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Effect of the mesons σ^* and Φ and the variety of $U_{\Sigma}^{(\mathrm{N})}$ on the transition density of hyperon stars*

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Abstract The effect of the mesons σ^* and Φ and the variety of $U_{\Sigma}^{(\mathrm{N})}$ on the transition density of hyperon stars is examined within the framework of relativistic mean field theory for the baryon octet {n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^- and Ξ^0} system. It is found that, compared with that without considering the mesons σ^* and Φ, the transition density of hyperon stars decreases, the critical baryon density that hyperons Σ^-, Σ^0, Σ^+, Ξ^- and Ξ^0 appears to decrease too, but for Λ the effect is not obvious. As $U_{\Sigma}^{(\mathrm{N})}$ goes up, the critical baryon density of Σ^+, Σ^0 and Σ^- increases, that of Ξ^0 decreases and that of Λ and Ξ^- is fixed. In addition, it is found that the variety of $U_{\Sigma}^{(\mathrm{N})}$ almost does not influence the transition density.

Key words neutron stars, hyperon stars, relativistic mean field theory

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1 Introduction

In 1968, Hewish et al first discovered radio pulsars [1], which were soon identified as neutron stars [2]. Since then they have become a hot topic.

Relativistic mean field theory (RMF) provides a good description of the bulk properties of nuclear matter as well as of a large number of single-particle properties of finite nuclei [3–5]. In 1985, RMF theory was extended and used to describe the properties of neutron star matter by Glendenning [6], who thought that there are interactions between nucleons or nucleons and hyperons. In order to describe the interaction between hyperons, Schaffner further extended the RMF model and suggested that interactions also exist between hyperons by exchanging the scalar mesons $f_0\,(975)$ (denoted as σ^*) and the vector mesons $\phi\,(1020)$ (denoted as ϕ) [7].

As the density of neutron star matter is near to the saturation density of normal nuclear matter, the neutron star matter is only composed of neutrons, protons, electrons and muons. With the baryon density increasing, the relative populations of neutrons in the neutron stars will decrease and those of hyperons will increase. At a certain baryon density, the neutron star will change into hyperon stars [8]. In 2001, Jia Huan-Yu et al investigated how the coupling constants influences the transition density of hyperon stars [9]. But it is well known that there might be a lot of other factors, i.e., the variety of $U_{\Sigma}^{(\mathrm{N})}$ (the potential well depth of Σ in nuclear matter), that affect the transition density too.

In this paper, the effect of the mesons $\sigma^*$and ϕ and the variety of $U_{\Sigma}^{(\mathrm{N})}$ on the transition density of hyperon stars is examined in the framework of relativistic mean field theory for the baryon octet {n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^- and Ξ^0} system.

2 Relativistic mean field theory (RMF)

The Lagrangian density of hadron matter reads as follows [10, 11],

$$\mathcal{L} = \sum_{\mathrm{B}}\overline{\Psi}_{\mathrm{B}}(\mathrm{i}\gamma_\mu\partial^\mu - m_{\mathrm{B}} + g_{\sigma\mathrm{B}}\sigma - g_{\omega\mathrm{B}}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho\mathrm{B}}\gamma_\mu\tau\cdot\rho^\mu)\Psi_{\mathrm{B}} + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right)$$

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$$-U_\sigma+\sum_{\lambda=\mathrm{e},\mu}\overline{\Psi_\lambda}\left(\mathrm{i}\gamma_\mu\partial^\mu-m_\lambda\right)\Psi_\lambda-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}$$

$$+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu-\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu}+\frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu+\mathcal{L}^{\mathrm{YY}}\ . \quad (1)$$

The last term representing the contribution of σ^* and ϕ mesons is

$$\mathcal{L}^{\mathrm{YY}}=\sum_{\mathrm{B}}g_{\sigma^*\mathrm{B}}\overline{\Psi}_{\mathrm{B}}\Psi_{\mathrm{B}}\sigma^*-\sum_{\mathrm{B}}g_{\phi\mathrm{B}}\overline{\Psi}_{\mathrm{B}}\gamma_\mu\Psi_{\mathrm{B}}\phi^\mu$$

$$+\frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^*-m_{\sigma^*\cdot}^2\sigma^{*2}\right)-\frac{1}{4}S_{\mu\nu}S^{\mu\nu}$$

$$+\frac{1}{2}m_\phi^2\phi_\mu\phi^\mu. \quad (2)$$

Here, $S_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu$.

