Low-lying states and isospin excitation in the Ge isotopes*

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Abstract The level structure of $^{64-70}$Ge isotopes has been studied within the framework of the interacting boson model-3 (IBM-3). The symmetry character in the proton and neutron degrees of freedom of the energy levels has been investigated. The isospin excitation states ($T > T_z$) have been assigned for the ^{64}Ge ($N = Z$) nucleus. Some intruder states in these nuclei have been suggested. The calculated energy levels and transition probabilities are in good agreement with recent experimental data. The study indicates that the Ge isotopes are in transition from γ-unstable to vibrational.

Key words IBM-3, Isospin, $Z \approx N$ nuclei, $^{64-70}$Ge isotopes

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1 Introduction

Highly improved detection capabilities have allowed the study of the nuclear structure in light and medium mass $N \approx Z$ nuclei in recent years[1—11]. Because the neutrons and protons are in the same major shells, the isospin effect plays an important role. Isospin excitation bands of nuclei in this area are characterized by the existence of large neutron-proton pairing[12—17]. The study of the nuclear structure of nuclei in this area is attracting more and more attention.

In Ge, Se and Kr isotopes, both valence protons and neutrons are in the same major-shell between shell-closures 28 and 50, and they were considered to be nearly spherical. Therefore their structure may be described by vibrational models, at least in the low energy region. However many experiments and theoretical work found that the low lying level structure of those nuclei is not a simple vibrator[18—21]. As a typical isotope in this area, the Ge isotopes present a useful testing ground for nuclear structure calculations. One interesting feature is that the structure is the strange behavior of the 0_2^+ states. The 0_2^+ and 2_2^+ states in the ^{70}Ge isotope are interesting cases. The 0_2^+ energy drops suddenly in ^{70}Ge, and continues to fall down at the ^{72}Ge isotope in which it is lower than the first $J = 2^+$ excited state. After dropping below the 2_1^+ in ^{72}Ge, its energy suddenly rises higher in ^{74}Ge. The strange behavior of this state is very rare in nuclei. It happens only in a few nuclei in the whole of the even-even isotopes in the nuclear chart, e.g. ^{72}Ge, ^{90,96,98}Zr and in ^{98}Mo nuclei and so on. The existence of the unusually low-lying excited 0^+ state around the first excited 2^+ state can not be ascribed simply as the 0^+ member of the two- phonon triplet (0^+, 2^+ and 4^+) states. Recently, Hasegawa et al.[22], have considered the configuration space ($p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $1g_{9/2}$), and carried out a systematical shell model calculation for the $^{68-82}$Ge isotopes. The calculations showed that the strong enhancement of $B(E2)$ and the unusually low excitation of the second 0^+ state near $N = 40$ can be explained only with sufficient occupation of protons and neutrons in the $g_{9/2}$ orbit. This mechanism interpreted the 0^+ state as an intruder state, and it is described in the IBM by $(N-1)$ normal bosons and

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one s′ boson[23]. Recently, high spin properties, backbending phenomena and $B(E2)$ transitions in the $^{66-70}$Ge isotopes have been investigated using the projected shell model[24].

There is evidence that there is deformation in nuclei in this region. Firstly, there exist low lying positive and negative parity states of $J = 3^-$ in some isotopes in this region, which is an indication of deformation[25]. Secondly, there is a considerable electric quadruple moment in the first excited 2^+ state in nuclei in this region. However, the deformation is not unique in this region, and it co-exists with the vibrational mode, and hence produces the interesting nuclear phenomena of shape coexistence. Evidence of the coexistence of two different shapes, vibrational and rotational, and shape transition between them have been studied by some authors[26—29]. Investigation of the even mass Ge isotopes by means of the interacting boson model with the fermiom pair model has been done by Hsieh et al[30]. In that study it was suggested that the complex shape coexistence was in the ^{68}Ge nucleus. More complicated structures of these nuclei were revealed when the reduced transition probabilities were studied. It was found that, in spite of the fact that the energies of 0_2^+, 2_2^+ and 4_1^+ support a vibrational character, the $B(E2)$ value and their relative ratios did not justify such an interpretation. Gangopadhyay[31], has performed systematic analysis of even-even $^{60—70}$Ge isotopes in relativistic mean field theory. The binding energy, electric quadrupole moment, and root mean square radius of a number of nuclei in these nuclei have been calculated and compared with the experimental data. Their studies show that there are deformations in these nuclei.

The aim of this paper is twofold. First, to carry out a systematic IBM-3 calculation of the even mass $^{64—70}$Ge isotopes and to search for the symmetry characters of the eigenstates by studying the correlations among the energy levels, wave function, F-spin values and electromagnetic transitions probabilities. Second, to identify the one-phonon and two-phonon mixed symmetry states and isospin excitation states.

2 The model Hamiltonian and the parameter

The building blocks of the interacting boson model (IBM) are nucleon pairs with angular momentum $L = 0$ and 2 which are mapped onto s and d bosons. In the early version of the model (IBM-1) there is no distinction made between proton and neutron bosons. The number of bosons is taken to be the number of nucleons outside the closed shell divided by two[32—34]. In the IBM-2 version[35] the numbers N_π and N_ν are obtained by counting neutrons and protons from the nearest closed shell. The IBM-2 predicts the existence of mixed-symmetry states, i.e. not completely symmetric states with respect to the proton-neutron boson exchange. In the present work the IBM-3 Hamiltonian has been used to produce the energy levels and the transition matrix elements. This model considers three types of bosons: the proton-proton boson (π), the neutron-neutron boson (ν) and the proton-neutron boson (δ). The (π), (ν) and (δ) bosons are the three members of a $T=1$ triplet and their inclusion is necessary to obtain an isospin invariant formulation of the IBM. This means that the Hamiltonian is not only dependent on the total number of bosons N but also on the isospins T. A mapping theory for the IBM-3 describing a subset of states of neutrons and protons with good isospin has been investigated in Refs. [36, 37]. The model Hamiltonian is of the form[38, 39]:

$$H = \epsilon_{\rm s}\hat{n}_{\rm s} + \epsilon_{\rm d}\hat{n}_{\rm d} + H_2, \tag{1}$$

