Analysis of the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with light-cone QCD sum rules[*]

WANG Zhi-Gang(王志刚)[1)]

Department of Physics, North China Electric Power University, Baoding 071003, China

Abstract In this article, we take the point of view that the light scalar meson $a_0(980)$ is a conventional $q\bar{q}$ state, and calculate the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with the light-cone QCD sum rules. The central value of the coupling constant $g_{a_0\eta\pi^0}$ is consistent with that extracted from the radiative decay $\phi(1020) \to a_0(980)\gamma \to \eta\pi^0\gamma$. The central value and lower bound of the decay width $\Gamma_{a_0\to\eta\pi^0} = 127^{+84}_{-48}$ MeV are compatible with the experimental data of the total decay width $\Gamma_{a_0(980)} = (50-100)$ MeV from the Particle Data Group with a very model dependent estimation (the decay width can be much larger), while the upper bound is too large. We give a possible explanation for the discrepancy between the theoretical calculation and experimental data.

Key words $a_0(980)$, light-cone QCD sum rules

PACS 12.38.Lg, 13.25.Jx, 14.40.Cs

1 Introduction

Light flavor scalar mesons present a remarkable exception for constituent quark models; the structures of these mesons have not been unambiguously determined yet [1–4]. Experimentally, the strong overlaps with each other and the broad widths (for the $f_0(980)$, $a_0(980)$, $f_0(1710)$, the widths are relatively narrow) are responsible for the fact that their spectra cannot be approximated by the Breit-Wigner formula. The numerous candidates with the same quantum numbers $J^{PC} = 0^{++}$ below $2\,\mathrm{GeV}$ cannot be accommodated in one $q\bar{q}$ nonet; some are supposed to be glueballs, molecules and multiquark states [2–4]. The more elusive things are the constituent structures of the mesons $f_0(980)$ and $a_0(980)$ with almost degenerate masses.

In the naive quark model, $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $f_0 = s\bar{s}$; while in the framework of the tetraquark models, the mesons $f_0(980)$ and $a_0(980)$ could either be compact objects (i.e. nucleon-like bound states of quarks with the symbolic quark structures $f_0 = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ and $a_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ [5, 6]) or spatially extended objects (i.e. deuteron-like bound states of hadrons: $K\bar{K}$ molecules [7, 8]). The hadronic

dressing mechanism takes the point of view that the mesons $f_0(980)$ and $a_0(980)$ have small $q\bar{q}$ cores of typical $q\bar{q}$ meson size, strong couplings to the intermediate hadronic states ($K\bar{K}$) enrich the pure $q\bar{q}$ states with other components and spend part (or most) of their lifetime as virtual $K\bar{K}$ states [9–11]. In the hybrid model, these mesons are tetraquark states $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ in the S-wave near the center, with some constituents $q\bar{q}$ in the P-wave, but further out they rearrange into $(q\bar{q})_1(q\bar{q})_1$ states and finally as meson-meson states [2, 4]. All these interpretations have both outstanding advantages and obvious shortcomings in one way or other.

We can study scalar mesons through their couplings to two pseudoscalar mesons, two-photon decays and radiative decays. The radiative decays $\phi(1020) \to \pi^0\pi^0\gamma$ and $\phi(1020) \to \eta\pi^0\gamma$ have been the subject of intense investigation [12–18]. From the invariant $\pi^0\pi^0$ and $\eta\pi^0$ mass distributions, we can obtain a lot of information about the nature of $f_0(980)$ and $a_0(980)$ respectively.

In this article, we take the scalar mesons $a_0(980)$ and $f_0(980)$ as the conventional $q\bar{q}$ states, and calculate the values of the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with the light-cone QCD sum rules. The cou-

pling constant $g_{a_0\eta\pi^0}$ is a basic parameter in studying the radiative decay $\phi(1020) \to a_0(980)\gamma \to \eta\pi^0\gamma$. In previous works, the mesons $f_0(980)$, $a_0(980)$, D_{s0}, D_{s1}, B_{s0} and B_{s1} were taken as the conventional $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ states respectively, and the values of the coupling constants g_{f_0KK}, g_{a_0KK}, $g_{D_{s0}DK}$, $g_{D_{s1}D^*K}$, $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ have been calculated using the light-cone QCD sum rules [19–24]. The large values of the coupling constants support the hadronic dressing mechanism. In Ref. [25], the authors study the coupling constant $g_{a_0\eta\pi^0}$ with the interpolating current

$$J_\mu^8 = \frac{1}{\sqrt{6}}\left[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s\right]$$

and a complex subtraction procedure is taken due to the asymmetric Borel parameters $M_1^2 \neq M_2^2$. In this article, we study the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ together and a simple subtraction procedure is taken. The decay $f_0(980) \to \pi\pi$ cannot occur at the tree level if the scalar meson $f_0(980)$ is a pure $s\bar{s}$ state. It should have some $n\bar{n}$ components and the coupling constant $g_{f_0\pi\pi}$ has also been calculated with the light-cone QCD sum rules [26].

The light-cone QCD sum rule approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$, while the nonperturbative matrix elements are parameterized by the light-cone distribution amplitudes instead of the vacuum condensates [27–31]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal [32–34].

The article is arranged as follows: in section 2, we obtain the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with the light-cone QCD sum rules; in section 3 numerical results are given; section 4 is reserved for the conclusion.

2　Coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with light-cone QCD sum rules

In the following, we write down the definitions for the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$,

$$\langle a_0|\eta\pi^0\rangle = ig_{a_0\eta\pi^0} = i\sqrt{\frac{2}{3}}g\,,$$

$$\langle a_0|\eta'\pi^0\rangle = ig_{a_0\eta'\pi^0} = i\frac{2}{\sqrt{3}}g\,, \quad (1)$$

where we have used the phenomenological Lagrangian $\mathcal{L} = g\mathrm{Tr}[SPP]$, the S and P stand for the light nonet scalar mesons and pseudoscalar mesons respectively.

