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Causality principle and nuclear dispersion anomaly in the elastic scattering for $\alpha+^{12}$C system

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Abstract The optical model analysis of the alpha particle elastic scattering on a carbon target was performed on the basis of the dispersion relation between the real and imaginary parts of the calculated volume integrals. A nuclear dispersion anomaly in an $\alpha+^{12}$C system was observed and interpreted clearly.

Key words nuclear dispersion anomaly, optical model potential, nuclear rainbow, airy minima

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1 Introduction

Due to the absence of the related scattering data and also the existence of ambiguities in the empirical determination of the optical potential from available data, the analysis of experimental elastic scattering data usually does not give reliable information on the energy dependence of the complex optical potential. The aim of this paper is to describe the existence of "anomalies" in the low-energy behaviors of $W(r,E)$ and $V(r,E)$. It has been observed that $|W(r,E)|$ decreases sharply when E falls below the top of the Coulomb barrier, and this decrease is accompanied by a bell-shaped maximum of $|V(r,E)|$. This behavior is due to the dispersion relation between V and W [1–5]. If

$$V(r,E)=V_0(r,E)+\Delta V(r,E), \tag{1}$$

the dispersion relation is

$$\Delta V(r,E)=\frac{P}{\pi}\int_0^{\infty}\frac{W(r,E')}{E'-E}\mathrm{d}E', \tag{2}$$

where P is the "Principal value". This dispersion relation is a consequence of the following causality principle: a scattered wave cannot be emitted before the interaction has occurred.

The real and imaginary parts of the potential are determined by the effective Nucleon-Nucleon (NN) of interaction and some open channels, respectively [4]. Instead of V and W, usually their volume integrals, J_V and J_W, are used. At the top of the Coulomb barrier, V or J_V decreases slowly, while W or J_W increases slowly [2].

2 Theoretical background

The single-particle wave equation is given by [6]

$$\left[\frac{-\hbar^2}{2m}\nabla^2+M(E)\right]\Phi_{\mathrm{E}}(\vec{r})=E\Phi_{\mathrm{E}}(\vec{r}), \tag{3}$$

where $M(E)=V(E)+\mathrm{i}W(E)$ is the non-hermitian "generalized optical potential" operator that can be constructed in such a way that the asymptotic behavior of $\Phi_{\mathrm{E}}(\vec{r})$ for large $|\vec{r}|$ yields the exact complex elastic scattering phase shift of a nucleus-nucleus or nucleon-nucleus collision. This depends upon energy. By introducing the Fourier transforms

$$\begin{aligned}U(t)&=(2\pi)^{-1}\int M(E)\exp(\mathrm{i}Et/\hbar)\mathrm{d}E,\\ \psi(\vec{r},t)&=\Phi_{\mathrm{E}}(\vec{r})\exp(\mathrm{i}Et/\hbar),\end{aligned} \tag{4}$$

the wave equation (3) becomes

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{r},t)+\int_{-\infty}^{\infty}U(t-t')\psi(\vec{r},t')\mathrm{d}t'=\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(\vec{r},t). \tag{5}$$

The causality principle corresponds to the requirement that

$$U(t-t')=0 \qquad \text{for} \quad t<t'.$$

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By integrating $\dfrac{M(E')}{E'-E}$ along with a contour C, which runs along with a large circle and the branch cuts, the following dispersion relation obtains,

$$V(E) = V_0+\frac{P}{\pi}\int_{-\infty}^{E_t^-}\frac{W(E')}{E'-E}\mathrm{d}E'+\frac{P}{\pi}\int_{E_t^+}^{+\infty}\frac{W(E')}{E'-E}\mathrm{d}E' +\sum_{\mathrm{p}}\frac{A_{\mathrm{p}}}{E-E_{\mathrm{p}}}+\sum_{\mathrm{h}}\frac{A_{\mathrm{h}}}{E-E_{\mathrm{h}}}, \tag{6}$$

where P is the principal value, E_t^- and E_t^+ are the branch-point of energies at which the left-hand and the right-hand cuts start, and E_{p} and E_{h} are poles that are located on the real axis between E_t^- and E_t^+, the residues A_{p} and A_{h} are real. The requirement to be $M(E)$ as an absorptive potential leads to

$$W(E)\leqslant 0.$$

The hermitian potential V_0 is given by the limit

$$V_0=\lim_{|E|\to\infty}V(E).$$

This is independent energy.