where

$$\begin{aligned} H_2 = & \frac{1}{2}\sum_{L_2T_2} C_{L_2T_2}((d^\dagger d^\dagger)^{L_2T_2}.(\tilde{d}\tilde{d})^{L_2T_2}) + \\ & \frac{1}{2}\sum_{T_2} B_{0T_2}((s^\dagger s^\dagger)^{0T_2}.(\tilde{s}\tilde{s})^{0T_2}) + \\ & \sum_{T_2} A_{2T_2}((s^\dagger d^\dagger)^{2T_2}.(\tilde{d}\tilde{s})^{2T_2}) + \\ & \frac{1}{\sqrt{2}}\sum_{T_2} D_{2T_2}((s^\dagger d^\dagger)^{2T_2}.(\tilde{d}\tilde{d})^{2T_2}) + \\ & \frac{1}{2}\sum_{T_2} G_{0T_2}((s^\dagger s^\dagger)^{0T_2}.(\tilde{d}\tilde{d})^{0T_2}). \end{aligned} \tag{2}$$

The symbols T_2 and L_2 represent the two-boson isospin and angular momentum, respectively. The parameters A, B, C, D and G are the two-body matrix elements and they have been studied microscopically by Evans et al[38]. The parameters A_1, C_{11} and C_{31} are similar to those of Majorana interaction in the IBM-2, which have a great effect on the energy shifting of the mixed symmetry states with respect to the symmetric states. The IBM-3 Hamiltonian contains many parameters which are functions of T and N functions, and it is hard to find the best fit with experimental data unless one follows a guideline, such as the shell model[39]. The fitting parameters were chosen according to the microscopic study of IBM-3 parameters in Ref. [38], which shows the dependence of the IBM-3 Hamiltonian on the isospin value as well as the boson number. The dependence on isospin is more dramatic than that on the boson number. In order to discuss the selected Ge isotopes, the Hamiltonian can be rewritten in terms of a linear combination of the Casimir operator as[40]:

$$\begin{aligned} H \;=\; & \lambda C_{2U_{\rm sd}(6)}+a_{\rm T}T(T+1)+\alpha_1 C_{1U_{\rm d}(5)}+ \\ & \alpha_2 C_{2O_{\rm sd}(6)}+\alpha_3 C_{2SU_{\rm sd}(3)}+\alpha_4 C_{2U_{\rm d}(5)}+ \\ & \alpha_5 C_{2O_{\rm d}(5)}+\alpha_6 C_{O_{\rm d}(3)}\,. \end{aligned} \quad (3)$$

The $\hat{C}_{\rm nG}$ denotes the nth order Casimir operator of the algebra G. The definition of all operators and the full Hamiltonian of the IBM-3 in terms of Casimir operators can be found in Ref. [41].

The coefficients of Casimir operators of groups show that these nuclei are closer to the $U(5)$ limits and the transitional nuclei $O(6)$. The λ parameter determines the position of the mixed symmetry states as well as the 1^+ state. The $a_{\rm T}$ parameter was fitted to the relative position of the first $T=2$ state. By using the data and the following Coulomb energy formula[42]:

$$E_{\rm Coulmob}=0.70\frac{Z^2}{A^{1/3}}(1-0.76Z^{-2/3}). \quad (4)$$

The energy of the $T=2$ isospin analogue state in ^{64}Zn is at 6.420 MeV, and it is close to the energy of 6.420 MeV in the present IBM-3 calculation with $a_{\rm T}=1.070$. In searching for the boson interaction parameters, it is helpful to take advantage of the experimental data for the ground state band and other symmetric states and by the guidance of group symmetry limits. In the Casimir operator form, the α_i ($i=1$—6) coefficients are fixed with respect to the fitting of the experimental low isospin states, therefore the low lying states for the even Ge isotopes are considered as follows:

$$\begin{aligned} H_{64} \;=\; & -0.350C_{2U_{\rm sd}(6)}+1.070T(T+1)+0.570C_{1U_{\rm d}(5)}+0.021C_{2O_{\rm sd}(6)}+0.013C_{2O_{\rm d}(5)}+0.025C_{O_{\rm d}(3)}, \\ H_{66} \;=\; & -0.220C_{2U_{\rm sd}(6)}+1.070T(T+1)+0.600C_{1U_{\rm d}(5)}+0.014C_{2O_{\rm sd}(6)}+0.005C_{2U_{\rm d}(5)}+0.010C_{2O_{\rm d}(5)}+0.030C_{O_{\rm d}(3)}, \\ H_{68} \;=\; & -0.181C_{2U_{\rm sd}(6)}+1.070T(T+1)+0.690C_{1U_{\rm d}(5)}+0.013C_{2O_{\rm sd}(6)}+0.006C_{2U_{\rm d}(5)}+0.002C_{2O_{\rm d}(5)}+0.021C_{O_{\rm d}(3)}, \\ H_{70} \;=\; & -0.115C_{2U_{\rm sd}(6)}+1.070T(T+1)+0.730C_{1U_{\rm d}(5)}+0.003C_{2U_{\rm d}(5)}+0.013C_{2O_{\rm d}(5)}+0.021C_{O_{\rm d}(3)}. \end{aligned} \quad (5)$$

Table 1. The parameters of the IBM-3 Hamiltonian used for the description of the $^{64-70}$Ge isotope.