We study the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ by means of the two-point correlation function $\Pi_\mu(p,q)$,

$$\Pi_\mu(p,q) = i\int d^4 x e^{-iq\cdot x}\langle 0|T\{J_\mu(0)J(x)\}|\pi^0(p)\rangle\,, \quad (2)$$

$$J_\mu(x) = \bar{u}(x)\gamma_\mu\gamma_5 u(x) + \bar{d}(x)\gamma_\mu\gamma_5 d(x)\,,$$

$$J(x) = \frac{\bar{u}(x)u(x) - \bar{d}(x)d(x)}{\sqrt{2}}\,, \quad (3)$$

where the currents $J_\mu(x)$ and $J(x)$ interpolate the pseudoscalar mesons η, η' and scalar meson $a_0(980)$, respectively; the external π^0 meson has the four momentum p_μ with $p^2 = m_\pi^2$. One may think that it is more convenient to take the octet current $J_\mu^8(x)$ and singlet current $J_\mu^0(x)$

$$J_\mu^8(x) =$$
$$\frac{\bar{u}(x)\gamma_\mu\gamma_5 u(x) + \bar{d}(x)\gamma_\mu\gamma_5 d(x) - 2\bar{s}(x)\gamma_\mu\gamma_5 s(x)}{\sqrt{6}}\,,$$

$$J_\mu^0(x) =$$
$$\frac{\bar{u}(x)\gamma_\mu\gamma_5 u(x) + \bar{d}(x)\gamma_\mu\gamma_5 d(x) + \bar{s}(x)\gamma_\mu\gamma_5 s(x)}{\sqrt{3}} \quad (4)$$

to interpolate the pseudoscalar mesons η and η' respectively. The $\bar{s}s$ components of the interpolating currents have no contributions at the level of the quark-gluon degrees of freedom and the octet current $J_\mu^8(x)$ and singlet current $J_\mu^0(x)$ lead to the same analytical expressions. $J_\mu(x)$ is a linear combination of the octet current $J_\mu^8(x)$ and singlet current $J_\mu^0(x)$; we choose it to interpolate the mesons η and η' together,

$$J_\mu(x) = \sqrt{\frac{2}{3}}J_\mu^8(x) + \frac{2}{\sqrt{3}}J_\mu^0(x)\,. \quad (5)$$

Irrespective of the chosen interpolating current, the coupling with $a_0(980)\pi^0$ takes place through the $u\bar{u}$ and $d\bar{d}$ components of the pseudoscalar mesons η and η' (not the $s\bar{s}$ component) at the level of the quark-gluon degrees of freedom. Although the coupling constant $g_{a_0\eta'\pi^0}$ has no direct phenomenological interest, we take into account the η' meson to facilitate subtractions of the continuum states and obtain more reliable QCD sum rules. We will revisit this subject at the end of this section.

The correlation function $\Pi_\mu(p,q)$ can be decomposed as

$$\Pi_\mu(p,q) = i\Pi(p,q)q_\mu + i\Pi_A(p,q)p_\mu \quad (6)$$

due to Lorentz covariance. We choose the tensor structure q_μ and study $\Pi(p,q)$.

According to the basic assumption of the quark-hadron duality in the QCD sum rules [32–34], we can insert a complete sets of intermediate hadronic states

with the same quantum numbers as the current operators $J_\mu(x)$ and $J(x)$ into the correlation function $\Pi_\mu(p,q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the mesons η, η' and $a_0(980)$, we get the following result (we present some technical details in the appendix),

$$
\begin{aligned}
\Pi_\mu =\ & \frac{\langle 0|J_\mu(0)\,|\,\eta(q+p)\rangle}{M_\eta^2-(q+p)^2-\mathrm{i}\epsilon}\langle\eta|J(0)|\pi^0\rangle + \frac{\langle 0|J_\mu(0)\,|\,\eta'(q+p)\rangle}{M_{\eta'}^2-(q+p)^2-\mathrm{i}\epsilon}\langle\eta'|J(0)|\pi^0\rangle + \cdots = \\[2mm]
& \frac{\mathrm{i}2f_\eta(q+p)_\mu}{\sqrt{6}\left[M_\eta^2-(q+p)^2-\mathrm{i}\epsilon\right]}\langle\eta|a_0\pi^0\rangle\frac{\mathrm{i}}{q^2-M_{a_0}^2+\mathrm{i}\epsilon}\langle a_0(q)|J(0)|0\rangle + \\[2mm]
& \frac{\mathrm{i}2f_{\eta'}(q+p)_\mu}{\sqrt{3}\left[M_{\eta'}^2-(q+p)^2-\mathrm{i}\epsilon\right]}\langle\eta'|a_0\pi^0\rangle\frac{\mathrm{i}}{q^2-M_{a_0}^2+\mathrm{i}\epsilon}\langle a_0(q)|J(0)|0\rangle + \cdots = \\[2mm]
& \left[\frac{\mathrm{i}2gf_\eta f_{a_0}M_{a_0}}{3\left[M_\eta^2-(q+p)^2-\mathrm{i}\epsilon\right]\left[M_{a_0}^2-q^2-\mathrm{i}\epsilon\right]} + \frac{\mathrm{i}4gf_{\eta'}f_{a_0}M_{a_0}}{3\left[M_{\eta'}^2-(q+p)^2-\mathrm{i}\epsilon\right]\left[M_{a_0}^2-q^2-\mathrm{i}\epsilon\right]}\right](p+q)_\mu + \cdots,
\end{aligned}
\tag{7}
$$

where the following definitions for the weak decay constants have been used,

$$
\begin{aligned}
\langle 0|J_\mu(0)|\eta(p)\rangle &= \frac{\mathrm{i}2f_\eta}{\sqrt{6}}p_\mu, \\[2mm]
\langle 0|J_\mu(0)|\eta'(p)\rangle &= \frac{\mathrm{i}2f_{\eta'}}{\sqrt{3}}p_\mu, \\[2mm]
\langle 0|J(0)|a_0(p)\rangle &= f_{a_0}M_{a_0}.
\end{aligned}
\tag{8}
$$

We have taken the ideal mixing limit for η and η' (i.e. $\eta = \left|\dfrac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}\right\rangle$, $\eta' = \left|\dfrac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}\right\rangle$) and neglected the anomaly contribution.