We solve the field equations in the usual RMF approximation. The equations of the baryon field are obtained as follows,

$$\left(\gamma_\mu k^\mu-m_{\mathrm{B}}+g_{\sigma\mathrm{B}}\sigma+g_{\sigma^*\mathrm{B}}\sigma^*-g_{\omega\mathrm{B}}\gamma_0\omega_0\right.$$

$$\left.-g_{\phi\mathrm{B}}\gamma_0\phi_0-g_{\rho\mathrm{B}}\gamma_0\tau_3\rho_{03}\right)\psi_{\mathrm{B}}(k,\lambda)=0, \quad (3)$$

and their eigenvalues are

$$e_{\mathrm{B}}(k)=g_{\omega\mathrm{B}}\omega_0+g_{\phi\mathrm{B}}\phi_0+g_{\rho\mathrm{B}}\rho_{03}I_{3\mathrm{B}}+\sqrt{k^2+m_{\mathrm{B}}^{*2}}\ , \quad (4)$$

here, m_{B}^* is the effective mass of baryons,

$$m_{\mathrm{B}}^*=m_{\mathrm{B}}-g_{\sigma\mathrm{B}}\sigma-g_{\sigma^*\mathrm{B}}\sigma^*\ . \quad (5)$$

The mesons field equations are given by

$$m_\sigma^2\sigma=-g_2\sigma^2-g_3\sigma^3+\sum_{\mathrm{B}}g_{\sigma\mathrm{B}}\rho_{\mathrm{sB}}, \quad (6)$$

$$m_{\sigma^*}^2.\sigma^*=\sum_{\mathrm{B}}g_{\sigma^*\mathrm{B}}\rho_{\mathrm{sB}}, \quad (7)$$

$$\omega_0=\sum_{\mathrm{B}}\frac{g_{\omega\mathrm{B}}}{m_\omega^2}\rho_{\mathrm{B}}, \quad (8)$$

$$m_\phi^2\phi_0=2\sum_{\mathrm{B}}g_{\phi\mathrm{B}}\rho_{\mathrm{B}}, \quad (9)$$

$$m_\rho^2\rho_{03}=\sum_{\mathrm{B}}g_{\rho\mathrm{B}}I_{3\mathrm{B}}\rho_{\mathrm{B}}. \quad (10)$$

The chemical potentials for the baryons and leptons, respectively, are

$$\mu_{\mathrm{B}}=e_{\mathrm{B}}(k),\ \mu_{\mathrm{e}}=\sqrt{k_{\mathrm{e}}^2+m_{\mathrm{e}}^2},\mu_\mu=\mu_{\mathrm{e}}$$

$$=\sqrt{k_\mu^2+m_\mu^2}, \quad (11)$$

$$\mu_{\mathrm{B}}=\mu_{\mathrm{n}}-q_{\mathrm{B}}\mu_{\mathrm{e}}. \quad (12)$$

The scalar density for baryon B and the condition of charge neutrality are given by

$$\rho=\sum_{\mathrm{B}}\rho_{\mathrm{B}}=\sum_{\mathrm{B}}\left(2J_{\mathrm{B}}+1\right)b_{\mathrm{B}}k_{\mathrm{B}}^3/\left(6\pi^2\right)=\mathrm{const}, \quad (13)$$

$$Q=\sum_{\mathrm{B}}Q_{\mathrm{B}}+\sum_{\lambda}Q_\lambda=\sum_{\mathrm{B}}\left(2J_{\mathrm{B}}+1\right)q_{\mathrm{B}}k_{\mathrm{B}}^3/\left(6\pi^2\right)$$

$$+\sum_{\lambda}2q_\lambda k_\lambda^3/\left(6\pi^2\right)=0. \quad (14)$$

The above equations will be used to solve the distributions of baryons in the neutron star matter.