isotope	^{64}Ge	^{66}Ge	^{68}Ge	^{70}Ge
$\epsilon_{\rm d\rho}$	0.917	1.735	1.973	2.373
$\epsilon_{\rm s\rho}$	0.145	0.890	1.119	1.450
$A_i(i=0,1,2)$	−4.938,−1.482,1.482	−4.692,−1.728,1.728	−4.616,−1.700,1.804	−4.510,−1.910,1.910
$C_{i0}(i=0,2,4)$	−5.552,−5.062,−4.712	−5.262,−4.642,−4.422	−5.002,−4.726,−4.432	−4.860,−4.604,−4.310
$C_{i2}(i=0,2,4)$	0.860,1.358,1.708	1.158,1.578,1.998	1.418,1.694,1.998	1.560,1.816,2.110
$C_{i1}(i=1,3)$	−1.758,−1.508	−2.058,−1.758	−2.700,−1.820	−2.152,−1.942
$B_i(i=0,2)$	−4.980,1.440	−4.7200,1.700	−4.620,1.778	−4.510,1.910
$D_i(i=0,2)$	0.000,0.000	0.000,0.000	0.000,0.000	0.000,0.000
$G_i(i=0,2)$	0.094,0.094	0.062,0.062	0.058,0.058	0.000,0.000
$\alpha_0=\beta_0$	0.110	0.065	0.075	0.065
$\alpha_1=\beta_1$	0.010	0.010	0.010	0.010
g_0	0.000	0.000	0.000	0.000
g_1	1.500	1.500	1.500	1.500

The corresponding parameters in the form of Eq. (2) are also given in Table 1. The group symmetry properties of Germanium isotopes have also been extensively studied. For example, Ge isotopes have provided a good example that reveals the transition from the gamma soft $O(6)$ limit to the vibrational $U(5)$ limit[20, 30, 38, 43]. Since the energy spectra of the Ge isotopes considered in this work are close to the $U(5)$ limit, the parameter α_1 is varied. The general trend of α_2 is in the decrease as we go away from the $O(6)$ limit.

3 Excitation energy

The Ge isotopes ($Z=32$) have been chosen with N_π=2 each relative to the $Z=28$ magic number. The neutron boson N_ν numbers go from 2 to 5. All the bosons are particles and this is one of the reasons that we stop the calculations, in the present work, at the ^{70}Ge isotope beyond which IBM-2 should be used. In examining the experimental data of the $^{64-70}$Ge isotopes, one is immediately struck by the virtual constancy of the energy of the first 2^+ state and the marked changes in energy of the 0_2^+ state. In the theoretical results, the 0_2^+ state remains at excitation below the 2_2^+ state. The calculated energy levels are shown in Figs. 1—4 together with the available experimental data, taken from Ref. [44—48]. The figures show good agreement between the calculated and available experimental data; only the predicted levels labelled with IBM-3c have no experimental observed counterparts. A detailed presentation is given in the figures where we draw the calculated energy levels and available experimental assignments (certain and uncertain spin and parity assignment). The J-sequence of the energy levels is in agreement with experiment.

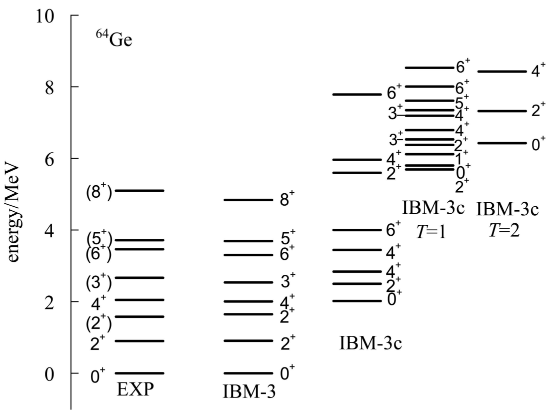

Fig. 1. Comparison between the lowest excitation energy bands ($T=T_z$, T_z+1 and T_z+2) of the IBM-3 calculation and the experimental data for ^{64}Ge.

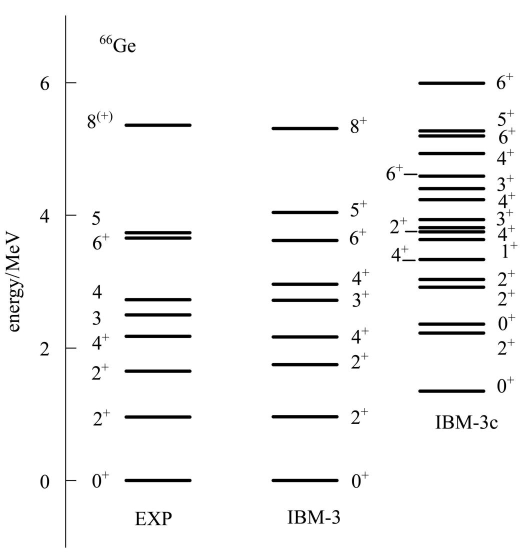

Fig. 2. Comparison between the lowest excitation energy bands ($T=T_z$) of the IBM-3 calculation and the experimental data for ^{66}Ge.

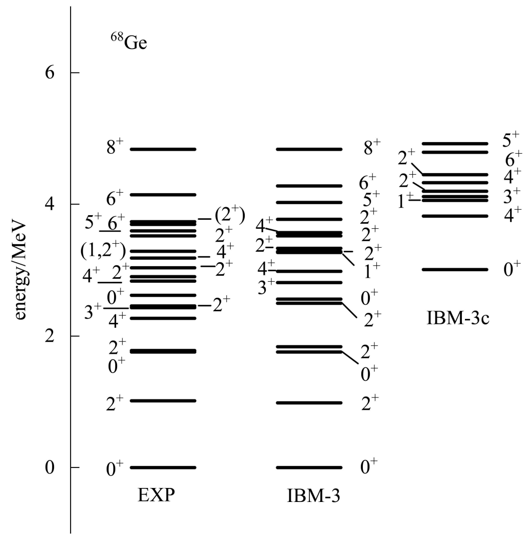

Fig. 3. Comparison between the lowest excitation energy bands ($T=T_z$) of the IBM-3 calculation and the experimental data for ^{68}Ge.

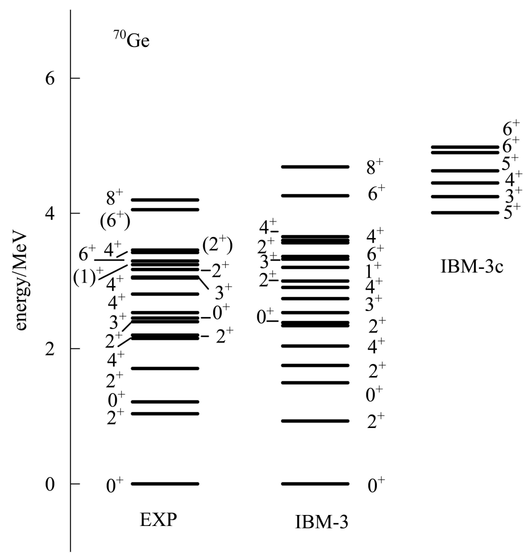

Fig. 4. Comparison between lowest excitation energy bands ($T=T_z$) of the IBM-3 calculation and the experimental data for ^{70}Ge.