In the following, we briefly outline the operator product expansion for the correlation function $\Pi_\mu(p,q)$ in perturbative QCD theory. The calculations are performed in the large space-like momentum regions $(q+p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge [35],

$$
S_{ij}(x_1,x_2) = \mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k(x_1-x_2)}\left\{\frac{\not{k}+m}{k^2-m^2}\delta_{ij} - \int_0^1\mathrm{d}v\,g_\mathrm{s}\,G_{ij}^{\mu\nu}(vx_1+(1-v)x_2)\right.
$$

$$
\left.\left[\frac{1}{2}\frac{\not{k}+m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x_1-x_2)_\mu\gamma_\nu\right]\right\}.
\tag{9}
$$

Substituting the u and d quark propagators and the corresponding π-meson light-cone distribution amplitudes into the correlation function $\Pi_\mu(p,q)$ and completing the integrals over the variables x and k, we obtain finally an analytical expression. In the calculation the two-particle and three-particle π-meson light-cone distribution amplitudes have been used [36–39]. The explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and are estimated with the QCD sum rules [36–39]. In this article, the energy scale μ is chosen to be $\mu = 1$ GeV.

After straightforward calculations we obtain the final expression of the double Borel transformed correlation function Π at the level of the quark-gluon degrees of freedom. The masses of the pseudoscalar meson and scalar meson are $M_{\eta'} = 0.958\,\mathrm{GeV}$ and $M_{a_0} = 0.985$ GeV respectively,

$$
\frac{M_{\eta'}^2}{M_{\eta'}^2+M_{a_0}^2} \approx 0.49.
\tag{10}
$$

There exists an overlapping working window for the two Borel parameters M_1^2 and M_2^2. It is convenient to take the value $M_1^2 = M_2^2$, $M^2 = \dfrac{M_1^2 M_2^2}{M_1^2+M_2^2}$. We introduce the threshold parameter s_0 and make the simple replacement,

$$
\mathrm{e}^{-\frac{m_u^2+u_0(1-u_0)m_\pi^2}{M^2}} \longrightarrow \mathrm{e}^{-\frac{m_u^2+u_0(1-u_0)m_\pi^2}{M^2}} - \mathrm{e}^{-\frac{s_0}{M^2}}
$$

to subtract the contributions from the high resonances and continuum states [35]. Finally we obtain the sum rule for the coupling constant g,

$$g = \frac{3\exp\left(\dfrac{M_{a_0}^2}{M_2^2}\right)}{2f_{a_0}M_{a_0}\left[f_{\eta}\exp\left(-\dfrac{M_{\eta}^2}{M_1^2}\right)+2f_{\eta'}\exp\left(-\dfrac{M_{\eta'}^2}{M_1^2}\right)\right]}\left\{\left[\exp\left(-\frac{\Xi}{M^2}\right)-\exp\left(-\frac{s_0}{M^2}\right)\right]\right.$$

$$\frac{f_{\pi}m_{\pi}^2 M^2}{2m_u}\left[\varphi_p(u_0)-\frac{\mathrm{d}\varphi_{\sigma}(u_0)}{6\mathrm{d}u_0}\right]+\exp\left(-\frac{\Xi}{M^2}\right)\left[-m_u f_{\pi}m_{\pi}^2\int_0^{u_0}\mathrm{d}tB(t)+\right.$$

$$f_{3\pi}m_{\pi}^2\int_0^{u_0}\mathrm{d}\alpha_u\int_{u_0-\alpha_u}^{1-\alpha_u}\mathrm{d}\alpha_g\varphi_{3\pi}(1-\alpha_u-\alpha_g,\alpha_g,\alpha_u)\frac{2(\alpha_u+\alpha_g-u_0)-3\alpha_g}{\alpha_g^2}-$$

$$\frac{2m_u f_{\pi}m_{\pi}^4}{M^2}\int_{1-u_0}^1\mathrm{d}\alpha_g\frac{1-u_0}{\alpha_g^2}\int_0^{\alpha_g}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\Phi(1-\alpha-\beta,\beta,\alpha)+$$

$$\frac{2m_u f_{\pi}m_{\pi}^4}{M^2}\left(\int_0^{1-u_0}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{u_0}\mathrm{d}\alpha_u\int_0^{\alpha_u}\mathrm{d}\alpha+\int_{1-u_0}^1\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}\mathrm{d}\alpha_u\int_0^{\alpha_u}\mathrm{d}\alpha\right)$$

$$\left.\left.\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}\right]\right\}, \tag{11}$$

where

<div align="left">

properly,</div>

$$\Phi(\alpha_i) = A_{\parallel}(\alpha_i)+A_{\perp}(\alpha_i)-V_{\parallel}(\alpha_i)-V_{\perp}(\alpha_i),$$
$$\Xi = m_u^2+u_0(1-u_0)m_{\pi}^2,$$
$$u_0 = \frac{M_1^2}{M_1^2+M_2^2}, \tag{12}$$

and we have taken the isospin limit $m_u=m_d$.

In Ref. [25] (also in Refs. [22, 23, 26]), a complex subtraction procedure is taken due to the asymmetry Borel parameters, $M_1^2\neq M_2^2$. In the light-cone QCD sum rules, we often take the technique developed in Ref. [35] to obtain the spectral densities at the level of the quark-gluon degrees of freedom,

$$\Pi = \int_0^1\frac{f(u)}{\Delta-(q+up)^2}\mathrm{d}u =$$
$$\int_{\Delta}^{\infty}\frac{\rho_{QCD}(s)}{[s-(p+q)^2][s-q^2]}\mathrm{d}s =$$
$$\int_{\Delta_1}^{\infty}\int_{\Delta_2}^{\infty}\frac{\rho_{QCD}(s_1,s_2)\delta(s_1-s_2)}{[s_1-(p+q)^2][s_2-q^2]}\mathrm{d}s_1\mathrm{d}s_2, \tag{13}$$

where $f(u)$ are functions of the two-particle light-cone distribution amplitudes, $u=\dfrac{\Delta-q^2}{s-q^2}$, Δ stands for the squared masses of the exchanged quarks, Δ_1 and Δ_2 are the corresponding thresholds. It works efficiently in the case where the threshold parameters s_1^0 and s_2^0 differ from each other slightly. If we take the values $s_1^0=s_{\eta}^0=(0.7\text{–}0.8)\,\mathrm{GeV}^2$ (in the case that the octet current $J_{\mu}^8(x)$ is chosen to interpolate the η meson, see Ref. [25]) and $s_2^0=s_{a_0}^0>M_{a_0}^2\approx 1\,\mathrm{GeV}^2$, the contributions from $a_0(980)$ are not taken into account

$$\Pi = \int_{\Delta_1}^{s_1^0}\int_{\Delta_2}^{s_2^0}\frac{\rho_{QCD}(s_1,s_2)\delta(s_1-s_2)}{[s_1-(p+q)^2][s_2-q^2]}\mathrm{d}s_1\mathrm{d}s_2+\cdots =$$
$$\int_{\Delta_1}^{s_1^0}\int_{\Delta_2}^{s_1^0}\frac{\rho_{QCD}(s_1,s_2)\delta(s_1-s_2)}{[s_1-(p+q)^2][s_2-q^2]}\mathrm{d}s_1\mathrm{d}s_2+\cdots. \tag{14}$$

