Low-lying spectra and E2 transition rates in $^{160-170}$Er isotopes in the interaction boson model*

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Abstract Spectra and E2 transition rates for the $^{160-170}$Er isotopes are studied in the framework of the interaction boson model. A schematic Hamiltonian able to describe their spectra and B(E2) transition is used. It is found that the $^{160-170}$Er isotopes are in the transition from the vibrational limit to rotational limit.

Key words spectra, electromagnetic transition, positive parity collective state

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1 Introduction

The interacting boson model(IBM)[1] of nuclei, introduced by Arima and Iachello, is phenomenologically successful in describing the spectra of medium heavy nuclei and heavy nuclei. This model treats pairs of valence nucleons(particles/holes) as bosons with angular momentum $l=0$(s bosons) or $l=2$(d bosons). In the original version of the interacting boson model, IBM-1, no distinction is made between neutron bosons and proton bosons. In the early work[1—3], D. D. Warner and R. F. Casten studied the low-lying states and E2 and M1 transition rates of ^{164,168}Er, and found that those isotopes are nearly perfect rotors. Recently, lots of experimental and theoretical studies have been done for Er isotopes[4—8]. A. Leviatan and I.Sinai[4] studied the structure of the lowest $K=0^+$collective excitation in nucleus ^{168}Er by partial dynamical $SU(3)$ symmetry(PDS). N. Minkov et al.[5] studied the ground-γ band mixing and odd-even staggering in $^{162-166}$Er isotopes. GRIGORIEV et al.[6] studied the positive and negative parity states of 170Er. L. Genilloud et al.[7] studied the negative parity states of ^{170}Er by the The interacting boson model with f boson(sdf IBM-1). V. E. Cer ó n and J. G. Hirsch[8] studied the properties of E2 and M1 transition in ^{162}Er by pseudo $SU(3)$ model. In this paper, we studied the positive parity collective states in the $^{160-170}$Er isotopes by IBM-1. The calculated values are in agreement with data. It is found that these even-even Er isotopes are in the transition from $U(5)$ to $SU(3)$ dynamical symmetry.

2 The schematic IBM Hamiltonian

The general IBM Hamiltonian contains 7 terms. For our study, we take the following schematic Hamiltonian[9]

$$\hat{H}=\varepsilon_{\rm d}\hat{n}_{\rm d}+K\hat{Q}\cdot\hat{Q}+K_{\rm L}\hat{L}\cdot\hat{L}, \tag{1}$$

Where

$$\hat{Q}_\mu=(\hat{s}^+\hat{\tilde{d}}+\hat{d}^+\hat{s})^2+\chi(\hat{d}^+\hat{\tilde{d}})^2_\mu,$$

$$\hat{L}_{\rm q}=\sqrt{10}(\hat{d}^+\hat{\tilde{d}})^{(1)}_{\rm q},\chi=-\sqrt{7}/2.$$

The Hamiltonian is able to give a transition from $U(5)$ to $SU(3)$, If $\varepsilon_{\rm d}=0$, then the Hamiltonian reduces to a $SU(3)$ limit Hamiltonian. If $K=0$, the Hamiltonian becomes a $U(5)$ limit, describing the vibrational collective motion. The term $K_{\rm L}(L\cdot L)$ is diagonal, it contributes the same to the energy levels with identical spin, and is a term adjusting energy level L. Therefore, the ratio of $K/\varepsilon_{\rm d}$ is a measure of the transition from $U(5)$ to $SU(3)$. If $K/\varepsilon_{\rm d}=0$,

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the Hamiltonian is vibrational, and if $K/\varepsilon_{\rm d}=\infty$, the Hamiltonian is rotational. If the ratio lies in between, the Hamiltonian is in the transition between $U(5)$ and $SU(3)$. The parameters in the Hamiltonian can be determined by fitting to the experimental spectra.