Now we present the obtained results for the nucleus ^{64}Ge. In fact, this isotope has $N_\pi = N_\nu=2$ and thus offers the opportunity to test directly the δ-boson component of the calculated wave functions. Our calculated levels are shown in Fig. 1. Clear reproduction of the low-lying structural features observed in the experimental data can be seen, especially those for the ground state band. The IBM-3 wave function of the first excitation states with $J=2^+$ and $T=T_z$, T_z+1, T_z+2 are as follows:

$$|2^+_{T=0}\rangle = -0.507\{|s_\nu^2 s_\pi d_\pi\rangle + |s_\nu s_\pi^2 d_\nu\rangle\} + 0.358\,|s_\nu s_\pi s_\delta d_\delta\rangle + 0.254\{|s_\pi s_\delta^2 d_\nu\rangle + |s_\nu s_\delta^2 d_\pi\rangle\} - 0.439\,|s_\delta^3 d_\delta\rangle + \cdots\cdots,$$

$$|2^+_{T=1}\rangle = 0.626\{|s_\nu s_\pi^2 d_\nu\rangle\} - |s_\nu^2 s_\pi d_\pi\rangle\} + 0.313\{|s_\pi s_\delta^2 d_\nu\rangle + |s_\nu s_\delta^2 d_\pi\rangle\} + 0.066\{|s_\nu d_\nu d_\pi^2\rangle - |s_\pi d_\pi d_\nu^2\rangle\} + \cdots\cdots,$$

$$|2^+_{T=2}\rangle = 0.428\{|s_\nu^2 s_\pi d_\pi\rangle + |s_\nu s_\pi^2 d_\nu\rangle\} + 0.151\,|s_\nu s_\pi s_\delta d_\delta\rangle + 0.107\,|s_\pi s_\delta^2 d_\nu\rangle - 0.742\,|s_\delta^3 d_\delta\rangle + 0.142\,|s_\delta d_\delta^3\rangle + \cdots\cdots.$$

From these wave function expressions, one can see that the three states have sizable components of the δ-boson. The $2^+_{T=1}$ state is a one d-boson mixed symmetry state with some admixture two d-boson and comes from [3, 1] $U(6)$ partition. The calculated wave function of the $2^+_{T=2}$ state contains four sizable amplitudes of the δ-boson. These structures of the IBM-3 wave function may be of interest with respect to possible shell model calculations. In ^{64}Ge, the calculated $3^+_1(T=0)$ and $5^+_1(T=0)$ states that appeared at 2.541 MeV and 3.691 MeV are close to experimental $J^\pi=(3^+)$ at 2.669 MeV and $J^\pi=(5^+)$ at 3.716 MeV, respectively[44]. In ^{66}Ge, the $J=3^+_2$ and $J=4^+_2$ states are predicted as 0.223 MeV and 0.233 MeV, higher than the observed $J^\pi=3$ at 2.495 MeV and $J^\pi=4$ at 2.725 MeV states, respectively[45].

The excitation energy of the second 0^+_2 state in ^{64}Ge is identified as 1.200 MeV. The present calculated energy is consistent with the recent shell model calculation at 1.353 MeV[49], and it will be very interesting to see if this model prediction can be confirmed in future experiments. The predicted 0^+_2 levels around 1.3 MeV in the ^{64,66}Ge have not been observed. In our calculation, there is not a $J^+=4^+$ state close to the experimental one $J^\pi=(4^+)$ at 2.154 MeV. There is no suitable solution in the present scheme for this problem, and one possible explanation is the effect of the g-boson.

In the ^{68}Ge isotope, the experimental 0^+_2, 0^+_3 and 0^+_4 states are: 1.755 MeV, 2.617 MeV and 3.204 MeV, respectively. The respective calculated energies are: 1.580 MeV, 2.596 MeV and 3.010 MeV. In the ^{70}Ge isotope, the 0^+_2 energy of the two phonon states equals 1.4960 MeV in IBM-3 while in the experimental data it is 1.215 MeV. This indicates that this state is outside the IBM-3 space, which is an "intruder state" as shown in Fig. 4. On the other hand, the calculated energies of the 0^+_3 and 0^+_4 states at 2.487 and 3.016 MeV are in good agreement with the experimental data at 2.306 and 2.880 MeV. The first 4^+_1 is fitted reasonably well in all nuclei. In ^{68,70}Ge there are many 4^+ states observed and the model predicted most of them. Presumably some of them are seniority-two states (two fermions coupled to $J=4$), which are outside the model space. The first 6^+_1 states have good agreement for $^{64-70}$Ge.

In order to observe the shape coexistence in the Ge isotopes one has to calculate the ratios $E_{4^+_1}/E_{2^+_1}$ and $E_{6^+_1}/E_{2^+_1}$ and then compare them with the experimental ratios. This comparison can give indications of the nuclear shape. It is well known that the nuclei shape varies smoothly from spherical near of closed shell, where $E_{4^+_1}/E_{2^+_1}=2.0$, to deformed near the mid-shell, where $E_{4^+_1}/E_{2^+_1}$=3.3, and in between it is gamma soft. As shown in Fig. 5, this ratio starts from 2.277, for the $N=Z$=32 isotope, and decreases to 2.07 for the ^{70}Ge isotope. The $E_{6^+_1}/E_{2^+_1}$ is equal to 3.8 in the ^{64}Ge and decreases to 3.16 in ^{70}Ge close to $U(5)$. These values lie between those for a good vibrator ($E_{6^+_1}/E_{2^+_1}$=3) and gamma soft ($E_{6^+_1}/E_{2^+_1}$=4.5). Indeed, most of the energy ratios in Fig. 5 are in good agreement with the γ-soft to vibrator transition.

