Imaging source with Gaussian proper time distribution*

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Abstract Making use of the maximum entropy method, we study the most probable source function in heavy ion collisions. An anisotropic Gaussian source is deduced by simply assuming that the particles are emitted within a finite proper-time. The general relations between the most probable source function and the minimal assumptions are discussed, which are instructive in constructing a self-consistent source function from observed Hanbury-Brown/Twiss(HBT) correlations.

Key words HBT correlation, source function, the maximum entropy method, proper-time distribution

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1 Introduction

A new form of matter — the quark gluon plasma (QGP) is expected to be formed at sufficient high energy density in relativistic heavy ion collisions. The exact estimation of energy density in heavy ion collisions requires the space-time information of the source. The quantum correlations between identical particles, or Hanbury-Brown/Twiss(HBT) correlations, are the only known way which can provide us with this important information[1—3].

The produced identical Bose-particle in relativistic heavy ion collisions, i.e., the most identical π mesons, have known quantum correlations in momentum space[4, 5]. Such correlations are experimentally measurable. Via the correlations of two identical particles, the emission function can be in principle determined by the relation[6—9],

$$C(q,K)\approx 1+\frac{\left|\displaystyle\int \mathrm{d}^4x\, S(x,K)\,\mathrm{e}^{\mathrm{i}qx}\right|^2}{\left|\displaystyle\int \mathrm{d}^4x\, S(x,K)\right|^2}, \qquad (1)$$

where, $q=p_1-p_2$, $K=\frac{1}{2}(p_1+p_2)$. $C(q,K)$ is the correlation function of two identical particles. $S(x,K)$ is the emission source function, which describes the probability of emitting a particle with four-momentum p at space-time point x.

However, the particles of final state are on mass-shell $E_i=\sqrt{m^2+p_i^2}$, or, $K\cdot q=0$, so that the time component of the four-momentum, q^0, can be expressed as $q^0=\beta_\perp q_o+\beta_l q_l$ with $\beta_\perp=\frac{K_\perp}{|K^0|}$, $\beta_l=\frac{K_l}{|K^0|}$, where q_l, q_o, q_s denote the components in space directions parallel to the beam ("longitudinal" or z-direction) and to the transverse components $K_\perp$ of K ("out" or x-direction), and in the remaining third Cartesian direction ("side" or y-direction), respectively[10, 11]. Only three-momentum components in four-momentum space are independent. While in coordinate space, four-dimensional space-time components must be independent of each other. The imaging space-time source cannot be uniquely determined by the inversion of the Fourier transform. To get the source function, a prior assumption is usually made.

Three typical methods are frequently used in constructing the source function in the market: (1) Suppose that the source function is Gaussian distribution with azimuthal symmetry. This is because the measured two-particle correlation function $C(q,K)$ is usually well parameterized by a Gaussian function[12]:

$$C(q,K)=1+\exp(-R_s^2q_s^2-R_o^2q_o^2-R_l^2q_l^2-2R_{ol}^2q_oq_l). \qquad (2)$$

From this measured correlation, the four model-independent constrains on source function can be

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deduced[13]:

$$\begin{cases} R_o^2 = \left\langle (x-\beta_\perp t)^2 \right\rangle - \langle x-\beta_\perp t \rangle^2 = \left\langle \left(\tilde{x}-\beta_\perp \tilde{t}\right)^2 \right\rangle; \\ R_s^2 = \langle y^2 \rangle - \langle y \rangle^2 = \langle \tilde{y}^2 \rangle; \\ R_l^2 = \left\langle (z-\beta_l t)^2 \right\rangle - \langle z-\beta_l t \rangle^2 = \left\langle \left(\tilde{z}-\beta_l \tilde{t}\right)^2 \right\rangle; \\ R_{ol}^2 = \left\langle \left(\tilde{x}-\beta_\perp \tilde{t}\right) \left(\tilde{z}-\beta_l \tilde{t}\right) \right\rangle; \end{cases} \tag{3}$$

where $\tilde{x}^\mu = x^\mu - \langle x^\mu \rangle$. However, in the case, there are still 7 un-determined parameters in 4×4 matrix of corresponding Gaussian distribution in four-dimensional space-time. In order to fully determine the source function, further assumptions that the matrix elements of $B_{\mu v}$ satisfy $B_{xx} = B_{yy}$ and $B_{xt} = B_{xz} = 0$[14] have to be made. This means that the source function is isotropic in transverse-plane, and the correlations between transverse-space and time, and between transverse and longitudinal directions are negligible.