3 Parameters

The coupling constants of the nucleons in our calculations are chosen as GL85 [6], GL97 [8], DD-MEI [12], NL1 and NL2 [13] constants, by which especially by the GL85 constants we will make a minute study on the effect of the σ^* and ϕ mesons on the transition density of a hyperon star.

We define the ratios: $x_{\sigma\mathrm{h}}=g_{\sigma\mathrm{h}}/g_\sigma$, $x_{\omega\mathrm{h}}=g_{\omega\mathrm{h}}/g_\omega$, $x_{\rho\mathrm{h}}=g_{\rho\mathrm{h}}/g_\rho$. For the coupling constants of meson ω, we use ratio

$$g_{\omega\mathrm{N}}/3=g_{\omega\Sigma}/2=g_{\omega\Lambda}/2=g_{\omega\Xi}, \quad (15)$$

which is given by the constituent quark model [$SU(6)$ symmetry] [8]. The coupling constants of meson σ are then determined by fitting the Λ, Σ and Ξ well depth in nuclear matter [8]. Here, we choose $U_\Lambda^{(\mathrm{N})}=-30$ MeV [14]. For the Ξ nuclear interaction, the experimental values indicate a nonrelativistic potential $U_\Xi^{(\mathrm{N})}$ of about -16 MeV [15] and -14 MeV [16] or less, respectively. Dover and Gal found the Ξ-nucleus potential well depth to be $\approx$21–24 MeV based on their analysis of emulsion data [17]. In our calculation, a more relativistic potential is chosen as $U_\Xi^{(\mathrm{N})}=-28$ MeV [18]. As for $U_\Sigma^{(\mathrm{N})}$, the attractive case is $U_\Sigma^{(\mathrm{N})}=-30$ MeV and the repulsive case is $U_\Sigma^{(\mathrm{N})}=+30$ MeV, as suggested in Refs. [19–23]. So we respectively choose $U_\Sigma^{(\mathrm{N})}=-30$, -20, -10, 0, 10, 20 and 30 MeV to examine how the variety of $U_\Sigma^{(\mathrm{N})}$ affects the transition density of hyperon stars.

In order to obtain the coupling constants of the strange mesons ϕ, we use the quark model relationship,

$$g_{\phi\Xi}=2g_{\phi\Lambda}=-2\sqrt{2}g_{\omega\mathrm{N}}/3. \quad (16)$$

For the σ^* mesons, we use the mass of the obtained $\mathrm{f}_0(975)$ meson, but treat its couplings purely phenomenologically so as to satisfy the equation of potential depths $U_\Lambda^{(\Xi)}\approx U_\Xi^{(\Xi)}\approx 2U_\Lambda^{(\Lambda)}\approx 40$ MeV. This yielded $g_{\sigma^*\Lambda}/g_{\sigma N}=g_{\sigma^*\Sigma}/g_{\sigma\mathrm{N}}=0.69$, $g_{\sigma^*\Xi}/g_{\sigma N}=1.25$ [7].

The transition density $\rho_{0\mathrm{H}}$ is defined as the lowest baryon density $\rho=\sum\limits_{\mathrm{B}}\rho_{\mathrm{B}}$ at which $\sum\limits_{\mathrm{H}}\rho_{\mathrm{H}}>\sum\limits_{\mathrm{N}}\rho_{\mathrm{N}}$, where ρ_{H} is the hyperon density and ρ_{N} is the nucleon density.

4 Theoretical results and analysis

The effect of σ* and ϕ mesons on the relative populations of hyperons is shown in Fig. 1. In this case, we choose the GL85 constants and $U_{\Lambda}^{(N)}=U_{\Sigma}^{(N)}=-30$ MeV, $U_{\Xi}^{(N)}=-28$ MeV. From Fig. 1 we can see that, compared with those without considering the contribution of σ* and ϕ mesons, the relative populations of neutrons decrease but those of hyperons increase. The reason is that, as the mesons σ* and ϕ are considered, more neutrons decay to hyperons. In addition, the existence of the mesons σ* and ϕ makes the critical density that Ξ^-, Σ^-, Ξ^0, Σ^0, Σ^+ appear decrease, while for Λ hyperons the effect is not obvious.