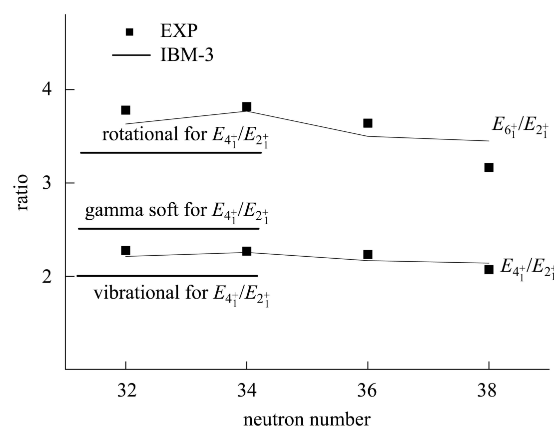

Fig. 5. Comparison between the experimental values of the ratios $E_{4^+_1}/E_{2^+_1}$ and $E_{6^+_1}/E_{2^+_1}$ and the IBM-3 prediction for $^{64-70}$Ge.

4 Mixed symmetry state

One of the important advantages of IBM-3 is the prediction of isospin excitation and mixed symmetry states. To identify the mixed symmetry states, one can make use of their electromagnetic transition properties: weak $E2$ and strong $M1$ decay to the ground state and the first 2^+ state, respectively[50, 51]. The mixed symmetry structure of wave functions can be seen by calculating the $\langle J \mid C_{2U(6)} \mid J\rangle$ value. Here we have the $U(6)$ labelling as it is a good quantum number approximately. For ^{64}Ge ($T_z=0$), the lowest mixed symmetry state comes from $[N-2,2]$ because $[N-1,1]$ does not contain $T=0$, while for ^{66}Ge it comes from $[N-1,1]$. This fact can be seen from Fig. 1 and 2 and therefore the lowest mixed symmetry state in ^{64}Ge has high energy. The lowest mixed symmetry state is $J^{\pi}=2^{+}$ coming from $[N-2,2]$ partition with $T=0$ at 5.618 MeV which is inconsistent with the calculated one given in Ref. [39] at 4.6 MeV. However, up to now no experimental evidence for such conclusions has been discovered. The first scissor mode state in ^{64}Ge is calculated at 6.161 MeV. Because IBM-3 has three charge states, for three kinds of boson, it is possible to have $U(6)$ partitions into three rows, namely the $[N1,N2,N3]$ states which are the characteristic of IBM-3. It is found that such states are produced at high energy, upwards at about 8.5 MeV, and the lowest example is a scissor mode at 8.922 MeV, which is predominantly the $[2,1,1]$ partition with $T=1$. These suggestions do not contradict the experimental data. In the ^{66}Ge isotope, the calculated 2_5^+ state at 3.030 MeV is the lowest mixed symmetry state and has $s^{N-1}d$ configuration. The IBM-3 calculations about $J=3^+$ clarified that the lowest 3_1^+ is a full symmetry state.

Since the ^{68,70}Ge isotopes have good experimental data, the detailed description of mixed symmetry states has been performed for these nuclei. The IBM-3 2_2^+ state is highly symmetric as its wave function is composed predominantly (94%) of the $2_2^+(s^{N-2}d^2)$ symmetric boson. In the present calculation, the influence of M, the ajorana parameters, on the energy levels has been studied. We take a more traditional approach of keeping all the parameters of the Hamiltonian and change only one term from the Majorana terms. According to the experimental data the 2_3^+ state of ^{68}Ge at 2.457 MeV decays by a 0.190 MeV $E2$ γ-ray transition to the first 4^+ state at 2.268 MeV with an intensity that is 2.1 times larger than the intensity of the 1.441 MeV transition to the 2_1^+ state. According to the Nuclear Data Sheets the 2_4^+ state of ^{68}Ge at 2.947 MeV decays by a 1.169 MeV transition to the second 2^+ state at 1.777 MeV, while the 2_5^+ decays by a 2.007 MeV to the 2_1^+ state. A dominant M1 decays to the 2_1^+ state, as is expected for a one-phonon mixed-symmetry state.

On the other hand, the 2_3^+ state of ^{70}Ge at 2.156 MeV decays by 0.450 MeV and 0.941 MeV transitions to the 0^+ and 2^+ two-phonon states, respectively. In the present calculation, the model symmetric predictions are consistent with the data on the 2_3^+ state in the two cases.

To identify the lowest mixed symmetry state in the ^{68,70}Ge isotopes we have analyzed the wave function for 2_3^+, 2_4^+ and 2_5^+ in these nuclei. The main components of the wave function for these states are given as follows;

$$
\begin{aligned}
|2_3^+\rangle_{68} =& -0.453\,|s_\nu^2 s_\pi d_\pi (d_\nu^2)_0\rangle - 0.261\,|s_\nu^3 d_\nu (d_\pi^2)_0\rangle - 0.382\,|s_\nu s_\pi^2 (d_\nu^3)_2\rangle - 0.387\,|s_\nu^2 s_\pi d_\pi (d_\nu^2)_4\rangle - \\
& 0.289\{|s_\nu^2 s_\pi d_\pi (d_\nu^2)_2\rangle - 0.223\,|s_\nu^3 d_\nu (d_\pi)_2\rangle \cdots\cdots, \\
|2_4^+\rangle_{68} =& -0.256\,|s_\nu s_\pi d_\pi (d_\nu^3)_3\rangle - 0.504\,|s_\nu s_\pi d_\pi (d_\nu^3)_2\rangle - 0.298[|s_\nu^2 (d_\nu^2)_0 (d_\pi^2)_2\rangle + |s_\nu^2 (d_\nu^2)_2 (d_\pi^2)_0\rangle] - \\
& 0.222\,|s_\pi^2 (d_\nu^4)_2\rangle - 0.249\,|s_\nu s_\pi (d_\nu^3)_4 d_\pi\rangle \cdots\cdots, \\
|2_5^+\rangle_{68} =& -0.612\,|s_\nu^4 s_\pi d_\pi\rangle + 0.426\,|s_\nu^3 s_\pi^2 d_\nu\rangle + 0.213[|s_\nu^3 s_\delta^2 d_\pi\rangle - |s_\nu^2 s_\delta^3 d_\delta\rangle] - 0.369\,|s_\nu^2 s_\pi s_\delta^2 d_\nu\rangle + \\
& 0.301[|s_\nu s_\delta^4 d_\nu\rangle + |s_\nu^3 s_\pi s_\delta d_\delta\rangle] \cdots\cdots,
\end{aligned}
$$

