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Influence of medium effects on the rotating hybrid stars*

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Abstract A hybrid star with a pure quark core, a hadron crust and a mixed phase between the two is considered. The relativistic mean field model for hadron matter and the effective mass bag model for quark matter are used to construct the equation of state for hybrid stars. The influences of medium effects that are parameterized by the strong coupling constant have been discussed on the configuration of rotating stars. The strong coupling constant is a prominent factor that influences the properties of rotating hybrid stars.

Key words hybrid stars, rotation, medium effects

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1 Introduction

Pulsars are generally accepted to be rotating compact stars. The investigation of rotating compact stars can provide the best cosmic laboratory to understand the behavior of nuclear matter under extreme conditions [1, 2]. Many configurational quantities of rotating compact stars are directly connected with observations, such as mass-radii relations, redshift and momentum of inertia. During the spin down evolution of compact stars, deconfinement phase transition can happen, which may be connected with the energy engine of a γ-ray burst [3] and thermal evolution of compact stars [4].

The chemical compositions and properties of matters in the compact stars depend upon the appropriate equation of state (EOS), which describes the crust and interior of compact stars. The existence of quark matter in the interior of compact stars has been discussed in the work of many authors [5, 6]. Compact stars that are made of hadronic matter in the outer region, but possess a pure quark core and mixed matter of hadron phase (HP) and quark phase (QP) between the two, are called hybrid stars. The EOS for the description of quark matter in the framework of the commonly used MIT bag model was improved by including medium effects [7]. This model was achieved by using a quasi-particle approach. The interaction of the quarks with the other quarks of the system is implemented by giving them density dependent effective quark masses. This should lead to a more realistic description of quark matter going beyond the free fermi gas approximation. The model is called the "effective mass bag model" [7]. In our work, we choose the effective mass bag model for quark matter and relativistic mean field theory (RMF) for hadronic matter [1].

The problem of rotation in the general theory of relativity was and remains one of central and complicated problems [1]. Hartle et al. [8, 9] introduced a method of perturbation theory to solve the problem of stationary gravitational fields and their sources. Some extensive studies have been reported in later researches [9–12]. These suggested that for compact stars in the stationary rotating regime without matter flux, perturbation theory gives a sufficiently good approximation [1, 13]. Using the method of perturbation theory, Chubarian et al. [13] discussed the development of deconfinement phase transition in the core of a compact star during its spin-down evolution.

With the EOS we choose and the perturbation theory, we are going to give out the star's gravitational mass, equatorial radius and the shape deformations, in both static and rotating sequences, with

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a different strong coupling constant g, which parameterized the influence of medium effects. The possible deconfinement phase transition during the spin-down evolution will also be discussed.

This paper will be organized as follows. In Section 2, we review the construction of hybrid stars and the rotating configuration which we choose. We discuss in detail the results of the influence of medium effects on the rotating hybrid stars in Section 3. A summary is presented in Section 4.

2 Equation of state

Here we discuss the EOS for a hybrid star. The details of the effective mass bag model for QP can be found in Ref. [7] and the details of the RMF theory for HP can be found in Ref. [1].

In the effective mass bag model, the quarks are considered as quasi-particles which acquire an effective mass generated by the interaction with the other quarks of the dense system. The effective quark masses can be written as [7]

$$m_{\rm q}^*(\mu)=\frac{g\mu}{\sqrt{6}\pi}\,, \tag{1}$$

$$m_{\rm s}^*(\mu)=\frac{m_{\rm s}}{2}+\sqrt{\frac{m_{\rm s}^2}{4}+\frac{g^2\mu^2}{6\pi^2}}\,, \tag{2}$$

where g is the strong coupling constant, μ is the quark chemical potential, and $m_{\rm s}$ is the s quark current mass.