Alternatively, it is supposed that the source function is the product of three independent parts[15, 16], i.e., $S(x,y,z,t) = I(x,y)G(\eta)H(\tau)$, where $H(\tau) = \dfrac{1}{N}\dfrac{\mathrm{d}n}{\mathrm{d}\tau}$ is the given distribution of the proper time at the frozen-out. $I(x,y)$ and $G(\eta)$ are the parameterized distributions in transverse-space and in pseudorapidity, respectively. (3) In addition, the source function can also be deduced from the hydrodynamical model[17, 18]. While such an obtained source function strongly depends on the hydrodynamical model and its boundary conditions, it makes the hydrodynamical model fail to describe the observed momentum correlations of two identical particles.

We see from those methods that in order to determine the source function, the assumptions for the type of source function are made subjectively in advance. In this paper, we try to determine the source function in a conversed way, i.e., starting from the observed correlation in momentum space, then adding the least assumptions to construct the source function, making use of the well-known maximum entropy method (MEM).

2 Application of the maximum entropy method

The information entropy of a random distribution is generally defined as[19],

$$\sigma = -\int p(x) \ln p(x) \, \mathrm{d}^4 x \,, \tag{4}$$

where $p(x)$ is normalized, i.e., $\int p(x)\mathrm{d}x = 1$. It describes the degree of the uncertainty of the system. The larger the information entropy, the more uncertain the observed system is.

The maximum entropy method is to choose the most probable distribution from all possible ones under known constraint conditions. Such an obtained distribution is a completely random one, which makes entropy maximum under given constraints. There are the least bias for the unknown part of the problem.

Even if the known constraint conditions alone are not sufficient for uniquely determining the distribution, like in the case we are concerned with, we can still add the least and basic constraint conditions to the unknown part and then deduce the distribution by MEM.

Generally, the constrained conditions are expressed as the average of multiple functions $g_i(x) (i = 1, \cdots, m)$,

$$\langle g_i(x) \rangle = C_i(K), \quad i = 1, \cdots, m \,. \tag{5}$$

where C_i is the known constants. The most probable distribution which makes the maximum entropy under the constraints can be deduced by the method of Lagrangian multipliers, i.e., from the extremum condition of the function,

$$\sigma + \lambda_0 N\mathrm{orm} + \sum_{i=1}^{m} \lambda_i \langle g_i(x) \rangle = \int \left[-p(x) \ln p(x) + \lambda_0 p(x) + \sum_{i=1}^{m} \lambda_i p(x) g_i(x) \right] \mathrm{d}x \,, \tag{6}$$

we can get,

$$p(x) = \exp \left[-\lambda_0 - \sum_{i=1}^{m} \lambda_i g_i(x) \right] , \tag{7}$$

where λ_0 and λ_i are determined by the normalization of $p(x)$ and constrain conditions Eq. (5).

The MEM has many applications in heavy ion collisions. As we know, people use the MEM to research the distributions of the charge multiplicity[20]. Corresponding to the HBT correlations in relativistic heavy ion collisions, we are interested in determining the space-time distribution of source function $S_K(x)$. Its corresponding information entropy is[21],

$$\sigma_K = -\int S(x,K) \ln S(x,K) \, \mathrm{d}^4 x \,. \tag{8}$$

The model-independent and experimentally measurable constraint conditions are given in Eq. (3). They are all the bilinear functions of $\tilde{x} (\tilde{x} = x - \langle x \rangle)$ and can be written as,

$$g_i(\tilde{x}) = \tilde{x}_\mu g_i^{\mu\nu} \tilde{x}_\nu, \quad i = 1, \cdots, 4 \,. \tag{9}$$

If we only take these constraints to construct the source function by MEM, the source function should also be a Gaussian distribution[21],

$$S(x,K) = \mathrm{e}^{-\lambda(K)}\exp\left(-\sum_{i=1}^{N}\lambda_i(K)\tilde{x}_\mu g_i^{\mu\nu}\tilde{x}_\nu\right) = \mathrm{e}^{-\lambda(K)}\exp\left(-\frac{1}{2}\tilde{x}_\mu B^{\mu\nu}(K)\tilde{x}_\nu\right), \quad (10)$$