for the ^{68}Ge isotope and

$$
\begin{aligned}
|2_3^+\rangle_{70} =& -0.464\,|s_\nu^3 s_\pi d_\pi (d_\nu^2)_0\rangle - 0.232\,|s_\nu^4 d_\nu (d_\pi^2)_0\rangle - 0.480\,|s_\nu^2 s_\pi^2 d_\nu^3\rangle - 0.148\,|s_\nu^4 d_\nu (d_\pi^2)_2\rangle - \\
& 0.296\,|s_\nu^3 s_\pi d_\pi (d_\nu^2)_2\rangle - 0.199\,|s_\nu^4 d_\nu (d_\pi^2)_4\rangle - 0.398\,|s_\nu^3 s_\pi d_\pi (d_\nu^2)_4\rangle + \cdots\cdots, \\
|2_4^+\rangle_{70} =& -0.690\,|s_\nu^5 s_\pi d_\pi\rangle + 0.436\,|s_\nu^4 s_\pi^2 d_\nu\rangle + 0.308\,|s_\nu^4 s_\pi s_\delta d_\delta\rangle - 0.327\,|s_\nu^3 s_\pi s_\delta^2 d_\nu\rangle - \\
& 0.189\,|s_\nu^3 s_\delta^3 d_\delta\rangle + 0.221[|s_\nu^4 s_\delta^2 d_\pi\rangle + |s_\nu^2 s_\delta^4 d_\nu\rangle] + \cdots\cdots, \\
|2_5^+\rangle_{70} =& -0.277\,|s_\nu^2 s_\pi d_\pi (d_\nu^3)_3\rangle + 0.545\,|s_\nu^2 s_\pi d_\pi (d_\nu^3)_2\rangle + 0.263[|s_\nu^3 (d_\nu^2)_0 (d_\pi^2)_2\rangle + |s_\nu^3 (d_\nu^2)_2 (d_\pi^2)_0\rangle] + \\
& 0.269[|s_\nu^2 s_\pi d_\pi (d_\nu^3)_4\rangle\,|s_\nu^3 (d_\pi^2)_4 (d_\nu^2)_4\rangle + 0.339\,|s_\nu s_\pi^2 (d_\nu^4)_2\rangle + \cdots\cdots,
\end{aligned}
$$

for the ^{70}Ge isotope.

In these expressions, we have labelled the angular momentum sub-total for the d-boson configurations. For instance, $\mid d_\nu(d_\pi^2)_0\rangle$ means that the 2 d_π bosons couple to $L_{d_\pi}=0$ and then they couple with a d_ν boson to form an $L=2$ basis state. The wave functions show that the 2_5^+ states at 3.311 MeV and 2_4^+ at 2.533 MeV are close to the experimental ones at 3.023 MeV and 2.534 MeV and are the one d-boson mixed symmetry states in ^{68,70}Ge, respectively. The two states are generated from the $[N-1,1]$ $U(6)$ partition. For the other 2^+ states, in ^{68}Ge the large mixed symmetry component is in the calculated 2_7^+ at 3.772 MeV and is close to 3.735 MeV in the data which was assigned (2^+)[46]. In ^{70}Ge, the state $(2)^+$ at 3.423 MeV in the experimental data[48] is closed to the mixed symmetry state $J=2_6^+$ at 3.362 MeV in our IBM-3 results (i.e $2_{2\mathrm{m}}^+$).

The level at 3.287 MeV in ^{68}Ge has possible $J=(1,2^+)$ assignments in the experimental data and is close to a level at 3.270 MeV with $J=1^+$ in the IBM-3 result with $C_{11}=-2.70$ MeV and $A_{21}=-1.70$ MeV, where the choice of the Majorana parameters plays a crucial role. As shown in Fig. 4 the first calculated scissor state in ^{70}Ge at 3.214 MeV is in good agreement with the observed one $J^\pi=1^{(+)}$ at 3.242 MeV. In both nuclei, the IBM-3 calculation about $J=3^+$ makes it clear that $3_2^+(s^{N-2}d^2)$ are mixed symmetry states. In ^{70}Ge, the second $J=3^+$ state at energy 3.424 MeV is close to the experimental data of 3.046 MeV. The existence of more experimental data gives us the opportunity to test the model prediction in this region.

5 Electrometric transitions

The $E2$ transition can be calculated by the following isoscalar and isovector transition operators

$$Q=Q^0+Q^1\,, \tag{6}$$

where,

$$Q^0 = \alpha_0\sqrt{3}[(s^+\hat{d})^{20}+(d^+\hat{s})^{20}]+\beta_0\sqrt{3}[(d^+\hat{d})^{20}, \tag{7}$$

$$Q^1 = \alpha_1\sqrt{2}[(s^+\hat{d})^{21}+(d^+\hat{s})^{21}]+\beta_1\sqrt{2}[(d^+\hat{d})]^{21}. \tag{8}$$

The $M1$ transition is also a one boson operator with isoscalar and isovector parts $M=M^0+M^1$ where,

$$M^0 = g_0\sqrt{3}(d^+\hat{d})^{10}=g_0L/\sqrt{10}, \tag{9}$$

$$M^1 = g_1\sqrt{2}(d^+\hat{d})^{11}, \tag{10}$$

where g_1 and g_0 are the isovector and isoscalar g-factors, respectively and L is the angular momentum operator.

Table 2 shows the IBM-3 calculations for the $B(E2)$ values. The α_i and β_i, $(i=0,1)$ were determined by fitting the exterminate data of $B(E2;2_1^+\rightarrow 0_1^+)$, as shown in Table 1. The agreement of IBM-3 with the available experimental data is good. The $E2$ transition $2_2^+\rightarrow 0_1^+$ is forbidden in the $U(5)$ limit, and this is why the value is so small and equals zero for ^{70}Ge.