In the case of non-equal threshold parameters $s_1^0\neq s_2^0$, we can take $s_0=\max(s_1^0,s_2^0)$ with s_0 small enough to avoid the contaminations from the high resonances in either of the two channels, or take $s_0=\min(s_1^0,s_2^0)$ with s_0 large enough to take into account the contributions from the ground states in either of the two channels. We have two choices in general, which can result in some uncertainties. In this article, we choose the current $J_{\mu}(x)$ to interpolate both the η and η' mesons to overcome the shortcoming, and take into account the contributions from the η' meson at the phenomenological side.

3   Numerical results and discussion

The input parameters of the light-cone distribution amplitudes are taken as $\lambda_3=0.0$, $f_{3\pi}=(0.45\pm 0.15)\times 10^{-2}\,\mathrm{GeV}^2$, $\omega_3=-1.5\pm 0.7$, $\omega_4=0.2\pm 0.1$, $a_1=0.0$, $a_2=0.28\pm 0.08$, $a_4=0.0$, $\eta_4=10.0\pm 3.0$ [36–39], $m_u=m_d=m_q=(5.6\pm 1.6)\,\mathrm{MeV}$, $f_{\pi}=0.130\,\mathrm{GeV}$, $m_{\pi}=0.135\,\mathrm{GeV}$, $M_{\eta}=0.547\,\mathrm{GeV}$, $M_{\eta'}=0.958\,\mathrm{GeV}$, $M_{a_0}=0.985\,\mathrm{GeV}$, $f_{\eta}=1.3f_{\pi}$, $f_{\eta'}=1.2f_{\pi}$ [40], and $f_{a_0}=(0.21\pm 0.01)\,\mathrm{GeV}$ [23].

The axial-vector current $J_{\mu}(x)$ has also non-vanishing couplings with both the pseudoscalar mesons $\eta(1295)$, $\eta(1405)$, $\eta(1475)$, etc and the axial-

vector mesons $f_1(1285)$, etc. The scalar current $J(x)$ also has non-vanishing couplings with the scalar mesons $a_0(1450)$, etc. The masses and widths of those mesons are $M_{\eta(1295)} = (1294 \pm 4)$ GeV, $\Gamma_{\eta(1295)} = (55\pm5)$ GeV; $M_{\eta(1405)} = (1409.8\pm2.5)$ GeV, $\Gamma_{\eta(1405)} = (51.1\pm3.4)$ GeV; $M_{\eta(1475)} = (1476\pm4)$ GeV, $\Gamma_{\eta(1475)} = (87\pm9)$ GeV; $M_{f_1(1285)} = (1281.8\pm0.6)$ GeV, $\Gamma_{f_1(1285)} = (24.2 \pm 1.1)$ GeV; $M_{a_0(1450)} = (1474 \pm 19)$ GeV and $\Gamma_{a_0(1450)} = (265 \pm 13)$ GeV from the Particle Data Group [41].

From the experimental data, we can see that the a_0 channel permits a larger threshold parameter than that of the η channel. If we take the value $s_0 = \max(s_\eta^0, s_{a_0}^0) = s_{a_0}^0 \leqslant 1.7$ GeV^2, the contaminations from the $\eta(1295)$ and $f_1(1285)$ are included. We have to take the other choice, $s_0 = \min(s_\eta^0, s_{a_0}^0) = s_\eta^0 \leqslant 1.6$ GeV^2. It happens to be the ideal choice and reproduces the mass of $a_0(980)$ with the conventional two-point QCD sum rules for the Borel parameter $M^2 = (1.0-1.6)$ GeV^2.

In this article, we take the threshold parameter and Borel parameter as $s_0 = (1.4 - 1.6)$ GeV^2 and $M^2 = (1.0 - 1.6)$ GeV^2 to avoid contaminations from the high resonances and continuum states as $\exp\left(-\dfrac{s_0}{M^2}\right) = 0.2 - 0.4$. In this region, the value of the coupling constant g is rather stable with respect to a variation of the Borel parameter, see Figs. 1–2.
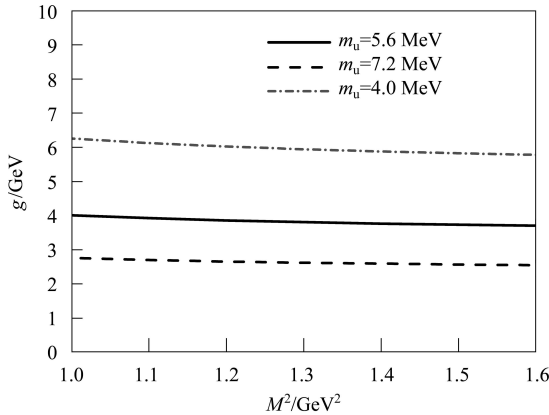

Fig. 1. The coupling constant g as a function of M^2 with m_u as a parameter.

