On η_c line shape in charmonium transitions

PING Rong-Gang(平荣刚)[1)]

(Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(1), Beijing 100049, China)

Abstract The line shapes to observe η_c in charmonium transitions, i.e., $\psi(2S)$, $J/\psi\to\gamma\eta_c$, are investigated. The η_c line shapes in exclusive decays or by observing the inclusive photon spectrum are given. The sensitivities to measure the η_c resonance parameters are also evaluated. With more than two thousand η_c events observed, the precision of the η_c decay width measurement will be improved by better than 3%. However, the uncertainties associated with the η_c modified line shapes will dominate the systematic errors and this will prohibit precision mass and width measurements.

Key words η_c, line shape, charmonium transition

PACS 13.25.Gv, 13.40.Hq, 14.40.Lb

1 Introduction

Charmonium decays and its spectrum have been an ideal place for studying the strong interactions in particle physics. In the past decades, much progress has been made in the measurement of its spectrum and decays. However, as the lightest charmonium state, η_c is not well studied experimentally or theoretically. Although various η_c measurements are available, the precise determination of its mass and width is still an open problem. Up to now, there exist significant discrepancies between the various measurements on the η_c mass and width. The measured mass varies from 2969 MeV[1] to 2984.1 MeV[2], and the width varies from 7.0 MeV to 28.0 MeV, and the confidence level values of fitting to its mass and width are quite small.

The magnetic dipole (M1) transition assumed in the $\psi(2S)$, $J/\psi\to\gamma\eta_c$ always overestimates the data in various potential model calculations. Under the nonrelativistic approximation, the M1 contribution to the decay width is explicitly given by:

$$\Gamma(n^3S_1\to\gamma n'^1S_0)=$$
$$\frac{3}{4}\alpha e_{\rm Q}^2\frac{k_\gamma^3}{m^2}\left|\int_0^\infty {\rm d}r r^2 R_{n'0}(r)R_{n0}(r)j_0\left(\frac{k_\gamma r}{2}\right)\right|^2. \quad (1)$$

Various potential model calculations based on the M1 transition show that the branching fraction is about 3%, which is twice as large as the experimental value (1.27 ± 0.36)%.

The hyperfine splitting between J/ψ and η_c has also been the subject of many potential model and lattice QCD calculations. In the potential model with the nonrelativistic approximation, the mass splitting is given by $M_{J/\psi}-M_{\eta_c}=\dfrac{32\pi\alpha_s(m_q)}{9m_q}|\Psi(0)|^2$, where $\Psi(0)$ is the wavefunction at the origin. The reliable way to estimate the mass splitting is the lattice QCD calculation. The lattice QCD with quench approximation yields $M_{J/\psi}-M_{\eta_c}=77$ MeV, which is quite a bit smaller than the experimental value 117 MeV[3].

To resolve these discrepancies between experiments and theoretical calculations, it is desirable to make a high precision measurement of the η_c mass and width. In the previous measurements performed with the processes of $\gamma\gamma$ fusion, pp collider, $p\bar{p}$ annihilation and charmonium transitions, the line shape of the η_c is naively assumed to have the shape described by the Breit-Wigner distribution. With the large data sample accumulated at CLEOc, however, it is found that the simple Breit-Wigner form can not describe well the η_c line shape[4]. In this work, we present some formulae to describe the η_c line shapes, and they are intended for using it to study the η_c transitions of J/ψ or $\psi(2S)$ at BESIII.

Received 22 December 2008, Revised 1 February 2009

1) E-mail: pingrg@mail.ihep.ac.cn

2 General description of η_c line shape

2.1 Exclusive decays

Exclusive decays of η_c with large branching fractions (about a few percent) can be used to reconstruct η_c signals and to measure the resonance parameters. From PDG08[5], these channels, for example, include $\eta_c\to\eta'(958)\pi\pi$, $\eta\pi\pi$, $\rho\rho$, $K^*(892)K^-\pi^+$+ c.c., $K\bar{K}\pi$, $K^{*0}\bar{K}^{*0}\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$, $2(\pi^+\pi^-)$ and $3(\pi^+\pi^-)$. Some other two-body decays, though with less branching fractions, can also be used due to the high detection efficiency they have, such as $\eta_c\to p\bar{p}$, $\Lambda\bar{\Lambda}$, $\phi\phi$ and so on.