This is not enough information about the microscopic theory of nucleus-nucleus scattering and the existence and properties of the corresponding generalized optical model potential. Existing models consider the projectile and target as objects that are either structureless or contain at most a few collective degrees of freedom. Antisymmetrization is not considered unless one can approximate to the energy-independent term V_0. Hence, the dispersion relation (6) becomes

$$V(\vec{r},\vec{r}\,',E) = V_0(\vec{r},\vec{r}\,')+\sum_{\mathrm{p}}\frac{A_{\mathrm{p}}(\vec{r},\vec{r}\,')}{E-E_{\mathrm{p}}} +\frac{P}{\pi}\int_{E_t}^{\infty}\frac{W(\vec{r},\vec{r}\,',E')}{E'-E}\mathrm{d}E', \tag{7}$$

where the pole terms arise from eigenstates in the excluded (nonelastic) channel space. Moreover, V_0 represents the average interaction of the two nuclei in the absence of nonelastic excitation, which can be interpreted as a generalized "Folded" potential that includes all of the exchange terms arising from antisymmetrization between the two nuclei. These exchange terms alone make V_0 nonlocal.

Since empirical model potentials are usually assumed to be local and M is nonlocal, some local equivalent to M has to be defined. Also it is necessary for some averaging over energy to approach an empirical optical potential. The average of the scattering amplitude over an energy interval size I is given by asymptotic behavior of the solution of the equation (3), in which $M(E)$ is replaced by $\bar{M}(E)=M(E-\mathrm{i}I)$ [7]. $\bar{M}(E)$ has the form

$$\bar{M}(\vec{r},\vec{r}\,',E)=V_0(\vec{r},\vec{r}\,')+\Delta V(\vec{r},\vec{r}\,',E)+\mathrm{i}W(\vec{r},\vec{r}\,',E). \tag{8}$$

In the nucleon-nucleus case, it obtains

$$\Delta V(\vec{r},\vec{r}\,',E) \approx \frac{P}{\pi}\int_{-\infty}^{E_{\mathrm{F}}^-}\frac{W(\vec{r},\vec{r}\,',E')}{E'-E}\mathrm{d}E' +\frac{P}{\pi}\int_{E_{\mathrm{F}}^+}^{\infty}\frac{W(\vec{r},\vec{r}\,',E')}{E'-E}\mathrm{d}E', \tag{9}$$

where E_{F}^+ and E_{F}^- are close to the nucleon separation energy of the ground state of the system with $(A+1)$ and $(A-1)$ nucleons, respectively. The dispersion integral, which runs from $-\infty$ to E_{F}^-, does not have the form that has been accepted for the nucleus-nucleus case. Finally, the dispersion relation is obtained as

$$\Delta\bar{V}(\vec{r},\vec{r}\,',E)\approx\frac{P}{\pi}\int_{E_{\mathrm{F}}}^{\infty}\frac{W(\vec{r},\vec{r}\,',E')}{E'-E}\mathrm{d}E'. \tag{10}$$

Here, we consider a local and spherically-symmetric empirical optical model potential in the form of $U(r,E)=V(r,E)+\mathrm{i}W(r,E)$. We assume that one can write V in the form of $V=V_0+\Delta V$, where V_0 is the independent energy, while ΔV has intrinsically dependent energy that can be evaluated as

$$\Delta V(r,E)=\frac{P}{\pi}\int\frac{W(r,E')}{E'-E}\mathrm{d}E'. \tag{11}$$

When strong absorption exists, as in the case of heavy ions, this equation is suitable. For the following volume integrals of potential per interacting pair of nucleons,

$$J_W(E)=\left[4\pi\int W(r,E)r^2\mathrm{d}r\right]\Big/(A_{\mathrm{p}}A_{\mathrm{t}}), \tag{12}$$

where A_{p} and A_{t} are the projectile and target mass numbers, respectively. The dispersion relation can be written in the form

$$J_{\Delta V}(E)=\frac{P}{\pi}\int\frac{J_W(E')}{E'-E}\mathrm{d}E'. \tag{13}$$

Adopting the extrapolation of $J_W(E)$, which is different for large positive and negative E, the value of $J_{\Delta V}(E_{\mathrm{F}})$ would be sensitive to these assumed high-energy extrapolations. However, the difference $J_{\Delta V}(E)-J_{\Delta V}(E_{\mathrm{F}})$ would remain rather stable. So it is only considered as

$$J_{\Delta V,E_{\mathrm{S}}}(E)=J_{\Delta V}(E)-J_{\Delta V}(E_{\mathrm{S}}), \tag{14}$$

where E_{S} is a reference energy that lies in the interest of energy domain. This difference is related to $J_W(E)$

by the following "subtracted dispersion relation",

$$J_{\Delta V,E_{\rm S}}(E)=(E-E_{\rm S})\frac{P}{\pi}\int\frac{J_W(E')}{(E'-E_{\rm S})(E'-E)}{\rm d}E', \tag{15}$$

which determines $J_{\Delta V}(E)$ up to a constant.