In this article, we take the values of the coefficients a_i of the twist-2 light-cone distribution amplitude $\varphi_\pi(u)$ from the conventional QCD sum rules [36, 39]. $\varphi_\pi(u)$ has been analyzed with the light-cone QCD sum rules and (non-local condensates) QCD sum rules and confronted with the high precision CLEO data on the $\gamma\gamma^* \to \pi^0$ transition form-factor [42–47]. We also study the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with the values $a_2 = 0.29$ and $a_4 = -0.21$

at $\mu = 1$ GeV, which are obtained via a one-loop renormalization group equation for the central values $a_2 = 0.268$ and $a_4 = -0.186$ at $\mu^2 = 1.35$ GeV^2 from the (non-local condensates) QCD sum rules with the improved model [47].
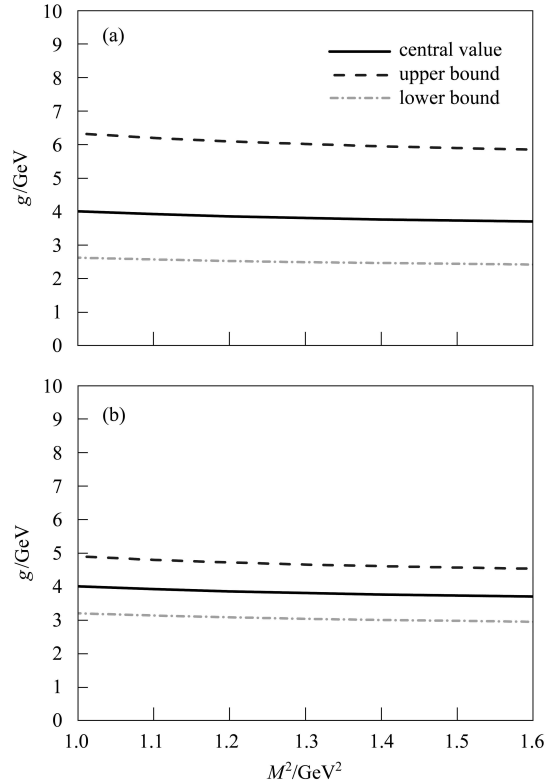

Fig. 2. Variation of the coupling constant g with M^2, (a) for $m_q = (5.6 \pm 1.6)$ MeV and (b) for $m_q = (5.6 \pm 0.6)$ MeV.

In the limit of a large Borel parameter M^2, the coupling constant g takes up the following behavior,

$$g \propto \frac{M^2}{m_u}\left[\varphi_p(u_0) - \frac{\mathrm{d}\varphi_\sigma(u_0)}{6\mathrm{d}u_0}\right]. \qquad (15)$$

It is not unexpected that the contributions from the two-particle twist-3 light-cone distribution amplitude $\varphi_p(u)$ are greatly enhanced by the large Borel parameter M^2; large uncertainties of the relevant parameters presented in the above equations have significant impact on the numerical results. The contribution from the two-particle twist-3 $\varphi_\sigma(u_0)$ is zero due to its symmetry property. If we take the value $m_u = m_d = m_q = (5.6 \pm 1.6)$ GeV [39], the uncertainty coming from m_q is very large, about (33–64)%, and the predictive ability is poor, see Fig. 1. From the Gell-Mann-Oakes-Renner relation, we can obtain $\dfrac{f_\pi^2 m_\pi^2}{m_u + m_d} = (0.027 \pm 0.003)$ GeV^3 [36], i.e. $m_q \approx (5.6 \pm 0.6)$ GeV, which may result in a much smaller uncertainty.

Taking into account all the uncertainties of the input parameters, finally we obtain the numerical values of the coupling constants, which are shown in Fig. 2,

$$g = 3.8^{+2.5}_{-1.4} \text{ GeV},$$

$$g_{a_0\eta\pi^0} = 3.1^{+2.0}_{-1.1} \text{ GeV}, \qquad (16)$$

$$g_{a_0\eta'\pi^0} = 4.4^{+2.9}_{-1.6} \text{ GeV},$$

for $m_q = (5.6 \pm 1.6)$ GeV and

$$g = 3.8^{+1.1}_{-0.8} \text{ GeV},$$

$$g_{a_0\eta\pi^0} = 3.1^{+0.9}_{-0.7} \text{ GeV}, \qquad (17)$$

$$g_{a_0\eta'\pi^0} = 4.4^{+1.3}_{-0.9} \text{ GeV},$$

for $m_q = (5.6 \pm 0.6)$ GeV. The parameters of the twist-2 light-cone distribution amplitude $\varphi_\pi(u)$ obtained in Ref. [47] can change the value of the coupling constant slightly, less than 0.1%.

In Table 1, we list some values (not all) of the coupling constant $g_{a_0\eta\pi^0}$ from different quark models and the experimental data. From the table, we see that the values of the early estimations with the $q\bar{q}$ model, tetraquark model and $K\bar{K}$ molecule model deviate greatly from the experimental data [52–54], i.e. we cannot use them to identify the structures of $a_0(980)$ with confidence. Compared with the values extracted from the radiative decay $\phi(1020) \to a_0(980)\gamma \to \eta\pi^0\gamma$ [50–54], the central value of our numerical result is reasonable and supports the $q\bar{q}$ model.

Table 1. The coupling constant $g_{a_0\eta\pi^0}$ from different quark models and experimental data.

quark models and experimental data	$g_{a_0\eta\pi^0}$/GeV
$q\bar{q}$ model [48]	2.03
tetraquark model [48]	4.57
$K\bar{K}$ molecule model [8, 49]	1.74
SND Collaboration [50, 51]	3.11
KLOE Collaboration [52, 53]	3.0 ± 0.2
KLOE Collaboration [54]	2.8 ± 0.1
light-cone sum rules ($q\bar{q}$ model) [25]	$2.6 - 3.4$
this work ($q\bar{q}$ model)	$3.1^{+0.9}_{-0.7}$

From the coupling constant $g_{a_0\eta\pi^0}$, we can obtain the decay width $\Gamma_{a_0 \to \eta\pi^0}$:

$$\Gamma_{a_0 \to \eta\pi^0} = \frac{p g^2_{a_0\eta\pi^0}}{8\pi M^2_{a_0}} = 127^{+222}_{-76} \text{ GeV for}$$

$$g = 3.8^{+2.5}_{-1.4} \text{ GeV}; = 127^{+84}_{-48} \text{ MeV for}$$

$$g = 3.8^{+1.1}_{-0.8} \text{ GeV}, \qquad (18)$$

$$p = \frac{\sqrt{\left[M^2_{a_0} - (M_\eta + m_\pi)^2\right]\left[M^2_{a_0} - (M_\eta - m_\pi)^2\right]}}{2M_{a_0}}.$$

Compared with the experimental data $\Gamma_{a_0(980)} = (50-100)$ GeV from the Particle Data Group with the very model dependent estimation (the decay width can be much larger) [41], the central value and lower bound of our numerical result $\Gamma_{a_0 \to \eta\pi^0} = 127^{+84}_{-48}$ GeV are reasonable; however, the upper-bound is too large. We should reduce the uncertainties of the input parameters $f_{3\pi}$ and m_q (the main uncertainties originate from them) before drawing a definite conclusion.