First, we consider the two-body decays of η_c, such as:

$$\begin{aligned}&\psi(2S),\ J/\psi(P_i,\epsilon_i)\to\gamma(p_3,\epsilon_\gamma)\eta_c(p_{12}),\\&\eta_c\to X_1(p_1,\epsilon_1)X_2(p_2,\epsilon_2),\end{aligned}\tag{2}$$

as shown in Fig. 1, where (p,ϵ) denote the vector particle's momentum and polarization vector. The amplitude corresponding to the sequential decay is written as:

$$\begin{aligned}\mathcal{M}\ =\ &\mathcal{M}_{J/\psi[\psi(2S)]\to\gamma\eta_c}\mathrm{BW}(m_{12},M,\Gamma)\times\\&\mathcal{M}_{\eta_c\to X_1X_2}\mathcal{F}(m_{12}),\end{aligned}\tag{3}$$

where $\mathrm{BW}(m_{12},M,\Gamma)$ is the η_c Breit-Wigner with resonance parameters M (mass) and Γ (width); $\mathcal{F}(m_{12})$ is the form factor to account for η_c production with the mass m_{12}, whose form will be given later on.

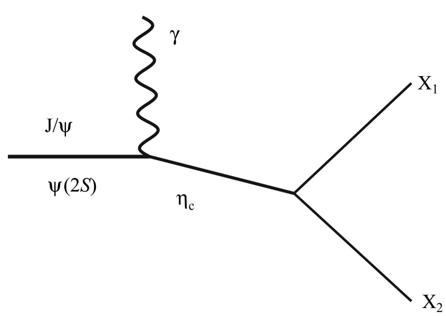

Fig. 1. To reconstruct η_c signals via two-body decays in J/ψ or $\psi(2S)$ radiative decays.

The standard partial decay rate is expressed by[5]:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}m_{12}}=\frac{1}{(2\pi)^5}\frac{1}{M_i^2}\int\bar{\Sigma}|\mathcal{M}|^2|\boldsymbol{p}_1^*||\boldsymbol{p}_3|\mathrm{d}\Omega_1^*\mathrm{d}\Omega_3\,,\tag{4}$$

where $\bar{\Sigma}$ denotes taking a sum of the amplitude over the total final state spins and averaging over the J/ψ or ψ polarizations. M_i is the mass of J/ψ or $\psi(2S)$, and $(|\boldsymbol{p}^*|,\ \Omega_1^*)$ is the momentum of particle X_1 in the rest frame of X_1 and X_2, and Ω_3 is the angle of the photon in the rest frame of J/ψ or $\psi(2S)$. The momenta magnitude of $|\boldsymbol{p}_1^*|$ and $|\boldsymbol{p}_3|$ are given by

$$\begin{aligned}|\boldsymbol{p}_1^*|&=\frac{\sqrt{[m_{12}^2-(m_1+m_2)^2][m_{12}^2-(m_1-m_2)^2]}}{2m_{12}},\\|\boldsymbol{p}_3|&=\frac{M_i^2-m_{12}^2}{2M_i}.\end{aligned}\tag{5}$$

In what follows, we will construct the amplitudes for J/ψ or $\psi(2S)\to\gamma\eta_c$ and η_c two-body decays using a covariant amplitude method. From the consideration of P-parity conservation, the amplitude for J/ψ or $\psi(2S)\to\gamma\eta_c$ is written as:

$$\mathcal{M}_{J/\psi[\psi(2S)]\to\gamma\eta_c}=\frac{g_1}{M_i}\epsilon_{\alpha\beta\mu\nu}p_3^\alpha\epsilon_\gamma^\beta P_i^\mu\epsilon_i^\nu\,,\tag{6}$$

where ϵ_γ and ϵ_i are the vectors of polarization for the photon and J/ψ $[\psi(2S)]$, respectively. The helicity value of J/ψ or $\psi(2S)$ is restricted to being $\lambda_i=\pm1$ due to their production from the e^+e^- beam. Similarly, for $\eta_c\to VV$ $(V=\rho,\ \phi)$, one has:

$$\mathcal{M}_{\eta_c\to VV}=\frac{g_2}{m_{12}}\epsilon_{\alpha\beta\mu\nu}p_1^\alpha\epsilon_1^\beta p_{12}^\mu\epsilon_2^\nu\,,\tag{7}$$

where $p_{12}=p_1+p_2$. For convenience, this amplitude can be calculated in the rest frame of η_c by replacing p_1 with p_1^*.