From the effective Hamiltonian the particle density ρ, energy density ϵ and pressure p at temperature $T=0$ of a fermi gas of free quasi-particles are given by [7]

$$\rho=\frac{d}{6\pi^2}(\mu^2-m^{*2}(\mu))^{3/2}\,, \tag{3}$$

$$\epsilon=\frac{d}{16\pi^2}\left[\mu k(2\mu^2-m^{*2})-m^{*4}\ln\left(\frac{k+\mu}{m^*}\right)\right]+B^*(\mu), \tag{4}$$

$$p=\frac{d}{48\pi^2}\left[\mu k(2\mu^2-5m^{*2})+3m^{*4}\ln\left(\frac{k+\mu}{m^*}\right)\right] -B^*(\mu), \tag{5}$$

where d is the degree of degeneracy and B^* is a function to maintain thermodynamic self-consistency,

$$\frac{{\rm d}B^*(\mu_i)}{{\rm d}m_i^*}=-\frac{d}{4\pi^2}\left[m_i^*\mu_i k_i-m_i^{*3}\ln\left(\frac{k_i+\mu_i}{m_i^*}\right)\right]. \tag{6}$$

The overall pressure $p_{\rm QP}$ and energy density $\epsilon_{\rm QP}$ of the quark phase can be written as

$$\epsilon_{\rm QP}=\epsilon_{\rm u}+\epsilon_{\rm d}+\epsilon_{\rm s}+\epsilon_{\rm e}+B, \tag{7}$$

$$p_{\rm QP}=p_{\rm u}+p_{\rm d}+p_{\rm s}+p_{\rm e}-B, \tag{8}$$

where B is the bag constant, which is supposed to mimic the influence of confinement and corresponds to the energy difference between the perturbation vacuum inside the deconfined quark matter phase and the "true" vacuum outside [7]. The most important parameters of the description of quark matter in the effective mass bag model are the bag constant B and the coupling constant g. Since in this work we mainly discuss the influence of medium effects on the rotating hybrid stars, we choose a bag constant of $B^{1/4}=170$ MeV. For the current quark masses, we assume $m_{\rm u}=m_{\rm d}=0$ and $m_{\rm s}=150$ MeV for s-quarks.

To describe the HP, we use an RMF model which was discussed by Glendenning [1]. This is one of the effective field theories describing hadron matter, where nucleons interact through the nuclear force mediated by the exchange of isoscalar and isovector mesons (σ, ω, ρ). In our work we choose a compression modulus of $K=300$ MeV as an input for RMF models of HP.

We consider the scenario of a mixed phase (MP) of hadronic and quark matter where the QP is described by the effective mass bag model and the HP is described by the RMF model. The MP is obtained by applying the global charge neutrality condition and the Gibbs conditions for HP and QP. The charge neutrality is imposed globally [1],

$$\chi q_{\rm QP}+(1-\chi)q_{\rm HP}=0, \tag{9}$$

where $q_{\rm QP}$ and $q_{\rm HP}$ are the charge densities of QP and HP, respectively. χ is the volume fraction of the QP, and $(1-\chi)$ is the volume fraction occupied by the HP. As usual, the phase boundary of the coexistence region between the HP and the QP is determined by Gibbs conditions. The Gibbs condition for mechanical and chemical equilibrium at zero temperature between the HP and the QP reads

$$\mu_{\rm HP,i}=\mu_{\rm QP,i}, \quad {\rm i}={\rm n,e} \tag{10}$$

$$p_{\rm HP}(\mu_{\rm HP})=p_{\rm QP}(\mu_{\rm QP}), \tag{11}$$

where $p_{\rm HP}$ is the pressure of the HP and $p_{\rm QP}$ is the pressure of the QP. The energy density and the total baryon number density in the MP read

$$\varepsilon=\chi\varepsilon_{\rm QP}+(1-\chi)\varepsilon_{\rm HP}, \quad \rho=\chi\rho_{\rm QP}+(1-\chi)\rho_{\rm HP}. \tag{12}$$

Once these quantities are obtained, we can construct the EOS for the MP. To construct the whole compact stars, we use the Baym-Pethick-Sutherland (BPS)

[14] EOS for subnuclear densities corresponding to the crust of the stars when densities at $\epsilon \approx 10^{14}$ g/cm^3.