where $B^{\mu\nu} = 2\sum_{i=1}^{N}\lambda_i g_i^{\mu\nu}$, $\tilde{x}_\mu = (\tilde{x},\tilde{y},\tilde{z},\tilde{t})$ and $\mathrm{e}^\lambda = \frac{\sqrt{\det B}}{4\pi^2}$. $B_{\mu v}$ behaves like the weight matrix and $B_{\mu v}^{-1} = \langle x_\mu x_v\rangle - \langle x_\mu\rangle\langle x_v\rangle$. The Lagrangian multipliers $\lambda_i(K)$ is determined by inserting the matrix elements of $B_{\mu v}^{-1}$ into constraint conditions, Eq. (3),

$$B_{\mu v} = \frac{1}{\Delta}\begin{pmatrix} R_l^2 & 0 & -R_{ol}^2 & R_{ol}^2\beta_l - R_l^2\beta_\perp \\ 0 & \dfrac{\Delta}{R_s^2} & 0 & 0 \\ -R_{ol}^2 & 0 & R_o^2 & R_{ol}^2\beta_\perp - R_o^2\beta_l \\ R_{ol}^2\beta_l - R_l^2\beta_\perp & 0 & R_{ol}^2\beta_\perp - R_o^2\beta_l & R_o^2\beta_l^2 - 2R_{ol}^2\beta_l\beta_\perp + R_l^2\beta_\perp^2 \end{pmatrix}, \quad (11)$$

where $\Delta = R_o^2R_l^2 - (R_{ol}^2)^2$. But the determinant of this matrix is zero. This means that the obtained four-dimensional space-time source function in fact does not depend on four independent variables, due to the above-mentioned mass-shell constraint in momentum space. So purely based on experimentally measured constraints, it is impossible to get a well-defined four-dimensional space-time source distribution. In order to get a four-dimensional source function, we have to assume at least one more constraint, in addition to the model-independent ones, Eq. (3).

3 Imaging source function with Gaussian proper time distribution

In model-independent constraints, Eq. (3), only three-spacial components of the momentum are independent. The time-component q_0 is not an independent variable. Therefore, the time-component of the source function cannot be independently determined. We know that the particles will freeze out from the source after a time period of the collisions, and the source has a finite lifetime. A good description for the time evolution of source is the proper time, which is the time in the rest frame of the moving source. It is independent of the observed reference and defined by time-space coordinate in any reference as,

$$\tau^2 = t^2 - x^2 - y^2 - z^2 . \quad (12)$$

In history, a number of models have tried to give the distribution of proper time.

In 1983, Bjorken proposed the distribution of proper time as[22],

$$H(\tau) = \delta(\tau - \tau_0) , \quad (13)$$

which means that all particles are hadronized simultaneously at the proper time τ_0. This is obviously a simple approximation. The particles should freeze-out within a proper-time range. So people further assumed various possible and simple distributions for proper time, such as[23]

$$H(\tau) = \frac{1}{(\tau-\tau_0)^2 + \sigma_\tau^2} , \quad (14)$$

and

$$H(\tau) = \tau^{-3/2}\exp(-\beta/\tau) . \quad (15)$$

These two kinds of distributions both satisfy the normalization condition. They reach their extremum at $\tau = \tau_0$ and $\tau = \frac{2}{3}\beta$, respectively. The distribution goes down sharply nearby the extremum. The effective range of proper time can be estimated by the width of its distribution, i.e., $\langle\tilde{\tau}^2\rangle$, However, they are both divergence, which means that the particles can be frozen out from the source endlessly. This is obviously impossible in practice.

In order to constrain the source to a finite lifetime, we simply assume that $H(\tau)$ has a Gaussian distribution with finite width σ_τ,

$$\langle\tilde{\tau}^2\rangle = \sigma_\tau^2 . \quad (16)$$

The definition of proper time in infinite longitudinal moment coordinate, Eq. (12), can be approximately written as,

$$\tau^2 = t^2 - z^2 . \quad (17)$$

In the case, the additional constraint can be taken as,

$$\langle\tilde{\tau}^2\rangle = \langle\tilde{t}^2\rangle - \langle\tilde{z}^2\rangle . \quad (18)$$

