Two-photon exchange to proton electromagnetic properties in time-like region*

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Abstract The contributions of two-photon exchange in the process $\mathrm{e}^+ + \mathrm{e}^- \to \mathrm{p} + \bar{\mathrm{p}}$ including N and Δ intermediate states are estimated in a simple hadronic model. The corrections to the unpolarized cross section as well as to the polarized observables P_x and P_z are evaluated. The results show the corrections to unpolarized cross section are small and the angle dependence becomes weak at small s after considering the N and $\Delta(1232)$ contributions simultaneously, while the correction to P_z is enhanced.

Key words two-photon exchange, nucleon, delta, form factor

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1 Introduction

The electromagnetic form factors in both space-like ($Q^2>0$) and time-like ($Q^2<0$) regions are essential to understand the intrinsic structures of hadrons. The experimental data of elastic form factors over several decades, including recent high precision measurement at Jefferson Lab[1, 2] and elsewhere[3], have provided considerable insight into the detail structure of the nucleon.

In the space-like region, the standard method that has been used to determine the electric and magnetic form factors, particularly those of the proton has been the Rosenbluth, or longitudinal-transverse (LT), separation method. The results of the Rosenbluth measurements for the proton form factor ratio $R=\mu_p G_E/G_M$ have generally been consistent with $R\approx 1$ for $Q^2\leqslant 6\ \mathrm{GeV}^2$[4—7]. Polarized lepton beams give another way to access the form factors[8] and has been applied only recently in Jefferson Lab[1], The result about the ratio of Sachs form factors[9, 10] is monotonically decreasing with increasing of Q^2, which strongly contradicts to the scaling ratio determined by the traditional Rosenbluth separation method[11].

In order to explain the discrepancy caused by different experimental measurement, radiative corrections, especially the two-photon contribution, have been involved[12—19]. From these calculations one can conclude that the two-photon exchange (TPE) corrections can, at least, partly explain the discrepancy of the two separation methods. Further more the amplitudes of TPE process have imaginary parts. In this case, the $1\gamma\otimes 2\gamma$ interference terms are supposed to be more important in time-like region, as the form factors are complex.

In the theoretical point of view, it seems unavoidable to check the TPE contributions to the nucleon form factors in the time-like region. In simple hadronic model, we calculate the TPE correction to the unpolarized differential cross section as well as the double spin polarization observables of $\mathrm{e}^+ + \mathrm{e}^- \to \mathrm{p} + \bar{\mathrm{p}}$ process.

2 TPE in simple hadronic model

Using the simple hadronic model[14, 15, 19] and including N and Δ as the intermediate states like Fig. 1, the unpolarized cross section can be written as

$$\mathrm{d}\sigma=\mathrm{d}\sigma_0(1+\delta_{\mathrm{N}}^{2\gamma}+\delta_{\Delta}^{2\gamma})\propto\sum|\mathcal{M}_0+\mathcal{M}_{\mathrm{N}}^{2\gamma}+\mathcal{M}_{\Delta}^{2\gamma}|^2,\quad(1)$$

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where $\mathcal{M}_0$ is the contribution from one-photon exchange diagram and $\mathcal{M}^{2\gamma}_{\mathrm{N},\Delta}$ denote the contribution from TPE diagrams with N and Δ as intermediate state.

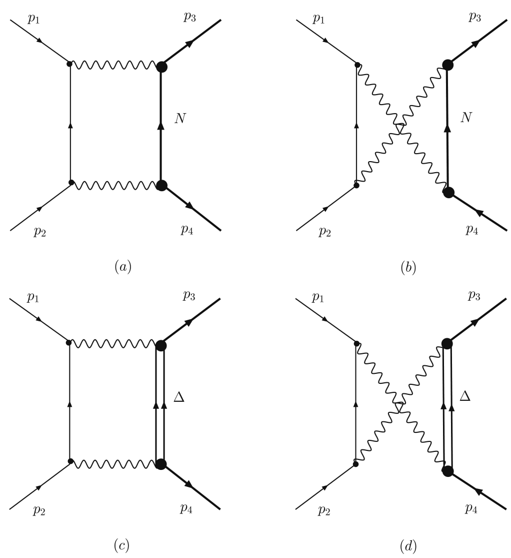

Fig. 1. Feynman diagrams included in present calculations.

