rm total}. \quad (44)$$

However the $\vec{\mathcal{P}}_g$ and $\vec{\mathcal{P}}_q$ used in this quark gluon momentum scale evolution calculation are not the real momentum operators, part of the quark momentum had been shifted to the gluon and gave the superficial large gluon momentum contribution to nucleon momentum.

The asymptotic nucleon spin structure[7] is obtained based on the decomposition Eq. (11), a QED analog of QCD angular momentum decomposition. The authors had pointed out that the quark and gluon orbital angular momentum operators are not gauge invariant. As we have mentioned in the beginning that in the present gauge field theory, an observable must be expressed in terms of a gauge invariant operator. The gauge dependent operators used in this analysis[7] is not measurable ones. Therefore this asymptotic limit of nucleon spin content should be reexamined.

Another nucleon internal structure parameters are the parton distribution function (PDF). Such as the quark PDF in a target A is defined as,

$$\mathscr{P}_{q/A}(\xi)=\frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}x^-}{2\pi}\mathrm{e}^{-\mathrm{i}\xi P^+x^-}\langle\bar{\psi}(0,x^-,0_\perp)\gamma^+ \mathcal{P}\exp\{\mathrm{i}g\int_0^{x^-}\mathrm{d}y^-A^+(0,y^-,0_\perp)\}\psi(0)\rangle_A, \quad (45)$$

where a gauge link (Wilson line) is inserted to achieve the gauge invariance. Based on our gauge field decomposition discussed in the third section that the above gauge link not only includes the necessary pure gauge $A_{\rm pure}$ part to achieve the gauge invariance, but also includes the physical part $A_{\rm phys}$ which induced a physical coupling and makes the PDF defined in Eq. (45) an interaction-involving one. The interaction term is more clear in the momentum relation,

$$\int_{-\infty}^{\infty}\mathrm{d}\xi\xi\mathscr{P}_{q/A}(\xi)=\frac{1}{2(P^+)^2}\langle\bar{\psi}\gamma^+\mathrm{i}D^+\psi\rangle_A. \quad (46)$$

This is just the + componetnt of $\vec{\mathcal{P}}_q(x)$ in Eq. (12). Here the gauge field in D^+ originates exactly from the gauge link in Eq. (45).

To obtain a gauge invariant quark PDF, a gauge link with the pure gauge part is enough,

$$P_{q/A}(\xi)=\frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}x^-}{2\pi}\mathrm{e}^{-\mathrm{i}\xi P^+x^-}\langle P|\bar{\psi}(0,x^-,0_\perp)\gamma^+ \mathcal{P}\exp\{\mathrm{i}g\int_0^{x^-}\mathrm{d}y^-A^+_{\rm pure}(0,y^-,0_\perp)\}\psi(0)|P\rangle_A, \quad (47)$$

this PDF will not includes the redundant physical gauge interaction and the integration gives the real quark momentum defined in Eq. (39).

$$\int_{-\infty}^{\infty}\mathrm{d}\xi\xi P_{q/A}(\xi)=\frac{1}{2(P^+)^2}\langle\bar{\psi}\gamma^+\mathrm{i}D^+_{\rm pure}\psi\rangle_A. \quad (48)$$

Analogously, the conventional gluon PDF

$$\mathscr{P}_{g/A}(\xi)=\frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{\mathrm{d}x^-}{2\pi}\mathrm{e}^{-\mathrm{i}\xi P^+x^-}\langle F^{+\nu}(0,x^-,0_\perp) \mathcal{P}\exp\{\mathrm{i}g\int_0^{x^-}\mathrm{d}y^-A^+(0,y^-,0_\perp)\}F_\nu{}^+(0)\rangle_A, \quad (49)$$

can be replaced accordingly to our strategy as

$$P_{g/A}(\xi)=\int_{-\infty}^{\infty}\frac{\mathrm{d}x^-}{2\pi}\mathrm{e}^{-\mathrm{i}\xi P^+x^-}\langle F^{+i}(0,x^-,0_\perp) \mathcal{P}\exp\{\mathrm{i}g\int_0^{x^-}\mathrm{d}y^-A^+_{\rm pure}(0,y^-,0_\perp)\}A^i_{\rm phys}(0)\rangle_A, \quad (50)$$

where besides the pure gauge link, the physical component $\vec{A}_{\rm phys}$ is used instead of F^+_ν as the gauge invariant variable. The second moments of $\mathcal{P}_{g/A}$ and $P_{g/A}$ relate to Poynting and the real gluon momentum in Eqs. (12) and (39), (the QCD quark and gluon momentums have exactly the same expression as those of QED, only the subscript e and ph are replaced by q and g).

