Transfer matrix for the open chain from giant gravitons*

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Abstract We construct the transfer matrix for the open chain with the centrally extended $SU(2|2)$ symmetry attached to the so called $Z=0$ giant graviton brane. Using the reflection equations, unitarity property and crossing property, we show that this model is integrable.

Key words $SU(2|2)$ symmetry, transfer matrix, integrability

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1 Introduction

Exactly solvable models have been studied for a long time leading to many important applications in the fields of theoretical physics and condensed matter physics. One of the most fascinating discoveries in recent years was the unravelling of integrable structures in planar $N=4$ SYM theory [1, 2] and in $AdS_5\times S^5$ super-string theory [3–5], which has lead to drastic simplifications in determining some quantities. For example, in planar $N=4$ SYM, the planar anomalous dimensions of local operators can be mapped to energies of quantum spin chain states, thus establishing some relation to topics of condensed matter physics. The Hamiltonian of this system is completely integrable at the one loop level and apparently even at higher loop levels. This remarkable feature shows promise that the planar spectrum might be described exactly by some sort of Bethe equations.

One way to obtain Bethe equations is to construct transfer matrices with different boundaries in the framework of the quantum inverse scattering method (QISM) [6, 7]. Hofman and Maldacena (HM) [8] recently considered open strings attached to maximal giant gravitons [9] in $AdS_5\times S^5$. They proposed boundary S-matrices describing the reflection of world-sheet excitations (giant magnons) for two cases, namely, the $Y=0$ and $Z=0$ giant graviton branes. For the $Y=0$ case, Murgan and Nepomechie constructed one transfer matrix [10]. Later the corresponding Bethe equations were obtained by using the algebraic [11] and the analytical ansatz method [12], respectively. However, for the $Z=0$ case, the transfer matrix has not been clearly constructed and the Bethe equations have not been achieved yet as far as we know. So, in this paper, we construct the transfer matrix for the open chain attached to $Z=0$ giant gravitons brane.

The outline of the paper is organized as follows. In section 2 we will introduce the bulk S matrix and boundary S matrix, including the right and left reflection equations. In section 3 we present the transfer matrix for the $Z=0$ giant gravition brane and show the integrability for the spin chain model defined by the transfer matrix. Some discussions are given in section 4.

2 Bulk *S*-matrix and boundary *S*-matrix

The bulk S-matrix based on $SU(2|2)$ symmetry can be found in Refs. [13, 14]. It satisfies the standard Yang-Baxter equation (YBE)

$$S_{12}(p_1,p_2)\,S_{13}(p_1,p_3)\,S_{23}(p_2,p_3)=S_{23}(p_2,p_3)\,S_{13}(p_1,p_3)\,S_{12}(p_1,p_2)\,, \quad (1)$$

where $S_{12}=S\otimes I$, $S_{23}=I\otimes S$ and $S_{13}=\mathcal{P}_{12}S_{23}\mathcal{P}_{12}$, $\mathcal{P}_{12}$ is the permutation matrix, I is a 4×4 identity

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matrix. The bulk S matrix has the unitarity property

$$S_{12}(p_1,p_2)S_{21}(p_2,p_1)=\mathcal{I}, \tag{2}$$

where $S_{21}=\mathcal{P}_{12}S_{12}\mathcal{P}_{12}$, $\mathcal{I}=I\otimes I$, as well as the crossing property [9, 15]

$$C_2(p_2)S_{12}(p_1,\bar{p_2})C_2(p_2)^{-1}S_{12}(p_1,p_2)^{t_2}=\mathcal{I}f(p_1,p_2), \tag{3}$$

$$C_1(\bar{p_1})S_{12}(\bar{p_1},p_2)C_1(\bar{p_1})^{-1}S_{12}(p_1,p_2)^{t_1}=\mathcal{I}f(p_1,p_2), \tag{4}$$

where $C_1(p)=C(p)\otimes I$, $C_2(p)=I\otimes C(p)$, t_i denotes the transpose in the $i^{\rm th}$ space, $\bar{p}$ denotes the antiparticle momentum with

$$x^{\pm}(\bar{p})=\frac{1}{x^{\pm}(p)}\ . \tag{5}$$

$f(p_1,p_2)$ is the scalar function

$$f(p_1,p_2)=\frac{\left(\frac{1}{x_1^+}-x_2^-\right)\left(x_1^+-x_2^+\right)}{\left(\frac{1}{x_1^-}-x_2^-\right)\left(x_1^- -x_2^+\right)} \tag{6}$$

and satisfies the property

$$f(p_1,p_2)=f(-p_2,-p_1)=f(\bar{p}_1,\bar{p}_2). \tag{7}$$

$C(p)$ is the following matrix

$$C(p)=\begin{pmatrix} 0 & \mathrm{i\,sign}(p) & 0 & 0\\ -\mathrm{i\,sign}(p) & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -1 & 0\end{pmatrix}. \tag{8}$$