3 Results and discussion

In Table 1, we give the parameters of the Hamiltonian and of the E2 transition operator in each nucleus studied. From table 1, all parameters change rather smoothly. In the lighter even Er isotopes, the value of $\varepsilon_{\rm d}$ decreases with increasing mass number, until ^{164}Er. In the heavier even-even Er isotopes, $\varepsilon_{\rm d}=0$ value increases with increasing mass number. It reflects the properties of the change of energy in excited states and of shape coexistence for Er isotopes.

Table 1. Parameters of energy level and B(E2) operator for Er isotopes.

nucleus	$\varepsilon_{\rm d}$/MeV	K/MeV	$K_{\rm L}$/MeV	e_2/eb
^{160}Er	0.300	−0.0120	0.021	
^{162}Er	0.100	−0.0125	0.020	0.234
^{164}Er	0.010	−0.0100	0.020	0.235
^{166}Er	0.015	−0.0100	0.015	0.160
^{168}Er	0.380	−0.0110	0.010	0.240
^{170}Er	0.420	−0.0110	0.010	0.180

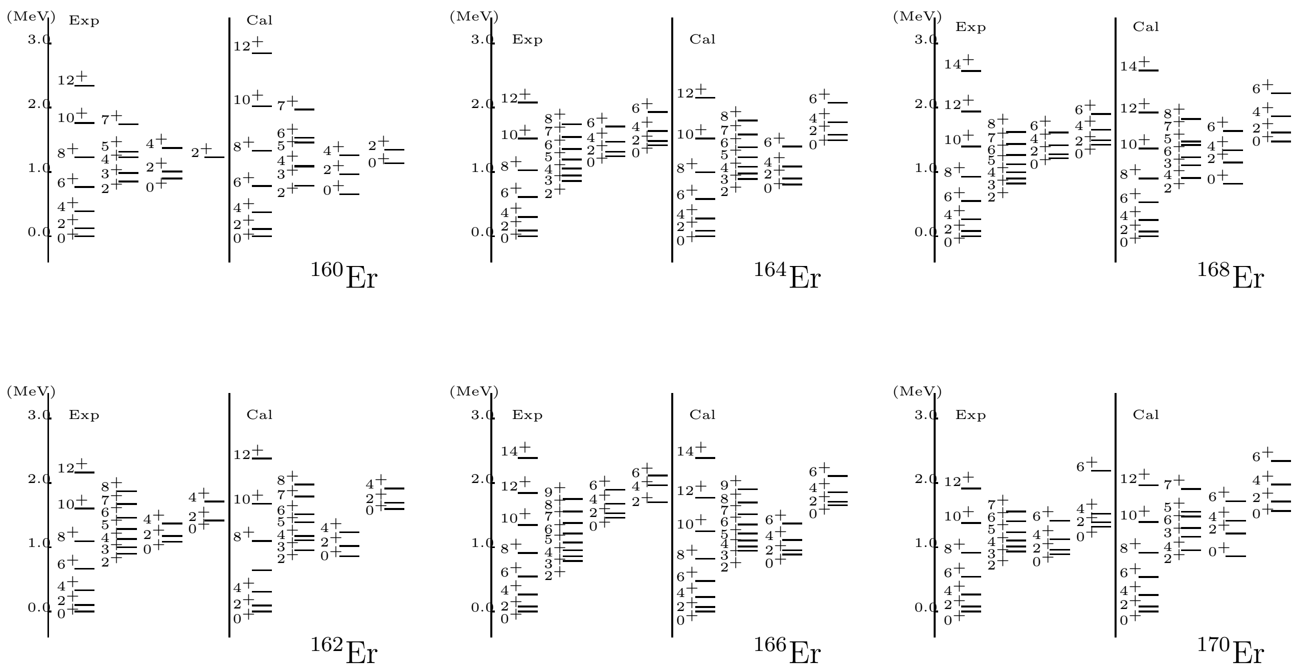

Fig. 1. Spectra $^{160-170}$Er.