Our approach is also convenient in constructing the gauge invariant polarized and transverse-momentum dependent PDFs with clear particle number interpretation, and 0ff-forward PDFs which can be measured to infer the real orbital angular momentums in Eq. (40). For example the polarized gluon PDF can be defined gauge invariantly as

$$P_{\Delta g/A}(\xi)=\int_{-\infty}^{\infty}\frac{\mathrm{d}x^-}{2\pi}\mathrm{e}^{-\mathrm{i}\xi P^+x^-}\langle F^{+i}(0,x^-,0_\perp) \mathcal{P}\exp\{\mathrm{i}g\int_0^{x^-}\mathrm{d}y^-A^+_{\rm pure}(0,y^-,0_\perp)\}\epsilon_{ij+}A^j_{\rm phys}(0)\rangle_A, \quad (51)$$

with a first moment relating to the gauge invariant gluon spin in Eq. (40).

5 Summary and prospect

Since the establishment of gauge invariance principle, we enjoy that the total momentum, angular momentum and the Lorentz boosting operators of a gauge system satisfy both the gauge invariance and Poincaré algebra, however we never have the separate momentum, orbital angular momentum operators of the fermion (electron in QED, quark in QCD) and boson (photon in QED, gluon in QCD) part which satisfy both the gauge invariance and the canonical momentum, angular momentum Lie algebra. We have the electron, quark spin operator but we never have the photon, gluon spin operator which satisfy both the gauge invariance and spin Lie algebra. Even it had been claimed in some textbooks that it is impossible to have a well defined photon spin[3]. The nucleon spin structure study needs the gluon spin operator, but after about ten years effort in searching a gluon spin operator since the so-called proton spin crisis such an effort has almost been given up for the last ten years. In this report we proposed a new set of quark (electron), gluon (photon) momentum, orbital angular momentum and spin operators which satisfy both the gauge invariance and the canonical momentum, angular momentum Lie algebra.

To achieve this a key point is to separate the gauge field into pure gauge and physical parts: the formal is unphysical and can be gauged away as in Coulomb gauge, it is used to compensate the induced unphysical gauge field due to the local gauge transformation of the Fermion field to keep the gauge invariance; the physical part is responsible for the physical coupling between Fermion and boson field. It is physical and should be gauge invariant (covariant). We provide a method to do this separation both for the Abelian $U(1)$ and the non-Abelian $SU(3)$ gauge field. Recently we found such an idea can be extended to gravitation field and help to get a general covariant energy-momentum tensor.

Our proposed momentum operators for the Fermion part are different from the canonical ones, the latter ones are not gauge invariant and so do not represent observables because they include the unphysical pure gauge field contribution. The new ones subtract the unphysical pure gauge field contribution and so they are gauge invariant and represent the observables.

We achieved to obtain a gauge invariant orbital angular momentum and spin operators of the photon and gluon by means of the physical part of the gauge field, Eq. (40), which provides the theoretical basis of the widely used multipole radiation analysis, the photon spin and orbital angular momentum used in quantum computation and communication study, the gluon spin contribution in the nucleon spin structure study.

The Poincaré algebra can not be fully maintained for the momentum, angular momentum and Lorentz boosting operators of the individual Fermion and boson part of an interacting gauge field system. What is the meaning of these observables if they are not Lorentz covariant? We have shown that the momentum and angular momentum algebra can be maintained simultaneously with the gauge invariance. How much part of the Poincaré algebra can be maintained for the operators of the interacting Fermion and boson separately, especially the Lorentz covariance can be maintained to what extent are left for further study.

The new asymptotic limit of quark and gluon parton momentums of a nucleon have been obtained, the immediate problem is the asymptotic limit of the quark and gluon orbital angular momentums and spins.

The gluon spin contribution to the nucleon spin is under measurement in different labs. A lattice QCD calculation with the gauge invariant gluon spin operator is called for.

To obtain the new PDFs the factorization theorems with respect to the new PDF formula should be examined.

In summary, the nucleon internal structure is better to be reexamined based on the new quark, gluon momentum, orbital angular momentum, spin operators and parton distribution functions and our picture of the nucleon internal structure might be modified.

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