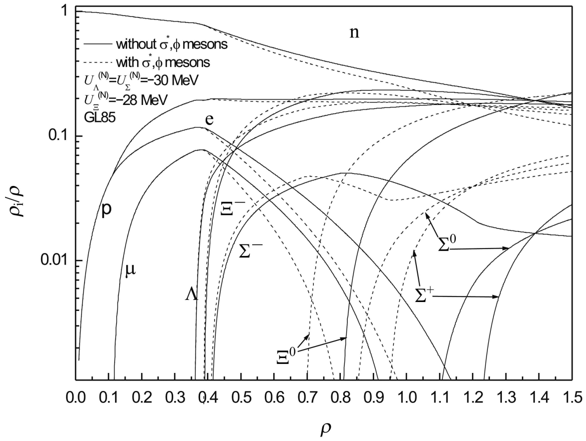

Fig. 1. The composition of neutron star matter using parameter GL85.

The effect of the σ* and ϕ mesons on the transition density of hyperon stars is plotted in Fig. 2. In this case, $U_{\Lambda}^{(N)}=U_{\Sigma}^{(N)}=-30$ MeV, $U_{\Xi}^{(N)}=-28$ MeV and the GL85, GL97, DD-MEI, NL1 and NL2 constants are chosen. From Fig. 2 we can see that, considering the contribution of σ* and ϕ mesons, for all the five sets of constants the transition density of hyperon stars decreases. This is because the existence of mesons σ* and ϕ makes more neutrons decay to hyperons.

For the case of GL85 constants, the contribution of hyperons of different kinds to the transition density of hyperon stars is given in Fig. 3, Fig. 4 and Table 1. From Fig. 3 we can see that at the transition density $\rho_{0\text{H}}$, the relative populations of neutrons are more than those of protons. It can also be seen that the contribution of hyperons Λ and Ξ^- to the transition density is more than that of hyperons Σ^- and Ξ^0 and for the hyperons Σ^- and Σ^+ the contribution is zero. Considering the mesons σ* and ϕ, at transition density, the numbers of neutrons, protons, Σ^- and Ξ^- decrease, while those of Λ and Ξ^0 increase. This means that at transition density $\rho_{0\text{H}}$, the existence of the mesons σ* and ϕ makes more nucleons decay to

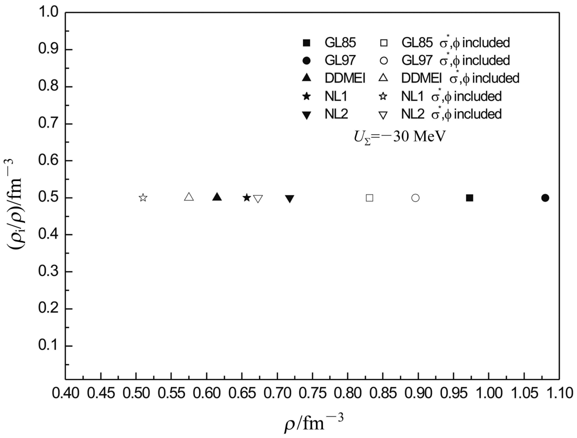

Fig. 2. Effect of the mesons σ* and ϕ on the transition density of hyperon stars.

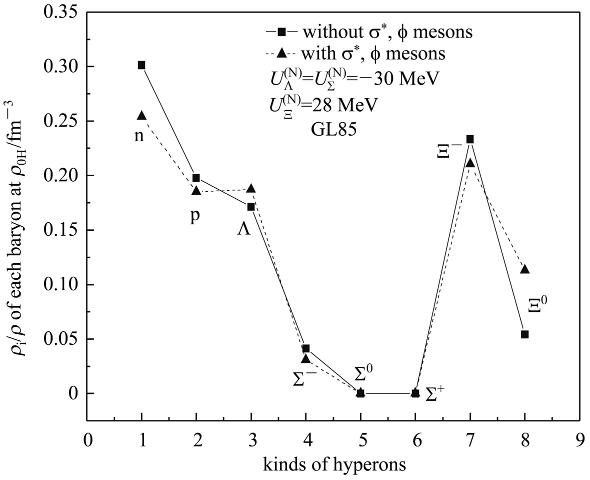

Fig. 3. The relative populations of baryons at the transition density of the hyperon star as a function of the kinds of hyperons.