In this article we take the point of view that the $a_0(980)$ is a scalar $q\bar{q}$ state. In Ref. [55] the light nonet scalar mesons are taken as tetraquark states, and the coupling constants among the light scalar mesons and pseudoscalar mesons are calculated with the QCD sum rules. The numerical results indicate that the values of the coupling constants for the tetraquark states are always smaller than the corresponding ones for the $q\bar{q}$ states [22, 23].

The predictions listed in Table 1 are obtained from the phenomenological (potential) quark models [8, 48, 49], and the resulting coupling constant g differs greatly from the corresponding ones from the QCD sum rules [22, 23, 55]. Furthermore, those predictions also differ significantly from the ones extracted from the experimental data [50–54]. In this article, we prefer the values from the QCD sum rules for consistency, i.e. if the nonet scalar mesons are tetraquark states, they have much smaller coupling constant g [22, 23, 55].

The scalar meson $a_0(980)$ may have a small $q\bar{q}$ kernel of the typical $q\bar{q}$ meson size. Strong coupling to the nearby $\bar{K}K$ threshold may result in some tetraquark components, i.e. either a nucleon-like bound state or a deuteron-like bound state. The tetraquark components may lead to a smaller decay width and smear the discrepancy between the (upper bound of) theoretical calculation and the experimental data.

4 Conclusion

In this article, we take the point of view that the scalar meson $a_0(980)$ is a conventional $q\bar{q}$ state and calculate the coupling constants $g_{a_0\eta\pi^0}$ and $g_{a_0\eta'\pi^0}$ with the light-cone QCD sum rules. Although the coupling constant $g_{a_0\eta'\pi^0}$ has no direct phenomenological interest, we take into account the η' meson to facilitate subtraction of the continuum states to obtain a more reliable sum rule. The central value of the coupling constant $g_{a_0\eta\pi^0}$ is consistent with the values extracted from the radiative decay $\phi(1020) \to a_0(980)\gamma \to \eta\pi^0\gamma$. The central value and lower bound

of the decay width $\Gamma_{a_0\to\eta\pi^0} = 127^{+84}_{-48}$ GeV are compatible with the experimental data of the total decay width $\Gamma_{a_0(980)} = (50-100)$ GeV from the Particle Data Group with a very model dependent estimation (the decay width can be much larger), while the upper bound is too large. The scalar meson $a_0(980)$ may have a small $q\bar{q}$ kernel of typical $q\bar{q}$ meson size.

Strong coupling to the nearby $\bar{K}K$ threshold may result in some tetraquark components, which can be a nucleon-like bound state or a deuteron-like bound state. The tetraquark components may lead to a smaller decay width and smear the discrepancy between the theoretical calculation and the experimental data.

Appendix A

We present some technical details in obtaining the spectral density at the phenomenological side,

$$
\begin{aligned}
\langle\eta|J(0)|\pi^0\rangle &= \langle\eta(p')|\sum_a\int\frac{d^3\vec{q}}{(2\pi)^3 2E}|a(q)\rangle\langle a(q)|J(0)|\pi^0(p)\rangle = \\
&\sum_a\int\frac{d^4q}{(2\pi)^4}\langle\eta(p')|\frac{i}{q^2-M_a^2+i\epsilon}|a(q)\rangle\langle a(q)|J(0)|\pi^0(p)\rangle = \\
&\sum_a\int\frac{d^4q}{(2\pi)^4}\langle\eta(p')|a(q)\pi^0(p)\rangle\frac{i}{q^2-M_a^2+i\epsilon}\langle a(q)|J(0)|0\rangle = \\
&\sum_a f_a M_a\int\frac{d^4q}{(2\pi)^4}\langle\eta(p')|i\int d^4y\,\mathcal{L}(y)|a(q)\pi^0(p)\rangle\frac{i}{q^2-M_a^2+i\epsilon} = \\
&\sum_a\int\frac{d^4q}{(2\pi)^4}\langle\eta(p')|\int d^4y\,g_a\eta(y)a(y)\pi^0(y)|a(q)\pi^0(p)\rangle\frac{f_a M_a}{M_a^2-q^2-i\epsilon} = \\
&\sum_a\int\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^4(p'-p-q)g_a\frac{f_a M_a}{M_a^2-q^2-i\epsilon} = \sum_a\frac{g_a f_a M_a}{M_a^2-(p'-p)^2-i\epsilon},
\end{aligned}
$$

where we have used the completeness relation,

$$
\sum_a\int\frac{d^3\vec{q}}{(2\pi)^3 2E}|a(q)\rangle\langle a(q)| = 1,
$$

which corresponds to the normalization condition $\langle a(q)|a(q')\rangle = (2\pi)^3 2E\delta^3(\vec{q}-\vec{q'})$. The a's are the intermediate hadronic states with the same quantum numbers as the current operator $J(0)$, the g_a denote the corresponding coupling constants among the η, a and π^0, and $\langle 0|J(0)|a(q)\rangle = f_a M_a$. In the light-cone QCD sum rules, we often use the economical form,

$$
\langle\eta|J(0)|\pi^0\rangle = \sum_a\langle\eta(p')|a(q)\pi^0(p)\rangle\frac{i}{q^2-M_a^2+i\epsilon}\langle a(q)|J(0)|0\rangle = \sum_a\frac{g_a f_a M_a}{M_a^2-q^2-i\epsilon},
$$

with a suitable definition $\langle\eta(p')|a(q)\pi^0(p)\rangle = ig_a$.