The corrections to the unpolarized cross section can be defined as

$$\delta^{2\gamma}_{\mathrm{N},\Delta}=\frac{\sum 2\mathrm{Re}\{\mathcal{M}^{2\gamma}_{\mathrm{N},\Delta}\mathcal{M}^{\dagger}_0\}}{\sum|\mathcal{M}_0|^2}. \quad (2)$$

In our present work, for $\mathrm{e}^++\mathrm{e}^-\to\mathrm{p}+\bar{\mathrm{p}}$ process, the incoming electron is longitudinally polarized, while the polarization of anti-proton in the final state is measured. Then a similar calculation can be applied to the polarized quantities P_x and P_z [20, 21] with the definitions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\mathrm{d}\sigma_{\mathrm{un}}}{\mathrm{d}\Omega}[1+P_y\xi_y+\lambda_e P_x\xi_x+\lambda_e P_z\xi_z]. \quad (3)$$

Here we define TPE corrections to double spin polarized observables as[21]:

$$\delta(P_{x,z})=P^{1\gamma\otimes2\gamma}_{x,z}/P^{1\gamma}_{x,z}. \quad (4)$$

To discuss the correction from Δ, we take the following matrix elements as[15, 23]

$$\Gamma^{\mu\alpha}_{\mathrm{N}\to\Delta\gamma}=\frac{-F_\Delta(q^2)}{M^2_{\mathrm{N}}}[g_1(g^{\alpha}_{\mu}\hat{k}\hat{q}-k_\mu\gamma^\alpha\hat{q}-\gamma_\mu\gamma^\alpha k\cdot q+\gamma_\mu\hat{k}q^\alpha)+g_2(k_\mu q^\alpha-k\cdot qg^\alpha_\mu)+g_3/M_{\mathrm{N}}(q^2(k_\mu\gamma^\alpha-g^\alpha_\mu\hat{k})+q_{1\mu}(q^\alpha\hat{k}-\gamma^\alpha k\cdot q))]\gamma_5T_3\ ,$$

$$\Gamma^{\mu\alpha}_{\gamma\to\Delta\bar{\mathrm{N}}}=\frac{-F_\Delta(q^2)}{M^2_{\mathrm{N}}}T^+_3\gamma_5[g_1(g^{\beta}_{\nu}\hat{q}\hat{k}-k_\nu\hat{q}\gamma^\beta-\gamma^\beta\gamma_\nu k\cdot q+\hat{k}\gamma_\nu q^\beta)+g_2(k_\nu q^\beta-k\cdot qg^\beta_\nu)-g_3/M_{\mathrm{N}}(q^2(k_\nu\gamma^\beta-g^\beta_\nu\hat{k})+q_{2\nu}(q^\beta\hat{k}-\gamma^\beta k\cdot q))], \quad (5)$$

where k (or q) is the momentum of the photon (or Δ) and T_3 is the third component of the $\mathrm{N}\to\Delta$ isospin transition operator. Here and after we have $\hat{k}\equiv\gamma\cdot k$.

For the propagator of Δ, the same form is employed as[15]

$$S^{\Delta}_{\alpha\beta}(k)=\frac{-\mathrm{i}(\hat{k}+M_\Delta)}{k^2-M^2_\Delta+\mathrm{i}\epsilon}P^{3/2}_{\alpha\beta}(k),$$

$$P^{3/2}_{\alpha\beta}(k)=g_{\alpha\beta}-\frac{\gamma_\alpha\gamma_\beta}{3}-\frac{(\hat{k}\gamma_\alpha k_\beta+k_\alpha\gamma_\beta\hat{k})}{3k^2}. \quad (6)$$