3 Results and discussion

We start to calculate the properties of the hybrid stars using the EOS discussed above. The stars are assumed to be rapidly rotating, relativistic and compact. The details of the rotation model are given in Ref. [8]. The metric can be written as

$$\begin{aligned} ds^2 =& -e^{\nu(r)}[1+2h(r,\theta)]dt^2 + e^{\lambda(r)}\Bigg[1 \\ &+\frac{2m(r,\theta)}{(r-2M(r))}\Bigg]dr^2 + r^2[1+2k(r,\theta)]\big\{d\theta^2 \\ &+\sin^2\theta[d\phi - w(r,\theta)dt]^2\big\} + O(\Omega^3), \qquad (13) \end{aligned}$$

where $e^{\nu(r)}$, $e^{\lambda(r)}$ and $M(r)$ are functions of r and describe the solution of the Tolman-Oppenheimer-Volkov (TOV) equations for non-rotating stars. The functions $h(r,\theta)$, $m(r,\theta)$, $k(r,\theta)$ and $w(r,\theta)$ are the perturbation corrections. The matter is assumed to be a perfect fluid so that the energy-momentum tensor is given by

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}, \qquad (14)$$

where ϵ, p, u and $g^{\mu\nu}$ are the energy density, pressure, four velocity and metric tensor, respectively. The details of the calculations for solving the field equations are given in Refs. [8, 12].

In Fig. 1, we have shown the mass-radius relation for the hybrid stars with different values of strong coupling constant g from 0 to 4 while keeping other EOS parameters fixed. The sequences shown in Fig. 1 terminate at the maximum mass point. The maximum mass is higher for larger g, the radius shift to the lower and the central density lowers while g increases. The data are given in Table 1. For comparison, van Kerkwijk and co-workers have obtained a mass of 1.86±0.32 for the X-ray pulsar Vela X-1 [15]. This shows that either static or rotating hybrid stars in our model favor lower strong coupling constant g. The dots in Fig. 1 show the corresponding mass and radius at which the MP exists. The crosses show the corresponding mass and radius at which the quark core exists. When g is higher, the maximum mass is reached before a quark core can possess. From these results, we can observe that the stability of hybrid stars does not favor the onset of pure quark core with the effective mass bag model we have used. Only a small range of mass and radii correspond to a hybrid star with a quark core. The influence of medium effect reflects the stability of the hybrid stars only through the MP.

Keeping the total baryon number conserved, we show the critical region of the phase transition in the inner structure of the star configuration and the equatorial and polar radii as a function of angular velocity in Fig. 3. It is obvious that with the increase in the angular velocity, the star is deforming its shape. The area at which the quark matter possessed is decreased while the star spins down.

The central baryon number density as a function of angular velocity for rotating hybrid star sequences with different strong coupling constant for the given total baryon numbers is shown in Fig. 2. The dashed

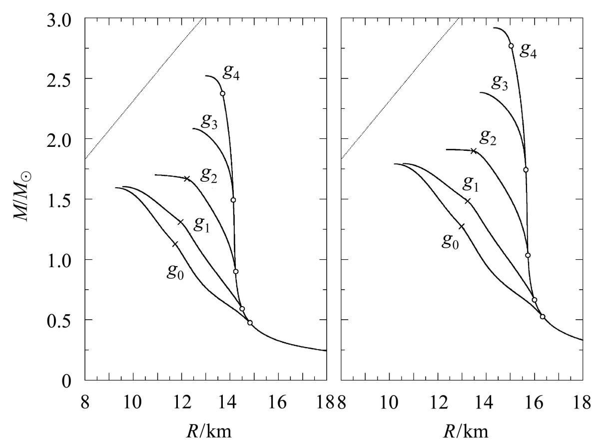

Fig. 1. Gravitational mass versus the equatorial radius. The left panel corresponds to the static model and the right panel corresponds to those with maximum angular velocity. The dotted lines refer to the causality limit.

Table 1. Maximum-mass for static models and those rotating with maximum angular velocity. The superscript r is denoted as the rotating configurations.

g	$M_{\max}/M_\odot$	R/km	$e_c/(\times 10^{15}\ \mathrm{g\,cm}^3)$	$M^r_{\max}/M_\odot$	R^r/km	$e^r_c/(\times 10^{15}\ \mathrm{g\,cm}^3)$
0	1.59	9.28	3.36	1.79	10.21	1.45
1	1.60	9.58	3.24	1.79	10.57	1.39
2	1.70	10.91	2.45	1.91	12.39	1.13
3	2.08	12.47	1.71	2.38	13.73	1.66
4	2.52	13.00	1.47	2.92	14.32	1.38