The dispersion correction, generally, can be written as

$$J_{\Delta V}(E)=J_{\Delta V}^{\rm CO}(E)+J_{\Delta V}^{\rm PO}(E), \tag{16}$$

where the "correlation" contribution is associated with a dispersion integral that runs from -∞ to $E_{\rm F}$ while the "polarization" contribution is given by an integral from $E_{\rm F}$ to $+\infty$.

3 Analysis of elastic α+^{12}C scattering

An optical model analysis of elastic α+^{12}C scattering was performed at the different laboratory energies that are available from the NNDC (EXFOR/CSISRS) database. The Woods-Saxon parameters were adjusted to obtain the best χ^2 fit to the scattering data by using the SPI-GENOVA program. The ratio of the experimental and calculated differential cross sections for elastic α+^{12}C scattering to the respective Rutherford cross sections are shown in Fig. 1.

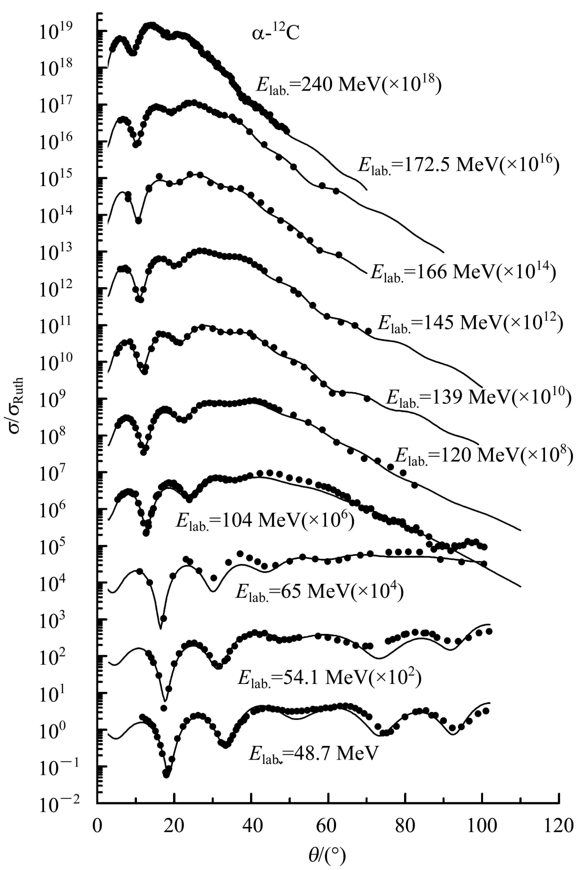

Fig. 1. Ratios of the differential cross sections for elastic α+^{12}C scattering at the different laboratory energies to the respective Rutherford cross sections.

Results show that the fits are clearly satisfactory. In this figure, a nuclear Rainbow structure is well indicated. Owing to this, we were able to construct the systematic energy of the positions of Airy minima (Fig. 2). This systematic energy confirmed the inverse-energy law and made it possible to select potentials [3]. In Fig. 2, the circles and triangles are indicated by experimental Airy minima, while the stars are calculated Airy minima.

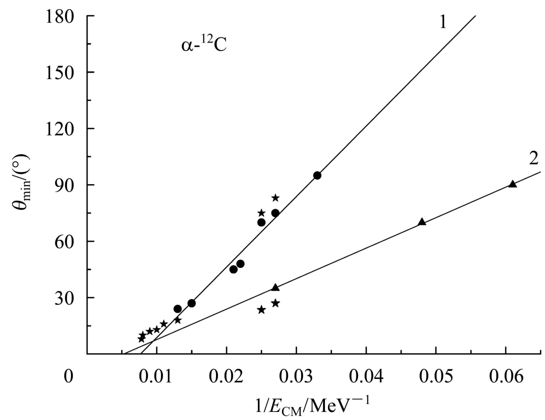

Fig. 2. Positions of Airy minima of experimental angular distributions for elastic α+^{12}C scattering versus the inverse CM energy.