For $\eta_c\to B\bar{B}$ $(B=p,\ \Lambda)$ decays, the amplitudes are written as

$$\mathcal{M}_{\eta_c\to B\bar{B}}=g_3m_{12}\bar{u}(p_1,s_1)\gamma_5v(p_2,s_2)\,,\tag{8}$$

where $u(p_1,s_1)$ and $v(p_2,s_2)$ are the Dirac spinors normalized as $u^\dagger u=-v^\dagger v=E_B/m_B$ for baryon and antibaryon, respectively. g_3 is a dimensionless coupling constant.

Inserting Eqs. (6)—(8) into Eq. (3), and using Eq. (4), one gets the mass distribution:

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}m_{12}}(\eta_c\to f)\ =\ &|\mathrm{BW}(m_{12},M,\Gamma)\mathcal{F}(m_{12})|^2\times\\&T_f^2|\boldsymbol{p}_3|^3|\boldsymbol{p}_1^*|^3\,;\end{aligned}\tag{9}$$

where f denotes the final states VV or $B\bar{B}$, and $T_{VV}^2=(2m^2+2mm_{12}-m_{12}^2)(2m^2-2mm_{12}-m_{12}^2)(m_{12}^2-M_i^2)^2g_1g_2/(2M_im_{12})^2$, and $T_{B\bar{B}}^2=(M_i+m_{12})^2(M_i-m_{12})^2m_{12}^4g_1g_3/(8mM_i^2(m+E))$, where m and E are the mass and energy of vector mesons or baryons.

Eq. (9) shows that to describe the observed mass distribution, besides the Breit-Wigner distribution, one needs a form factor $\mathcal{F}(m_{12})$ to account for η_c production and another factor T_f to account for the η_c decay.

The multi-body decays are commonly believed to proceed dominated by the quasi-two-body decays, since the multi-body decays have less probability to create more quark pairs to form hadrons than two body decays. In experiments, η_c decays into the $KK\pi$, $K^*K\pi$, $\eta\pi\pi$, and $\eta'(958)\pi\pi$ have large branching fractions of about a few percent. However, no indications about the two body decays are seen in these channels due to the low statistics.

2.2 Inclusive decays

For the photon inclusive decays, the partial decay rate reads

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}m_{12}} \propto |\mathrm{BW}(m_{12},M,\Gamma)\mathcal{F}(m_{12})|^2 \times \int \bar{\Sigma}|\mathcal{M}_{J/\psi[\psi(2S)]\to\gamma\eta_c}|^2|\boldsymbol{p}_3^*|\mathrm{d}\Omega_3 \times \sum_{n,\lambda_n}\int|\mathcal{M}_{\eta_c\to X_n(\lambda_n)}|^2\mathrm{d}\Omega_n\,, \tag{10}$$

where $\mathcal{M}_{\eta_c\to X_n(\lambda_n)}$ is the amplitude for $\eta_c \to X_n$ decays with the n-body phase space $\mathrm{d}\Omega_n$. The sum runs over all decay channels and spins of the final particle states λ_n. Note that here we neglect the interference effects between η_c decay channels, so that the second integral can be approximately replaced by the η_c decay width Γ. Hence one has

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}m_{12}} \propto |\mathrm{BW}(m_{12},M,\Gamma)\mathcal{F}(m_{12})|^2|\boldsymbol{p}_3^*|^3\Gamma\,. \tag{11}$$

In experiment, the inclusive photon spectrum is also used to reconstruct the decay J/ψ, $\psi(2S)\to\gamma\eta_c$. The energy distribution of the transition photon is expressed by:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} \propto \frac{\mathcal{F}(\tilde{m}_{12})\Gamma M_i}{(M_i^2-2M_iE_\gamma-m_{\eta_c}^2)^2+\Gamma^2m_{\eta_c}^2} \times \frac{E_\gamma^3}{\sqrt{M_i^2-2M_iE_\gamma}}\,, \tag{12}$$

where $\tilde{m}_{12}=\sqrt{M_i^2-2M_iE_\gamma}$ and M_i is the mass of J/ψ or $\psi(2S)$.