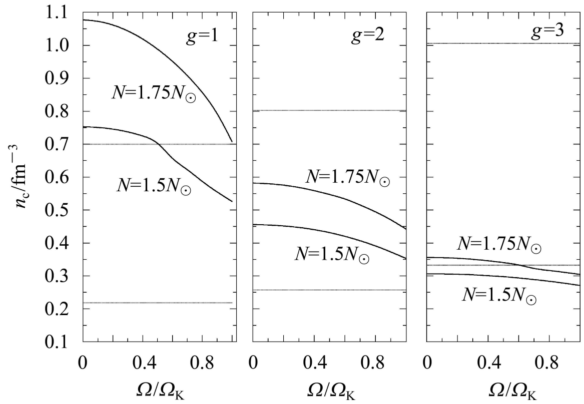

Fig. 2. The central baryon number density as a function of rotation frequency for rotating hybrid stars with a different strong coupling constant g. The total baryon number is $N/N_{\odot}=1.5, 1.75$.

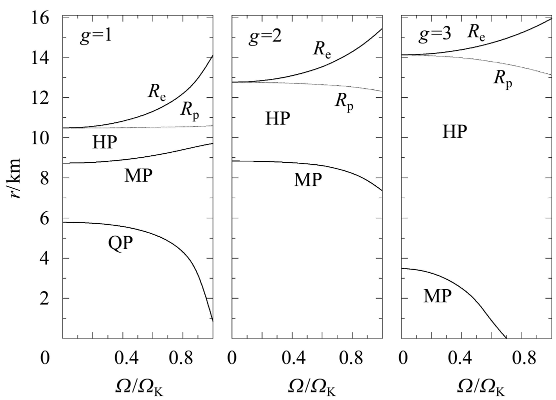

Fig. 3. The phase structure of rotating hybrid stars in the equatorial direction as a function of the rotation frequency with a different strong coupling constant. The total baryon number is $N/N_{\odot}=1.75$.

horizontal lines indicate the density where the QP or the MP exists. The effect of a strong coupling constant on the phase transition is obvious in Fig. 3. We can find that the onset of the MP and the QP is varied with different g. With lower g, the stars restrain a pure quark core in the hybrid stars. At the center of some stars, the matter can be gradually converted from relatively incompressible HP to compressible MP when the stars spin down. At the same time, the region of MP decreased and a deconfinement phase transition occurred in the hybrid stars.

4 Summary

We have studied compact stars with a mixed matter of HP and QP with the effective mass bag model for QP and RMF model for HP under static and rotating configuration. The results show that the strong coupling constant g has a prominent influence on the configuration of rotating stars and various properties of a rotating sequences, such as maximum mass, radius and shape deformation. We can conclude that the hybrid stars favor low coupling constant and possess quark matter as a mixed phase. A pure quark core is constrained. When the stars spin down, a deconfinement phase transition can happen in such a model. The coupling constant g influences the gress of the phase transition. The critical regions of the phase transition are affected by a different strong coupling constant. Moreover, these findings have to be more carefully constrained by astrophysical observations and by solving the full set of Einstein's equation of rotation numerically and nonpertubatively.

References

1 Glendenning N K. Compact Stars, Nuclear Physics, Particle Physics and General Relativity. Second Edition: New York: Springer-Verlag, 2000

2 Shapiro S L, Teukolsky S A. Black Holes, White Dwarfs and Neutron Stars. New York: Wiley, 1983

3 Drago A, Lavagno A, Pagliara G. Nucl. Phys. A, 2006, **774**: 823

4 Iida K, Sato K. Astrophys. J, 1997, **477**: 294

5 Glendenning N K. Phys. Rev. D, 1992, **46**: 1274

6 Prakash M, Cooke J R, Lattimer J M. Phys. Rev. D, 1995, **52**: 661

7 Schertler K, Greiner C, Thoma M H. Nucl. Phys. A, 1997, **616**: 659

8 Hartle J B. Astrophys J, 1967, **150**: 1005

9 Hartle J B, Thorne K S. Astrophys J, 1968, **153**: 807

10 Datta B. Nature, 1984, **308**: 338

11 Weber F, Glendenning N K. Astrophys J, 1992, **390**: 541

12 Benhar O, Ferrari V, Gualtieri L et al. Phys. Rev. D, 2005, **72**: 044028

13 Chubarian E, Grigorian H, Poghosyan G et al. Astron. Astrophys., 2000, **357**: 968

14 Baym G, Pethick C J, Sutherland P. Astrophys. J, 1971, **170**: 299

15 Barziv O, Kaper L, van Kervijk M H et al. Astron. Astrophys, 2001, **377**: 925