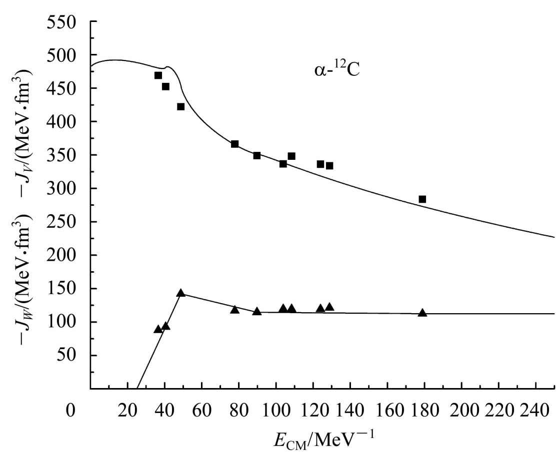

Fig. 3. Dispersion analysis of volume integrals of phenomenological potentials for the α+^{12}C system.

Assuming the volume integrals determined are unique, we used the calculated volume integrals for dispersion analysis of the α+^{12}C system. In order to approximate the dependence of $J_W(E)$, we employed "the schematic linear model" proposed in [2].

In the lower part of Fig. 3, we show (triangles) the empirical values of $J_W(E_{\rm CM})$ and (curve) the approximation of the energy-dependence of this quantity in the schematic linear model. In the upper part are shown the corresponding values for the volume integrals $J_V(E_{\rm CM})$, which were calculated by using the dispersion relation.

One can see the imaginary part of potential, which decreases sharply when the energy falls below the top of the Coulomb barrier, and this decrease is accompanied by a bell-shaped maximum of real part of potential. This is the same as the diagram of the dispersion analysis for other systems. This anomaly in the behavior of the low energy of real and imaginary parts of potential is characteristic for this analysis and relates to “spurious” energy-dependence due to nonlocality of the imaginary part of the potential. The values found for the real and imaginary parts of the volume integrals and also values of χ^2/N (N is the number of experimental points in a given angular distribution) and cross sections are given in Table 1. It should be noted that, as a rule, the analysis was performed by using a constant experimental error of 10%. Moreover, the angular range in the analysis at energies of 48.7, 54.1, 65 and 104 MeV was constrained from above by a value of 100° in order to reduce the effect of other mechanisms, such as elastic transfer at large angles. This is the reason why the χ^2/N value calculated for the total angular range 48.7, 54.1, 65 and 104 MeV is so large.

Table 1. The parameters found for the α+^{12}C system.

$E_{\rm lab.}$/MeV	$-J_V$/(MeV·fm^3)	$-J_W$/(MeV·fm^3)	χ^2/N	σ/mb
48.7	469.02	87.94	13.89	875.7
54.1	452.37	92.77	14.03	868.1
65	422.22	142.24	7.643	766.6
104	366.37	117.14	6.515	862.7
120	349.1	114.71	1.95	813.5
139	336.53	119.5	2.84	761.7
145	348.23	119.53	0.811	781.1
166	336.26	119.4	2.313	743.0
172.5	333.73	121.43	0.438	751.9
240	283.75	112.54	1.209	682.4

4 Conclusion

The nuclear Rainbow is a very good phenomenon for finding the potential of the light heavy ions elastic scattering. The basic of the problem for this phenomenon in finding the correct potential is uncertain. But this problem can be solved by some tests. One of these is to use the dispersion relation between the real and imaginary parts of potential. The dispersion relation reproduced the energy dependence of the volume integrals of the real and imaginary parts of the potential at the region of an “anomaly”. In this work, this anomaly is simulated for α+^{12}C scattering, and it was shown that the imaginary part of the potential decreases toward zero when E falls below the top of the Coulomb barrier rapidly, while the real part of the potential increases sharply.

The behavior of the imaginary part can be interpreted as inelastic channels that are decreased by the Coulomb barrier effectively. By using the dispersion relation, for this reason, $|\,V\,|$ must have a bell-shaped maximum in the same region.

It should be emphasized that the dispersion relation has a connection only with intrinsic energy dependence, while the empirical local potentials result by fitting the experimental data, which may include “spurious” energy dependence due to nonlocality potentials.

This study proves that the dispersion relation can be applied as a test for selecting the correct potential among the available potentials too.

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