The light-cone distribution amplitudes of the π meson are defined as,

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 d(x)|\pi(p)\rangle = \mathrm{i}f_\pi p_\mu \int_0^1 \mathrm{d}u\, e^{-\mathrm{i}up\cdot x}\left\{\varphi_\pi(u)+\frac{m_\pi^2 x^2}{16}A(u)\right\}+\mathrm{i}f_\pi m_\pi^2 \frac{x_\mu}{2p\cdot x}\int_0^1 \mathrm{d}u\, e^{-\mathrm{i}up\cdot x}B(u),$$

$$\langle 0|\bar{u}(0)\mathrm{i}\gamma_5 d(x)|\pi(p)\rangle = \frac{f_\pi m_\pi^2}{m_u+m_d}\int_0^1 \mathrm{d}u\, e^{-\mathrm{i}up\cdot x}\varphi_p(u),$$

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu}\gamma_5 d(x)|\pi(p)\rangle = \mathrm{i}(p_\mu x_\nu - p_\nu x_\mu)\frac{f_\pi m_\pi^2}{6(m_u+m_d)}\int_0^1 \mathrm{d}u\, e^{-\mathrm{i}up\cdot x}\varphi_\sigma(u),$$

$$\langle 0|\bar{u}(0)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)d(x)|\pi(p)\rangle = f_{3\pi}\left\{(p_\mu p_\alpha g_{\nu\beta}^\perp - p_\nu p_\alpha g_{\mu\beta}^\perp)-(p_\mu p_\beta g_{\nu\alpha}^\perp - p_\nu p_\beta g_{\mu\alpha}^\perp)\right\}$$
$$\int \mathcal{D}\alpha_i \varphi_{3\pi}(\alpha_i)e^{-\mathrm{i}p\cdot x(\alpha_d+v\alpha_g)},$$

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)d(x)|\pi(p)\rangle = p_\mu\frac{p_\alpha x_\beta - p_\beta x_\alpha}{p\cdot x}f_\pi m_\pi^2 \int \mathcal{D}\alpha_i A_\parallel(\alpha_i)e^{-\mathrm{i}p\cdot x(\alpha_d+v\alpha_g)}+$$
$$f_\pi m_\pi^2(p_\beta g_{\alpha\mu}-p_\alpha g_{\beta\mu})\int \mathcal{D}\alpha_i A_\perp(\alpha_i)e^{-\mathrm{i}p\cdot x(\alpha_d+v\alpha_g)},$$

$$\langle 0|\bar{u}(0)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)d(x)|\pi(p)\rangle = p_\mu\frac{p_\alpha x_\beta - p_\beta x_\alpha}{p\cdot x}f_\pi m_\pi^2 \int \mathcal{D}\alpha_i V_\parallel(\alpha_i)e^{-\mathrm{i}p\cdot x(\alpha_d+v\alpha_g)}+$$
$$f_\pi m_\pi^2(p_\beta g_{\alpha\mu}-p_\alpha g_{\beta\mu})\int \mathcal{D}\alpha_i V_\perp(\alpha_i)e^{-\mathrm{i}p\cdot x(\alpha_d+v\alpha_g)}, \tag{A1}$$

where $\tilde{G}_{\alpha\beta}=\frac{1}{2}\epsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$ and $\mathcal{D}\alpha_i = \mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$.

The light-cone distribution amplitudes are parameterized as

$$\varphi_\pi(u) = 6u(1-u)\left\{1+a_1 C_1^{\frac{3}{2}}(2u-1)+a_2 C_2^{\frac{3}{2}}(2u-1)+a_4 C_4^{\frac{3}{2}}(2u-1)\right\},$$

$$\varphi_p(u) = 1+\left\{30\eta_3-\frac{5}{2}\rho^2\right\}C_2^{\frac{1}{2}}(2u-1)+\left\{-3\eta_3\omega_3-\frac{27}{20}\rho^2-\frac{81}{10}\rho^2 a_2\right\}C_4^{\frac{1}{2}}(2u-1),$$

$$\varphi_\sigma(u) = 6u(1-u)\left\{1+\left[5\eta_3-\frac{1}{2}\eta_3\omega_3-\frac{7}{20}\rho^2-\frac{3}{5}\rho^2 a_2\right]C_2^{\frac{3}{2}}(2u-1)\right\},$$

$$\varphi_{3\pi}(\alpha_i) = 360\alpha_u\alpha_d\alpha_g^2\left\{1+\lambda_3(\alpha_u-\alpha_d)+\omega_3\frac{1}{2}(7\alpha_g-3)\right\},$$

$$V_\parallel(\alpha_i) = 120\alpha_u\alpha_d\alpha_g(v_{00}+v_{10}(3\alpha_g-1)),$$

$$A_\parallel(\alpha_i) = 120\alpha_u\alpha_d\alpha_g a_{10}(\alpha_d-\alpha_u),$$

$$V_\perp(\alpha_i) = -30\alpha_g^2\left\{h_{00}(1-\alpha_g)+h_{01}\left[\alpha_g(1-\alpha_g)-6\alpha_u\alpha_d\right]+h_{10}\left[\alpha_g(1-\alpha_g)-\frac{3}{2}(\alpha_u^2+\alpha_d^2)\right]\right\},$$

$$A_\perp(\alpha_i) = 30\alpha_g^2(\alpha_u-\alpha_d)\left\{h_{00}+h_{01}\alpha_g+\frac{1}{2}h_{10}(5\alpha_g-3)\right\},$$

$$A(u) = 6u(1-u)\left\{\frac{16}{15}+\frac{24}{35}a_2+20\eta_3+\frac{20}{9}\eta_4+\left[-\frac{1}{15}+\frac{1}{16}-\frac{7}{27}\eta_3\omega_3-\frac{10}{27}\eta_4\right]C_2^{\frac{3}{2}}(2u-1)+\right.$$
$$\left.\left[-\frac{11}{210}a_2-\frac{4}{135}\eta_3\omega_3\right]C_4^{\frac{3}{2}}(2u-1)\right\}+\left\{-\frac{18}{5}a_2+21\eta_4\omega_4\right\}$$
$$\left\{2u^3(10-15u+6u^2)\log u+2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\log\bar{u}+u\bar{u}(2+13u\bar{u})\right\},$$

$$g_\pi(u) = 1+g_2 C_2^{\frac{1}{2}}(2u-1)+g_4 C_4^{\frac{1}{2}}(2u-1),$$

$$B(u) = g_\pi(u)-\varphi_\pi(u), \tag{A2}$$

where

$$
\begin{aligned}
h_{00} &= v_{00} = -\frac{\eta_4}{3}\,, \\
a_{10} &= \frac{21}{8}\eta_4\omega_4 - \frac{9}{20}a_2\,, \\
v_{10} &= \frac{21}{8}\eta_4\omega_4\,, \\
h_{01} &= \frac{7}{4}\eta_4\omega_4 - \frac{3}{20}a_2\,, \\
h_{10} &= \frac{7}{2}\eta_4\omega_4 + \frac{3}{20}a_2\,, \\
g_2 &= 1 + \frac{18}{7}a_2 + 60\eta_3 + \frac{20}{3}\eta_4\,, \\
g_4 &= -\frac{9}{28}a_2 - 6\eta_3\omega_3\,,
\end{aligned}
\tag{A3}
$$

$C_2^{\frac{1}{2}}(\xi)$, $C_4^{\frac{1}{2}}(\xi)$, $C_2^{\frac{3}{2}}(\xi)$ and $C_4^{\frac{3}{2}}(\xi)$ are Gegenbauer polynomials, $\eta_3 = \dfrac{f_{3\pi}}{f_\pi}\dfrac{m_{\mathrm{u}}+m_{\mathrm{d}}}{m_\pi^2}$ and $\rho^2 = \dfrac{(m_{\mathrm{u}}+m_{\mathrm{d}})^2}{m_\pi^2}$.