3 Form factor

3.1 Phenomenology

In studying resonance state production and decays, the monopole/dipole form factors are always used to account for the off-shell effects of the intermediate state[6]. For example, the monopole form factor is used in studying η_c production from two-photon collisions at LEP[7]. In the case of the M1 transition of $J/\psi\to\gamma\eta_c$, we take the phenomenological form as

$$\mathcal{F}(m_{12}) = \begin{cases} \left(\dfrac{\Lambda^2}{\Lambda^2+|m_{12}^2-m_{\eta_c}^2|}\right)^n \\ \exp(-|m_{12}^2-m_{\eta_c}^2|/\Lambda^2) \end{cases}, \tag{13}$$

conventionally, the first expression is called the monopole ($n=1$) or dipole ($n=2$) form factor with an energy cut-off parameter Λ, and the second one is called the exponential form factor, which accounts for the overlap of two charmonium wavefunctions as arises in the lattice QCD calculations[8].

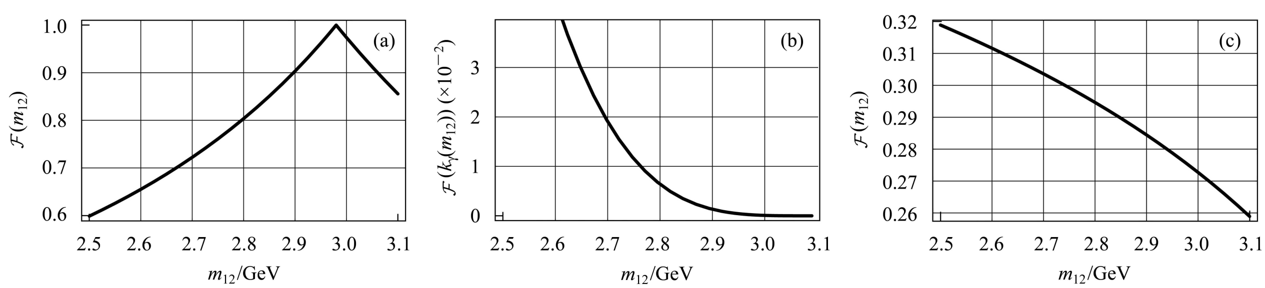

Fig. 2. The shapes of the form factor: (a) the dipole form factor with $\Lambda=3.0$ GeV; (b) M1 transition for $\psi(2S)\to\gamma\eta_c$ [see Eq. (16)]; (c) hadronic loop contributions.

3.2 M1 transition

If the M1 transition is assumed to dominate the decay $J/\psi[\psi(2S)]\to\gamma\eta_c$, the decay width by a nonrelativistic calculation is given by Eq. (1). From this equation, one gets the form factor as:

$$\mathcal{F}(k_\gamma) \propto \int_0^\infty \mathrm{d}r r^2 R_{n'0}(r)R_{n0}(r)j_0\left(\frac{k_\gamma r}{2}\right), \tag{14}$$

where k_γ is the photon momentum. The small photon momentum allows one to expand the spherical Bessel function as:

$$j_0\left(\frac{k_\gamma r}{2}\right) = 1-\frac{(k_\gamma r)^2}{24}+\cdots\,. \tag{15}$$

Note that for the orthogonality condition of the R_n, to the leading order accuracy, one gets

$$\mathcal{F}(k_\gamma)\propto k_\gamma^m, \quad \text{with} \quad m=0 \quad \text{for} \quad \mathrm{J}/\psi, \\ \text{and} \quad m=2 \quad \text{for} \quad \psi(2S). \tag{16}$$

3.3 Hadronic loop contributions

The quark model predicts the branching fraction for the M1 transition $\mathrm{J}/\psi\to\gamma\eta_c$ to be 2.4—2.9 keV, which is comparable with the measured value 1.85±0.17 keV[4]. However, for the hindered M1 transition $\psi(2S)\to\gamma\eta_c$, the theoretical value ~ 9.7 keV is larger than the experimental value $\sim 0.88\pm0.13$ keV. Recently, the D-meson loop contribution to the M1 transition was investigated in Ref. [9]. It turns out that the interfering between the D-meson loop and the M1 transition naturally reproduces the experimental values. For the transition J/ψ, $\psi(2S)\to\gamma\eta_c$, we define the amplitude with the form factor as:

$$\mathcal{M}_{fi}=\frac{1}{M_i}\mathcal{F}(m_{12})\epsilon_{\alpha\beta\mu\nu}p_3^\alpha\epsilon_3^\beta P_i^\mu\epsilon_i^\nu\,, \tag{17}$$

where the form factor $\mathcal{F}_{12}$ receives contributions from the quark model prediction and the $\mathrm{D\bar{D}(D)}$ loops with three configurations, together with the interfering effects between them. A numerical calculation using the loop integral expression in Ref. [9] is easily performed to get the the form factor $\mathcal{F}(m_{12})$ as shown in Fig. 2(c). We find it can be fitted with the following polynomial in 2.5—3.1 GeV,

$$\mathcal{F}(m_{12})=1-0.82m_{12}+0.31m_{12}^2-0.04m_{12}^3\,. \tag{18}$$

3.4 Sensitivity evaluation

We use the maximum likelihood to estimate the experimental sensitivities. The joint likelihood is defined by:

$$\mathcal{L}=\prod_{i=1}^{N}f(m_i,\alpha)=\prod_{i=1}^{N}\frac{|\mathcal{M}(m_i,\alpha)|^2}{\int \mathrm{d}m_i|\mathcal{M}(m_i,\alpha)|^2}\,, \tag{19}$$

where $f(m_i,\alpha)$ is the probability to observe the ith event with the amplitude $\mathcal{M}(m_i,\alpha)$, and N is the number of events observed, and α is the parameter in question, e.g. η_c mass or width.

The sensitivity to measure the parameter α is defined by:

$$\delta(\alpha)=\frac{\sqrt{V(\alpha)}}{\alpha}\,, \tag{20}$$

where $V(\alpha)$ is the variance of parameter α, which is calculated by:

$$V^{-1}(\alpha)=N\int\frac{1}{f(m_i,\alpha)}\left[\frac{\partial f(m_i,\alpha)}{\partial\alpha}\right]^2\mathrm{d}m_i\,, \tag{21}$$

where N is the total number of events.

If we use the non-relativistic Breit-Wigner to describe the η_c line-shape, i.e.

$$\mathrm{BW}(m_{12},M,\Gamma)=\frac{\sqrt{\Gamma}}{m_{12}-M-\mathrm{i}\Gamma/2}\,, \tag{22}$$

and the amplitude is taken as $|\mathcal{M}(m_{12},M,\Gamma)|^2=|\mathrm{BW}(m_{12},M,\Gamma)|^2$, then the sensitivity of measurement with N events observed can be evaluated by:

$$\delta(M)\equiv\frac{\sigma_M}{M}=\frac{1}{\sqrt{2N}}\frac{\Gamma}{M}, \\ \delta(\Gamma)\equiv\frac{\sigma_\Gamma}{\Gamma}=\sqrt{\frac{2}{N}}. \tag{23}$$

If the mass and width of η_c are, respectively, taken as M = 2.98 GeV and Γ = 0.0267 GeV[5], then the sensitivity of mass and width measurement versus the number of observed events are given in Fig. 3. Over two thousand events are observed, the mass and width sensitivity got are better than 0.02% and 3%, respectively.

4 Discussion and summary

The line shape of the bare Breit-Wigner is significantly modified by the form factors as shown in Fig. 4. Since all form factors have the same behavior to suppress the η_c line shape on deviating from its nominal mass point, the modified line shapes become thinner than the bare Breit-Wigner form. This implies that

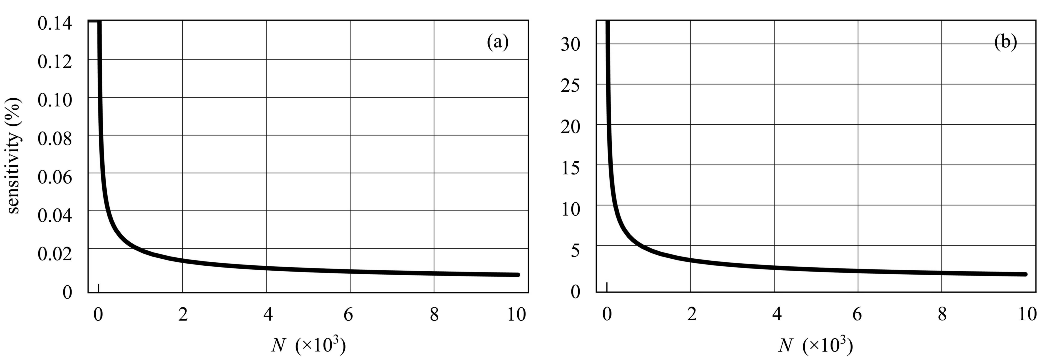

Fig. 3. The estimation of sensitivity for the η_c mass (a) and width (b) measurement.