Then the source function with Gaussian proper time distribution can be deduced by the maximum entropy method,

$$S(x,K)=\frac{1}{4\pi^2R_sT\sqrt{\Delta}}\exp\left[-\frac{\tilde{y}^2}{2R_s^2}-\frac{R_l^2(\tilde{x}-\beta_\perp\tilde{t})^{2+R_o^2(\tilde{z}-\beta_l\tilde{t})^2}}{2\Delta}-\frac{R_{ol}^2(\tilde{x}-\beta_\perp\tilde{t})(\tilde{z}-\beta_l\tilde{t})}{\Delta}-\frac{1}{2}\frac{\beta_l^2\tilde{z}^2+2\beta_l\tilde{z}\tilde{t}+\tilde{t}^2}{R_l^2+\sigma_\tau^2-\sigma_\tau^2\beta_l^2}\right],\quad(19)$$

where the weight matrix of Gaussian function B is

$$B_{\mu v}=\frac{1}{\Delta}\begin{pmatrix} R_l^2 & 0 & -R_{ol}^2 & R_{ol}^2\beta_l-R_l^2\beta_\perp \\ 0 & \dfrac{\Delta}{R_s^2} & 0 & 0 \\ -R_{ol}^2 & 0 & \dfrac{\beta_l^2\Delta}{R_l^2+\sigma_\tau^2-\sigma_\tau^2\beta_l^2}+R_o^2 & \Delta_1 \\ R_{ol}^2\beta_l-R_l^2\beta_\perp & 0 & \Delta_1 & \Delta_2 \end{pmatrix},\quad(20)$$

with

$$\Delta_1=\frac{-\beta_l\Delta}{R_l^2+\sigma_\tau^2-\sigma_\tau^2\beta_l^2}+R_{ol}^2\beta_\perp-R_o^2\beta_l$$

and

$$\Delta_2=\frac{\Delta}{R_l^2+\sigma_\tau^2-\sigma_\tau^2\beta_l^2}+R_o^2\beta_l^2-2R_{ol}^2\beta_l\beta_\perp+R_l^2\beta_\perp^2\ .$$

This matrix has a non-zero determinant which means the source function is well-defined.

From this matrix, we can see the following important properties of the source function: (1) The different diagonal elements in transverse plane, B_{yy} and B_{xx}, show that the Gaussian source is anisotropy in transverse plane. (2) The non-zero non-diagonal elements of B_{xz}, B_{xt} and B_{zt} indicate that there are correlations in $x\sim z$, $x\sim t$ and $z\sim t$ for Gaussian proper time distribution, but no correlation in $x\sim y$ and $y\sim t$. (3) The lifetime of the source function is

$$\langle\tilde{t}^2\rangle=\frac{\sigma_\tau^2}{\beta_\perp^2}+\frac{R_l^2(1+\beta_l^2)}{\beta_\perp^4}\ ,\quad(21)$$

which is related to the finite width of proper-time distribution as known, and also related to longitudinal radii and momentum in both transverse and longitudinal directions.

Comparing matrix elements in Eq. (11) and Eq. (20), we can find that additional constraints, Eq. (16) and Eq. (18), only change the elements, B_{zz}, B_{tt}, and B_{zt} in Eq. (11). The additional constraint related variables change, while the unrelated ones keep unchanged. This is because the MEM determines the source function only according to the given constraint relation.

If the proper-time is exactly given by Eq. (12) and the additional constraint for Gaussian proper-time distribution turns to

$$\langle\tilde{\tau}^2\rangle=\langle\tilde{t}^2\rangle-\langle\tilde{x}^2\rangle-\langle\tilde{y}^2\rangle-\langle\tilde{z}^2\rangle\ .\quad(22)$$

All elements of B in Eq. (11) will be changed and not be zero. The appearance of B_{xy} and B_{yt} is due to the small transferring of transverse momentum. So they should be smaller than B_{xz}, B_{xt} and B_{zt}. From these discussions, we see clearly and directly how the additional constraint controls the source function by the MEM.

This is instructive in reconstructing the source function which well describes the observed data. The newly measured correlations show a non-Gaussian tail[24]. These make model-independent constraints no longer covariance like. By MEM, a non-Gaussian source function should be expected.

4 Conclusions

Making use of the maximum entropy method, we study the most probable source function in relativistic heavy ion collisions. This is the opposide of the conventional way. An anisotropic Gaussian source is deduced by simply assuming that the particles are emitted within a finite proper time. In principle, all kinds of correlations can appear in source function. But the correlations between two directions in transverse plane are smaller than those between transverse and longitudinal directions, and between transverse (or longitudinal) direction and time. The relations between the most probable source function and the minimal assumptions are discussed, which is instructive in reconstructing a self-consistent source function from the observed HBT correlations.

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