References

1 Godfrey S, Napolitano J. Rev. Mod. Phys., 1999, **71**: 1411–1462
2 Close F E, Tornqvist N A. J. Phys. G, 2002, **28**: R249–R267
3 Jaffe R L. Phys. Rept., 2005, **409**: 1–45
4 Amsler C, Tornqvist N A. Phys. Rept., 2004, **389**: 61–117
5 Jaffe R L. Phys. Rev. D, 1977, **15**: 267–280, 281–289
6 Jaffe R L. Phys. Rev. D, 1978, **17**: 1444–1446
7 Weinstein J D, Isgur N. Phys. Rev. D, 1990, **41**: 2236–2257
8 Weinstein J D, Isgur N, Phys. Rev. D, 1983, **27**: 588–599
9 Tornqvist N A. Z. Phys. C, 1995, **68**: 647–660
10 Boglione M, Pennington M R. Phys. Rev. Lett., 1997, **79**: 1998–2001
11 Boglione M, Pennington M R. Phys. Rev. D, 2002, **65**: 114010
12 Bramon A, Colangelo G, Franziniet P J et al. Phys. Lett. B, 1992, **287**: 263–266
13 Franzini P J, Kim W, Lee-Franzini J. Phys. Lett. B, 1992, **287**: 259–262
14 Colangelo G, Franzini P J. Phys. Lett. B, 1992, **289**: 189–193
15 Achasov N N, Gubin V V, Solodov E P. Phys. Rev. D, 1997, **55**: 2672–2682
16 Bramon A, Grau A, Pancheri G. Phys. Lett. B, 1992, **289**: 97–102
17 Bramon A, Escribano R, Lucio M J L et al. Eur. Phys. J. C, 2002, **26**: 253–260
18 Marco E, Hirenzaki S, Oset E et al. Phys. Lett. B, 1999, **470**: 20–26
19 WANG Z G, WAN S L. Phys. Rev. D, 2006, **73**: 094020
20 WANG Z G. J. Phys. G, 2007, **34**: 753–765
21 WANG Z G, WAN S L. Phys. Rev. D, 2006, **74**: 014017
22 Colangelo P, Fazio F D. Phys. Lett. B, 2003, **559**: 49–59
23 WANG Z G, YANG W M, WAN S L. Eur. Phys. J. C, 2004, **37**: 223–231
24 WANG Z G. Phys. Rev. D, 2008, **77**: 054024
25 Gokalp A, Sarac Y, Yilmaz O. Mod. Phys. Lett. A, 2004, **19**: 3011–3017
26 Gokalp A, Sarac Y, Yilmaz O. Phys. Lett. B, 2005, **609**: 291–297
27 Balitsky I I, Braun V M, Kolesnichenko A V. Nucl. Phys. B, 1989, **312**: 509–550
28 Chernyak V L, Zhitnitsky I R. Nucl. Phys. B, 1990, **345**: 137–172
29 Chernyak V L, Zhitnitsky A R. Phys. Rept., 1984, **112**: 173–318
30 Braun V M, Filyanov I E. Z. Phys. C, 1989, **44**: 157–166
31 Braun V M, Filyanov I E. Z. Phys. C, 1990, **48**: 239–248
32 Shifman M A, Vainshtein A I, Zakharov V I. Nucl. Phys. B, 1979, **147**: 385–447
33 Shifman M A, Vainshtein A I, Zakharov V I. Nucl. Phys. B, 1979, **147**: 448–518
34 Reinders L J, Rubinstein H, Yazaki S. Phys. Rept., 1985, **127**: 1–97
35 Belyaev V M, Braun V M, Khodjamirian A et al. Phys. Rev. D, 1995, **51**: 6177–6195
36 Ball P. JHEP, 1999, **9901**: 010
37 Ball P, Zwicky R. Phys. Lett. B, 2006, **633**: 289–297
38 Ball P, Zwicky R. JHEP, 2006, **0602**: 034
39 Ball P, Braun V M, Lenz A. JHEP, 2006, **0605**: 004
40 Burakovsky L, Goldman J T. Phys. Lett. B, 1998, **427**: 361–370
41 YAO W M et al. J. Phys. G, 2006, **33**: 1–1232
42 Schmedding A, Yakovlev O I. Phys. Rev. D, 2000, **62**: 116002
43 Bakulev A P, Mikhailov S V, Stefanis N G. Phys. Lett. B, 2001, **508**: 279–289
44 Bakulev A P, Mikhailov S V, Stefanis N G. Phys. Rev. D, 2003, **67**: 074012
45 Bakulev A P, Mikhailov S V, Stefanis N G. Phys. Lett. B, 2004, **578**: 91–98
46 Bakulev A P, Pimikov A V. Acta. Phys. Polon. B, 2006, **37**: 3627–3634
47 Bakulev A P. hep-ph/0611139
48 Achasov N N, Ivanchenko V N. Nucl. Phys. B, 1989, **315**: 465–476
49 Achasov N N, Gubin V V. Phys. Rev. D, 1997, **56**: 4084–4097
50 Achasov M N et al. Phys. Lett. B, 2000, **485**: 349–356
51 Achasov M N et al. Phys. Lett. B, 2000, **479**: 53–58
52 Aloisio A et al. Phys. Lett. B, 2002, **536**: 209–216
53 Aloisio A et al. Phys. Lett. B, 2002, **537**: 21–26
54 Ambrosino F et al. arXiv:0707.4609
55 Brito T V, Navarra F S, Nielsen M et al. Phys. Lett. B, 2005, **608**: 69–76