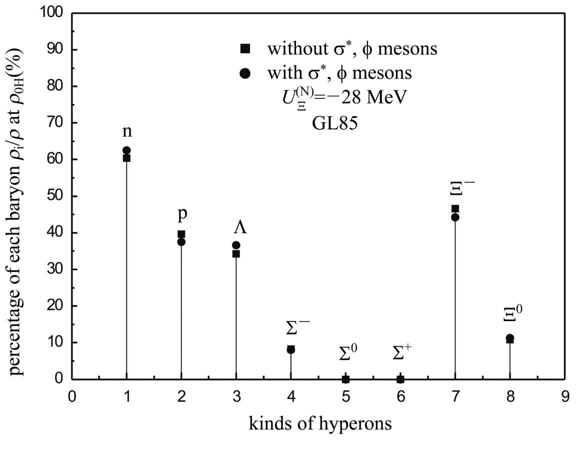

Fig. 4. The percentage of relative populations of baryons at $\rho_{0\text{H}}$.

hyperons Λ and Ξ^0 but mitigates against the appearance of Σ^- and Ξ^-.

The more detailed results are shown in Fig. 4 and Table 1. At the transition density $\rho_{0\mathrm{H}}$ of the hyperon stars, compared with those without considering the mesons σ* and ϕ, the percentage of neutron density increases from 60.38% to 62.45%, while that of protons decreases from 39.62% to 37.55%. As for hyperons, the percentage of Λ and Ξ^0 increases from 34.27% and 10.82% to 36.56% and 11.26%, respectively, but that of Σ^- and Ξ^- decreases from 8.27% and 46.64% to 7.99% and 44.19%, separately. It is thus clear that, at transition density $\rho_{0\mathrm{H}}$, the star is mainly composed of neutrons, protons, Λ, Ξ^- and a small amount of Σ^- and Ξ^0, but the hyperons of Σ^0 and Σ^+ don't appear. The existence of the mesons σ* and ϕ makes the number of neutrons Λ and Ξ^0 increase but that of Σ^- and Ξ^- decrease.

The effect of the variety of $U_{\Sigma}^{(\mathrm{N})}$ on the transition density of hyperon stars is shown in Fig. 5, Fig. 6 and Table 2. Here, the GL85 constant is adopted.

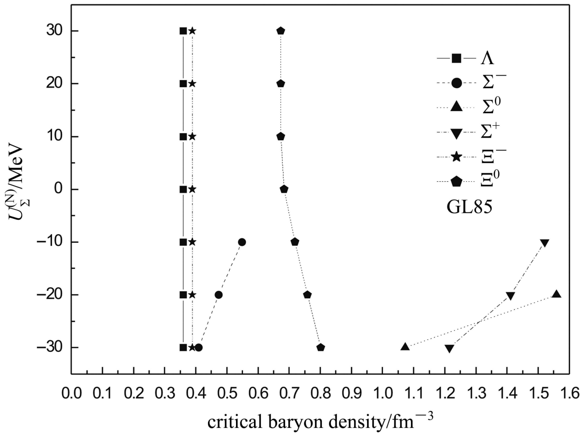

Fig. 5. Effect of the variety of $U_{\Sigma}^{(\mathrm{N})}$on the critical baryon density.