3.1 Energy levels

The comparisons between calculated and experimental values[6] of energy levels for each Er nucleus are shown in Figs. 1—6, respectively. In general, the agreement is good, especially for the ground-state band levels and γ- band levels. However, there exist some discrepancies. The main reason is that the mix of many bands is not considered.

For nucleus ^{160}Er, the quality of agreement between theory and experimental data is good. This nucleus exhibits staggering phenomenon in the gamma band. It is noticed that the agreement between the calculated and experimental staggering is not good. This deviation may be improved by the use of cubic terms. For isotopes $^{162-168}$Er, the agreement is quite good, especially for ground-state band and gram-state band levels. To observe the transition between limits, we show the ratios $R=({\rm E}(4_1^+)/{\rm E}(2_1^+)$ for isotopes $^{160-170}$Er in table 2. It is obvious that the ratios R reflect a transition from vibration-like nuclei to more deformed ones. The values of R of ^{168,170}Er are 3.1, so the ^{168,170}Er isotopes close to rotational nuclei.

Table 2. The ratio of R for $^{160-170}$Er isotopes, where the A is the nucleon number.

A	160	162	164	166	168	170
R	3.10	3.23	3.28	3.29	3.31	3.31

3.2 E2 Transition

After the determination of the spectra, the wave function is determined. The electric and magnetic transition properties can then be obtained accordingly. For example, the E2 transition operator is

$$\hat{T}(\text{E2})^2_\mu = e_2[(\hat{s}^+\tilde{d}+\hat{d}^+\hat{s})^2_\mu + \chi(\hat{d}^+\tilde{d})^2_\mu],$$

The meaning of the symbols is the same as those in other papers about IBM. Table 3 gives the comparison between calculated and experimental B(E2) values for $^{160-170}$Er isotopes. Results obtained in the present work are in good agreement with experiments. This reflects a transition from $U(5)$ to $SU(3)$.

4 Conclusion

We have given a detailed study of the energy levels and E2 transitions in $^{160-170}$Er isotopes in IBM. The results indicate that $^{160-170}$Er isotopes are the $U(5)$ to $SU(3)$ transitional nuclei. Meanwhile, the discrepancy between calculated and experimental data is found, which means that other factors must be introduced into Hamiltonian, such as pair interacting, isospin effect, high angular momentum boson and so on.

Table 3. Comparison of B(E2) values in $^{162-170}$Er isotopes.

nucleus	J_i	J_f	Expt/($e^2\cdot\text{fm}^4$)	Calc/($e^2\cdot\text{fm}^4$)
^{162}Er	2^+_1	0^+_1	11630	11720
	2^+_2	0^+_1	330	268
^{164}Er	2^+_1	0^+_1	11615	11920
	4^+_1	2^+_1	13746	16830
	8^+_1	6^+_1	18275	18410
	10^+_1	8^+_1	19074	18120
	12^+_1	10^+_1	14120	17450
	2^+_2	2^+_1	607	596
	2^+_2	0^+_1	277	410
^{166}Er	2^+_1	0^+_1	11630	11815
	4^+_1	2^+_1	16900	16710
	6^+_1	4^+_1	18860	18070
	8^+_1	6^+_1	19840	18410
	10^+_1	8^+_1	20160	18227
	2^+_2	4^+_1	36	55
	2^+_2	2^+_1	527	853
^{168}Er	2^+_1	0^+_1	11400	12490
	4^+_1	2^+_1	17490	17660
	6^+_1	4^+_1	24200	19110
	8^+_1	6^+_1	19250	19490
	10^+_1	8^+_1	16610	19340
	12^+_1	10^+_1	18370	18840
	8^+_2	8^+_1	99	122
	2^+_2	0^+_1	264	260
^{170}Er	2^+_1	0^+_1	11630	12490
	8^+_1	6^+_1	20700	19800
	10^+_1	8^+_1	17900	19800
	12^+_1	10^+_1	20980	19485
	2^+_4	0^+_1	15	12

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