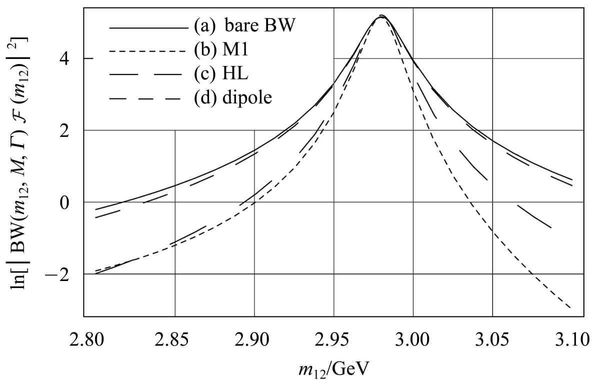

Fig. 4. The modified line shapes of η_c Breit-Wigner $\ln[|\mathrm{BW}(m_{12},M,\Gamma)\mathcal{F}(m_{12})|^2]$ by various form factors: (a) Bare Breit-Wigner with $(M,\Gamma)=(2.98,0.0267)$ GeV; (b) M1 transition form factor for $\psi(2S)\to\gamma\eta_c$ [see Eq. (16)]; (c) hadronic loop form factor; (d) dipole form factor with $\Lambda=3.0$ GeV.

the modified line shapes are necessary to fit the experimental data for extracting the actual η_c decay width. However, due to the uncertainties associated with the form factors in theoretical aspect, the modified line shapes will result in large uncertainties in the η_c measurement. For example, CLEOc reported that the systematic uncertainty from the η_c modified line shapes was about 7% in measuring $Br[\psi(2S)\to\gamma\eta_c]$, which is larger than the statistic error (about 4%)[4].

To summarize, in this work η_c line shapes in charmonium transition, i.e., $\psi(2S)$, $J/\psi\to\gamma\eta_c$, have been investigated. In η_c exclusive decays, besides the Breit-Wigner distribution, one needs a η_c birth factor $\mathcal{F}(m_{12})$, together with a decay factor T_f to reproduce the η_c line shape observed. The line shape by observing the inclusive photon spectrum is also given. As for the η_c birth factor, we present several forms, including the dipole or exponential phenomenological form, M1 transition and hadronic loop contribution forms. The sensitivities to measure the η_c resonance parameters are also evaluated. With more than two thousand η_c events observed, the precision of η_c decay width will be improved by better than 3%. The η_c modified line shape by the form factors has a significant impact on the measurements of η_c resonance parameters or the branching fraction. The uncertainties associated with the line shapes will dominate the systematic errors in the η_c measurement and this will prohibit precision mass and width measurements.

The author would like to thank Dr. Zhu Kai for his useful comments.

References

1 BAI Z et al. Phys. Rev. Lett., 1990, **65**: 1309
2 Ambrogiani M et al. Phys. Lett. B, 2003, **556**: 45
3 Choe S et al. JHEP, 2003, **308**: 22
4 Mitchell R E, Shepherd M R et al. Phys. Rev. Lett., 2009, **102**: 011801; arXiv:0805.0252 [hep-ex]
5 Particle Data Group. Phys. Lett. B, 2008, **667**: 1
6 Shklyar V, Lenske H, Mosel U. Phys. Rev. C, 2005, **72**: 015210; XIE J J, Wilkin C, ZOU B S. Phys. Rev. C, 2008, **77**: 058202
7 Acciarri M, Achard P et al. Phys. Lett. B, 1999, **461**: 155
8 Dudek Jozef J, Edwards Robert G, Richards David G. Phys. Rev. D, 2006, **73**: 074507
9 LI Gang, ZHAO Qiang. Phys. Lett. B, **670**: 55; arXiv: 0709.4639 [hep-ph]