In Fig. 5, the effect of the variety of $U_{\Sigma}^{(\mathrm{N})}$ on the critical baryon density that hyperons appear is given. In this case, the σ* and ϕ are not considered and the $U_{\Sigma}^{(\mathrm{N})}$ are chosen as −30, −20, −30, 0, 10, 20 and 30 MeV. From Fig. 5 we can see that, with the raise in $U_{\Sigma}^{(\mathrm{N})}$, the critical baryon density of Σ^+, Σ^0 and Σ^- increases, that of Ξ^0 decreases and that of Λ and Ξ^- is fixed. That is to say, the higher potential depth $U_{\Sigma}^{(\mathrm{N})}$ will be conducive to the appearance of Ξ^0 but mitigate against the appearance of Σ^-, Σ^0 and Σ^+.

From Fig. 6 and Table 2 we can see that the transition density $\rho_{0\mathrm{H}}$ is fixed as $U_{\Sigma}^{(\mathrm{N})}$goes up from −30 MeV to 30 MeV, i.e., the variety of $U_{\Sigma}^{(\mathrm{N})}$ almost does not influence the transition density $\rho_{0\mathrm{H}}$. Compared with those without considering the contribution of mesons σ* and ϕ, the transition density $\rho_{0\mathrm{H}}$ will greatly decrease. The reason is that the existence of the mesons σ* and ϕ makes more nucleons decay to hyperons.

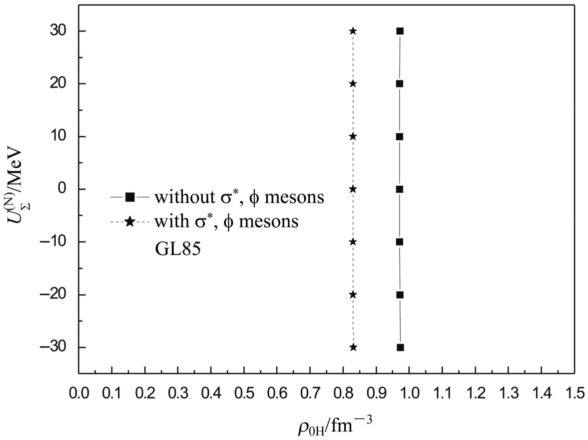

Fig. 6. Effect of the variety of $U_{\Sigma}^{(\mathrm{N})}$ and the mesons σ* and ϕ on the transition density $\rho_{0\mathrm{H}}$.

Table 1. Relative populations of the baryons at the transition density of hyperon stars. Here, $U_{\Sigma}^{(\mathrm{N})} = -30$ MeV.

kinds of hyperons	n	p	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0
number	1	2	3	4	5	6	7	8
with σ*, ϕ	0.3117	0.1874	0.1831	0.0400	0	0	0.2213	0.0564
percentage(%)	62.45	37.55	36.56	7.99	0	0	44.19	11.26
without σ*, ϕ	0.3014	0.1978	0.1716	0.0414	0	0	0.2336	0.0542
percentage(%)	60.38	39.62	34.27	8.27	0	0	46.64	10.82

Table 2. The transition density $\rho_{0\mathrm{H}}$ of hyperon stars.

$U_{\Sigma}^{(\mathrm{N})}$/MeV	−30	−20	−10	0	10	20	30
$\rho_{0\mathrm{H}}$ (with σ*, ϕ)/fm^{-3}	0.831	0.830	0.830	0.830	0.830	0.830	0.830
$\rho_{0\mathrm{H}}$ (without σ*, ϕ)/fm^{-3}	0.973	0.972	0.971	0.971	0.971	0.971	0.971

5 Summary

In conclusion, in this paper the effect of the mesons σ^* and Φ and the variety of $U_{\Sigma}^{(N)}$ on the transition density of hyperon stars is examined within the framework of relativistic mean field theory for the baryon octet {n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^- and Ξ^0} system. It is found that, compared with that without considering the mesons σ^* and Φ, the transition density of hyperon stars decreases, the critical baryon density that hyperons Σ^-, Σ^0, Σ^+, Ξ^- and Ξ^0 appear decreases too, but for Λ the effect is not obvious. As $U_{\Sigma}^{(N)}$ goes up, the critical baryon density of Σ^+, Σ^0 and Σ^- increases, that of Ξ^0 decreases and that of Λ and Ξ^- is fixed. In addition, it is also found that the variety of $U_{\Sigma}^{(N)}$ almost dose not influence the transition density.

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