In the practical calculation, we take the form factor F_Δ in the monopole form as G_E in N case[21]

$$F_\Delta(q^2)=G_E(q^2)=G_M(q^2)/\mu_p=\frac{-\Lambda^2_1}{q^2-\Lambda^2_1}, \quad (7)$$

and the coupling parameters and cut-offs are[21, 23]

$$g_1=1.91,\ g_2=2.63,\ g_3=1.58,\ \Lambda_1=0.84\ \mathrm{GeV}. \quad (8)$$

3 Numerical results and discussion

Taking the Eqs. (7) and (8) as input, the TPE corrections can be calculated directly. We use the package FeynCalc[24] and LoopTools[25] to carry out the calculation. Practically, for the interaction of the outgoing hadrons, the time-like form factors have a phase structure, which means the form factors are complex in the time-like region. But the phase structures of the form factors keep unknown at present. In the IJL model with two components fits[26], one can reproduce the latest JLab data in the space-like region. After continued to the time-like region, we find the imaginary parts of the form factors are much smaller than the real parts. Further more, what we concern in this work are the ratio $\delta_{2\gamma}$ and double spin polarization observables P_x and P_z. The phenomenological form factors appear in both denominator and numerator of these physical observables. In such cases, the form of form factors varies the ratio and polarization observables in a very limited extension. The same conclusion can be drawn from the results of two-photon exchange corrections to space-like form factors[17].

The IR divergence only exist in the N intermediate case and it is exact in the soft calculations. In amplitudes of TPE process, after replacing one of the photon momentum in the numerator by zero, one can get the results with soft approximation. For example, the TPE correction to unpolarized differential cross section is,

$$\delta^{2\gamma}_{\rm soft} = -2\frac{\alpha}{\pi}\ln\left|\frac{s-M_{\rm N}^2}{s+t-M_{\rm N}^2}\right|\ln\left|\frac{t}{\lambda^2}\right|. \qquad (9)$$

with $s=(p_1-p_3)^2$, $t=(p_1+p_2)^2=q^2$ and λ is infinitesimal photon mass, which has been introduced in the photon propagator to regulate the IR divergence. The finite corrections we calculated are the full calculations of TPE corrections minus the IR part of soft approximation and these finite corrections are independent with the λ. Our numerical results verify such cancelation.

The numerical results for $\delta^{2\gamma}_{\rm N,\Delta}$ are showed in Fig. 2. The correction $\delta^{2\gamma}_{\Delta}$ is found to be always opposite to the corrections $\delta^{2\gamma}_{\rm N}$ in all the angle region. This behavior is similar to the ep scattering case[15]. Detailedly, at $q^2=4$ GeV^2 the absolute magnitude of $\delta^{2\gamma}_{\Delta}$ is so close to $\delta^{2\gamma}_{\rm N}$ that results in the large cancelation and small total correction to unpolarized cross section. The small $\delta^{2\gamma}_{\rm N+\Delta}$ and its weak angle dependence suggest the Rosenbluth method will work well in this region. This conclusion is some different with the ep scattering case where the cancelation is much smaller and the total correction still strongly depend on the scattering angle. At $q^2=5$ GeV^2, the absolute magnitude of $\delta^{2\gamma}_{\Delta}$ becomes larger than $\delta^{2\gamma}_{\rm N}$ which suggests the important roles played by $\Delta(1232)$ intermediate state in the process of $e^+ + e^- \rightarrow p + \bar{p}$.