Table 2. Experimental[45—49] and calculated $B(E2)$ for $^{64-70}$Ge. The units of the $B(E2)$ values are given by 10^{-2} e^2b^2.

$J_i^+\rightarrow J_f^+$	$B(E2)$							
	^{64}Ge		^{66}Ge		^{68}Ge		^{70}Ge	
	Exp.	IBM-3	Exp.	IBM-3	Exp.	IBM-3	Exp.	IBM-3
$2_1^+\rightarrow 0_1^+$	4.10(60)	3.826	1.896(362)	1.879	2.912(329)	3.096	3.593(68)	3.360
$2_2^+\rightarrow 0_1^+$	0.015(5)	0.036	0.016(6)	0.008	0.023(4)	0.010	0.171(85)	0.000
$2_3^+\rightarrow 0_1^+$		0.001		0.000		0.000		0.000
$2_4^+\rightarrow 0_1^+$		0.000		0.000		0.041		0.046
$2_2^+\rightarrow 2_1^+$	6.20(210)	5.995	2.686(1264)	3.102	0.086(34)	5.289	4.97(189)	5.760
$2_3^+\rightarrow 2_1^+$		0.013		0.003		0.004		0.000
$2_4^+\rightarrow 2_1^+$		0.001		0.000		0.006		0.007
$2_3^+\rightarrow 2_2^+$		1.901		0.905		1.546		1.371
$3_1^+\rightarrow 2_1^+$		0.052		0.011	0.003(1)	0.015		0.000
$3_1^+\rightarrow 2_2^+$		4.563		2.577		4.627		5.143
$4_1^+\rightarrow 2_1^+$		5.991	>1.517	3.102	2.287(29)	5.292	4.112(11)	5.760
$4_2^+\rightarrow 2_1^+$		0.038		0.009	0.077^{+29}_{-65}	0.011		0.000
$4_2^+\rightarrow 2_2^+$		3.346		1.907	3.949^{+1481}_{-3455}	3.449	4.961(2053)	3.772
$5_1^+\rightarrow 3_1^+$		2.541		1.772		3.548		4.032
$6_1^+\rightarrow 4_1^+$		6.389	>0.189	3.624	1.975(658)	6.534	5.817(1197)	7.200

Table 3. $B(E2)$ ratios relative to the $B(2_1^+ \to 0_1^+)$ transition for selected transitions in $^{64-70}$Ge.

Ratio	$B(E2)$							
	^{64}Ge		^{66}Ge		^{68}Ge		^{70}Ge	
	Exp.	IBM-3	Exp.	IBM-3	Exp.	IBM-3	Exp.	IBM-3
$\frac{2_2^+\to 0_1^+}{2_1^+\to 0_1^+}$	0.004	0.009	0.017	0.004	0.007	0.003	0.047	0.000
$\frac{2_3^+\to 0_1^+}{2_1^+\to 0_1^+}$		0.000		0.000		0.000		0.000
$\frac{2_2^+\to 2_1^+}{2_1^+\to 0_1^+}$	1.512	1.566	1.417	1.650	0.0294	1.708	1.382	1.714
$\frac{2_3^+\to 2_1^+}{2_1^+\to 0_1^+}$		0.003		0.001		0.001		0.000
$\frac{2_3^+\to 2_2^+}{2_1^+\to 0_1^+}$		0.496		0.481		0.499		0.408
$\frac{3_1^+\to 2_1^+}{2_1^+\to 0_1^+}$		0.013		0.001	0.005	0.001		0.000
$\frac{3_1^+\to 2_2^+}{2_1^+\to 0_1^+}$		1.193		1.371		1.494		1.530
$\frac{4_1^+\to 2_1^+}{2_1^+\to 0_1^+}$		1.563	>0.607	1.650	0.783	1.709	1.149	1.714

In the ^{68}Ge isotope, the transition $2_2^+ \to 2_1^+$ differs from the experiment by two orders of magnitude. There is no suitable solution in the present scheme for this problem, where some of the $U(5)$ forbidden transitions are non-zero in the experiment. The mixing of different d-boson numbers in the wave function is necessary in order to allow the $U(5)$ forbidden transitions to occur. The calculations produce the large experimental $B(E2)$ value for the $2_2^+ \to 2_1^+$ transition in ^{66,70}Ge. The relative ratios are also calculated and listed in Table 3 together with available experimental values. A small ratio for transitions from the second 2^+ gives a second indication that this state is a band head of a quasi γ- band. The transitions ratio from 4_1^+, agrees well with the experimental ratio. However, in all cases where the $B(E2)$ value in the numerator is very small, we expect to get substantial disagreement.

To produce $M1$ matrix elements, the isoscalar g_0 factor is taken to be zero, for all isotopes, and the isovector factor g_1 is taken to be 1.5 μ_N for all isotopes. We found that for small experimental value the model gives zero $M1$ matrix elements, which means that the model assumed the state to be purely symmetric in the boson space. The importance of the mixed symmetry component in the electromagnetic transitions from 2_4^+ and 3_2^+ states is affirmed in $B(M1)$ to symmetric states. The $B(M1)$ values are larger than the $B(E2)$ ones from these states to full symmetry 2_1^+ and 2_2^+, respectively. In ^{68,70}Ge, the $B(M1; 2_{1\text{ms}}^+ \to 2_1^+)$ is equal to 0.2202 μ_N^2 and 0.2170 μ_N^2, respectively. The first scissor mode state in these nuclei has dominant $M1$ decay to 2_2^+ with the calculated $B(M1)$ being equal to 0.3034μ_N^2 and 0.2954 μ_N^2, respectively.