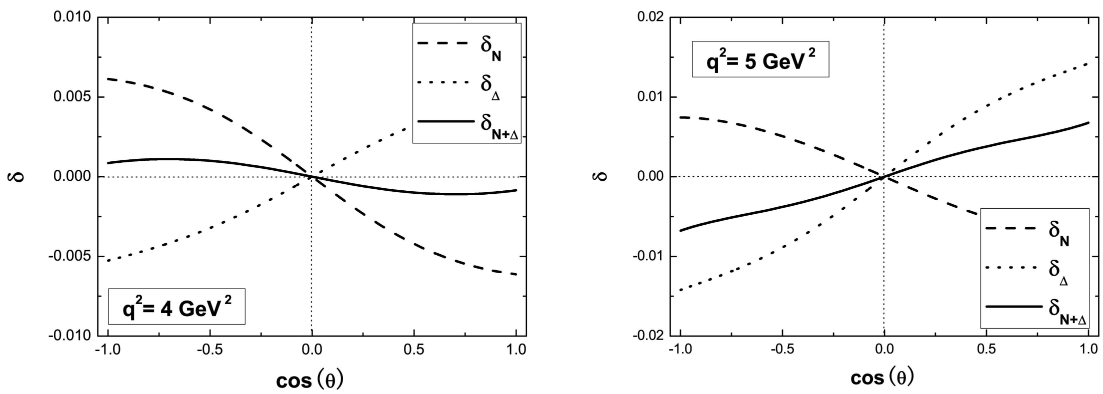

Fig. 2. TPE contributions to unpolarized differential cross section. The dashed and dotted lines are the contributions from nucleon and Δ intermediate states separately, while the solid line are the total contributions.

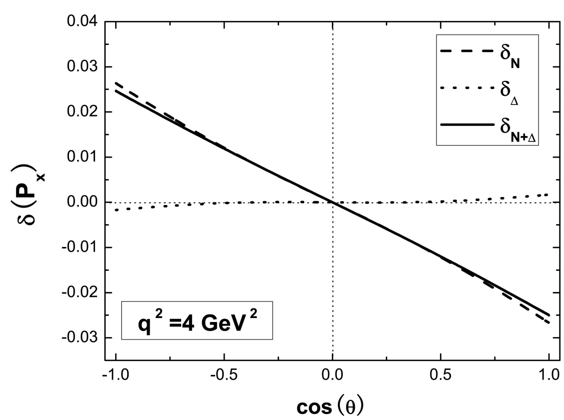

Fig. 3. TPE contributions to polarized observable P_x. The different lines have the same meanings as those in Fig. 2.

The results of the corrections to P_x and P_z are presented in Fig. 3 and Fig. 4. In our previous results[22], when discussing the TPE corrections to polarized observables, only the contributions in term $\frac{{\rm d}\sigma}{{\rm d}\Omega}$ are considered, while the corrections in $\frac{{\rm d}\sigma_{\rm un}}{{\rm d}\Omega}$ are neglected. Here the calculations are improved to include both corrections.

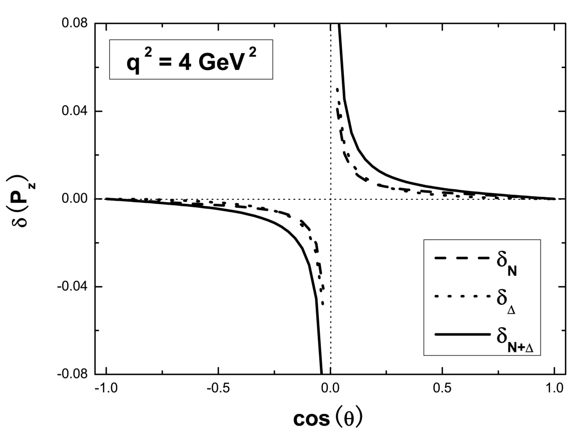

Fig. 4. The same as Fig. 3 but for polarized observable P_z .

For the polarized observables, Fig. 3 shows the corrections to P_x from Δ and N are in opposite sign, but the former is much smaller than later. The total correction to P_x is about $\pm 2.5\%$ at $\cos\theta = \pm 1$. In Fig. 4, we show the correction to P_z. One can see the contributions from Δ and N are in same sign and similar magnitude. This increases the TPE corrections to P_z which enhances our previous suggestion that the nonzero P_z at $\theta = \pi/2$ may be a good place to measure the two-photon exchange like effects directly.

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