6 Conclusion

A systematic investigation of even-even Ge-isotopes in the framework of IBM-3 for energy levels and electromagnetic transitions has been carried out. The IBM-3 wave functions have been analyzed in detail, and the electromagnetic transition has been calculated and compared with experiments. Good agreement between the calculations and experiments has been achieved. In ^{64}Ge, one with $Z = N$ in the pf-shell, the IBM-3 calculation predicted that the isospin excitation states with $T = T_z + 1$ and $T = T_z + 2$ are the $J = 2^+$ and $J = 0^+$ states at 5.694 MeV and 6.420 MeV, respectively. It is found by inspecting the wave function, that the 2_5^+ and 2_4^+ states in ^{68,70}Ge are the first, and 2_7^+ in ^{68}Ge and 2_6^+ in ^{70}Ge the second mixed symmetry 2^+ states. The calculated structure also supports the shape coexistence in these nuclei. In ^{70}Ge, the 0_2^+ state is an intruder state. It is found that these even-even Ge isotopes are in the transition from $O(6)$ to $U(5)$ dynamical symmetry.

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References

1 Bonneau L, Quentin P, Sieja K. Phys. Rev. C, 2007, **76**: 014304
2 Scholl C et al. Phys. Rev. C, 2007, **75**: 064321
3 Lisetskiy F A, Schmidt A, Schneider I, Friessner C, Pietralla N, von Brentano P. Phys. Rev. Lett., 2002, **89**: 012502
4 Al-Khudair H F, LI Y S, LONG G L. J. Phys. Nucl. Part Phys. G, 2004, **30**: 1287
5 War A T, Devi R, Khosa K S. Eur. Phys. J A, 2004, **22**: 13
6 Lenzi M S et al. Z. Phys. A, 1996, **354**: 117
7 Brandolini F et al. Nucl. Phys. A, 1998, **642**: 387
8 Brandolini F et al. Nucl. Phys. A, 2001, **693**: 517
9 Brandolini F et al. Phys. Rev. C, 1999, **60**: 041305
10 Brandolini F et al. Phys. Rev. C, 2001, **64**: 044307
11 Jänecke J, ODonnell W T, Goldanskii I V. Phys. Rev. C, 2002, **66**: 024327
12 Satula W, Wyss R. Phys. Lett. B, 1996, **393**: 1
13 Ur A C. Eur. Phys. J. A, 2004, **20**: 113
14 Al-Khudair F H, LI Y S, LONG G L. Phys. Rev. C, 2007, **75**: 054316
15 LV L J, BAI H B, ZHANG J F. Chin. Phys. C (HEP & NP), 2008, **32**: 177
16 ZHANG J F, BAI H B, Chin. Sci. Bull., 2007, **52**: 165
17 Al-Khudair F H, LI Y S, LONG G L. HEP & NP, 2004, **28**: 370 (in Chinese)
18 Landulfo E, Saxena N R, Zamboni B C, Lapolli L A. Phys. Rev. C, 1994, **50**: 733
19 Sen S, Darden E S, Luhen C R, Gaiser O N. Phys. Rev. C, 1985, **31**: 787
20 Padilla E, Castanos O, Bijker R, Uribarri G A. 2006 nucl-th/0601060
21 Hasegawa M, Kaneko K, Mizusaki T. Phys. Rev. C, 2004, **70**: 031301
22 Hasegawa M, Mizusaki T, Kaneko K, Sun Y. Nucl. Phys. A, 2007, **789**: 46
23 Joubert J, Hahne W F, Navrati P, Geyer B H. Phys. Rev. C, 1994, **50**: 177
24 Dar A P, Devi R, Khosa k S, Sheikh A J. Phys. Rev. C, 2007, **75**: 054315
25 Cottle D P. Phys. Rev. C, 1990, **42**: 1264
26 Arodowin D, Tamisier R, Vergnes M, Kalifa G. Phys. Rev. C, 1975, **12**: 1745
27 Lecomte R, Kajrys G, Landsberger S, Paradis P, Monaro S. Phys. Rev. C, 1982, **25**: 2812
28 Carchidi M, Fortune T H, Stephans F G, Bland C L. Phys. Rev. C, 1984, **30**: 1293
29 Sugawara M et al. Eur. Phys. J. A, 2003, **16**: 409
30 Hsieh S T, Chiang H C, Chuu Der-San. Phy. Rev. C, 1992, **46**: 195
31 Gangopadhyay G. Phys. Rev. C, 1999, **59**: 2541
32 Arima A, Iachello F. Ann. Phys., 1976, **99**: 253
33 Arima A, Iachello F. Ann. Phys., 1978, **111**: 201
34 Arima A, Iachello F. Ann. Phys., 1979, **123**: 468
35 lachello F, Arima A. The Interacting Boson Model. Cambrideg: Cambrideg University Press, 1987
36 Hasegawa M. Prog. Theor Phys., 1989, **81**: 104
37 Hasegawa M. Prog. Theor Phys., 1991, **85**: 69
38 Evans J A, Elliott J P, Lac V S, LONG G L. Nucl. Phys. A, 1995, **593**: 85
39 Elliott J P, Evans J A, Lac V S, LONG G L. Nucl. Phys. A, 1996, **609**: 1
40 Al-Khudair H F, LONG G L. Chin. Phys., 2004, **13**: 1230
41 Garcia-Ramos J E, Van Isacker P. Ann. Phys., 1999, **274**: 45
42 Audi G, Wapstra H A. Nucl. Phys. A, 1993, **565**: 1
43 Chuu Der-San, Hsieh T S, Chiang C H. Phys. Rev. C, 1993, **47**: 183
44 Singh B. Nucl. Data Sheets, 2007, **108**: 197
45 Bhat R M. Nucl. Data Sheets, 1998, **83**: 789
46 Burows W T. Nucl. Data Sheets, 2002, **97**: 1
47 Tulj K J. Nucl. Data Sheets, 2004, **103**: 389
48 Evaluated Nuclear Structure Data File (ENSDF). http://www.nndc.bnl.gov/ensdf/ 2008
49 Starosta K et al. Phys. Rev. Lett., 2007, **99**: 042503
50 Hamilton D W, Irbäck A, Elliott J P. Phys. Rev. Lett., 1984, **53**: 2469
51 Van Isacker P, Heyde K, Jolie J, Sevrin A. Ann. Phys. (N.Y), 1986, **171**: 253
52 Elliott J P, Evans J A, Lac V S. Nucl. Phys. A, 1996, **597**: 341