The $x^{\pm}(p)$ are defined by

$$x^+(p)+\frac{1}{x^+(p)}-x^-(p)-\frac{1}{x^-(p)}=\frac{\mathrm{i}}{g},\quad \frac{x^+(p)}{x^-(p)}=\mathrm{e}^{\mathrm{i}p} \tag{9}$$

with the property

$$x^{\pm}(-p)=-x^{\mp}(p). \tag{10}$$

Moreover, exchanging space 1 and space 2 in Eqs. (3, 4) yields

$$C_1(p_1)S_{21}(p_2,\bar{p_1})C_1(p_1)^{-1}S_{21}(p_2,p_1)^{t_1}=\mathcal{I}f(p_2,p_1), \tag{11}$$

$$C_2(\bar{p}_2)S_{21}(\bar{p}_2,p_1)C_2(\bar{p}_2)^{-1}S_{21}(p_2,p_1)^{t_2}=\mathcal{I}f(p_2,p_1). \tag{12}$$

The $Z=0$ giant graviton brane has a boundary degree of freedom and full $SU(2|2)$ symmetry [8, 9]. We use a 16×16 matrix $R^{\rm R}$ to denote the right boundary S-matrix, which satisfies the right boundary reflecting equation (BYBE) [8, 9]

$$S_{12}(p_1,p_2)R^{\rm R}_{13}(p_1)S_{21}(p_2,-p_1)R^{\rm R}_{23}(p_2)= R^{\rm R}_{23}(p_2)S_{12}(p_1,-p_2)R^{\rm R}_{13}(p_1)S_{21}(-p_2,-p_1). \tag{13}$$

Referring to the work of Nepomechie [10, 12], for the $Z=0$ case, we propose that the left BYBE has the following form

$$S_{21}(p_2,p_1)^{t_1t_2}R^{\rm L}_{31}(p_1)^{t_1}C_1(-p_1)\times S_{21}(p_2,\overline{-p_1})^{t_2}C_1(-p_1)^{-1}R^{\rm L}_{32}(p_2)^{t_2}= R^{\rm L}_{32}(p_2)^{t_2}C_2(-p_2)S_{12}(p_1,\overline{-p_2})^{t_1}\times C_2(-p_2)^{-1}R^{\rm L}_{31}(p_1)^{t_1}S_{12}(-p_1,-p_2)^{t_1t_2}, \tag{14}$$

where the bulk S-matrix obeys the unitarity and crossing property. Making a full transpose in space 1,2,3 on both sides of Eq. (14), we get

$$S_{12}(p_1,p_2)M_1R^{\rm L}_{31}(-p_1)^{t_3}S_{21}(p_2,-p_1)M_2R^{\rm L}_{32}(-p_2)^{t_3}= M_2R^{\rm L}_{32}(-p_2)^{t_3}S_{12}(p_1,-p_2)M_1R^{\rm L}_{31}(-p_1)^{t_3}S_{21}(-p_2,-p_1), \tag{15}$$

where

$$M=C(-p)C(p)^{-1}=\mathrm{diag}(-1,-1,1,1)=M^{-1} \tag{16}$$

and the crossing property Eqs. (4, 12), the identity equation Eq. (7), and the following property

$$[S_{12}(p_1,p_2),M\otimes M]=0 \tag{17}$$

have been used. Comparing Eq. (15) with the right BYBE Eq. (13), we get:

$$R^{\rm L}_{21}(p)=M_1R^{\rm R}_{12}(-p)^{t_2}. \tag{18}$$

3 The transfer matrix and its integrability

Referring to the work of Refs. [10, 12], we propose that in the $Z=0$ case, the transfer matrix is constructed as

$$t(p;\{q_i\})=\mathrm{tr_a}R^{\rm L}_{0\rm a}(p)\mathcal{T}^{\rm R}_{\rm a}(p\ ;\{q_i\})= \mathrm{tr_a}R^{\rm L}_{0\rm a}(p)T_{\rm a1\cdots L}(p\ ;\{q_i\})R^{\rm R}_{\rm aL+1}(p)\hat{T}_{\rm a1\cdots L}(p;\{q_i\}), \tag{19}$$

where

$$\mathcal{T}^{\rm R}_{\rm a}(p\ ;\{q_i\})=T_{\rm a1\cdots L}(p\ ;\{q_i\})R^{\rm R}_{\rm aL+1}(p)\hat{T}_{\rm a1\cdots L}(p\ ;\{q_i\}) \tag{20}$$

satisfying

$$S_{\rm ab}(p_{\rm a},p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a}\ ;\{q_i\})S_{\rm ba}(p_{\rm b},-p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b}\ ;\{q_i\})= \mathcal{T}^{\rm R}_{\rm b}(p_{\rm b}\ ;\{q_i\})S_{\rm ab}(p_{\rm a},-p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a}\ ;\{q_i\})S_{\rm ba}(-p_{\rm b},-p_{\rm a}), \tag{21}$$

and

$$T_{\rm a1\cdots L}(p\ ;\{q_i\})\ =\ S_{\rm aL}(p,q_{\rm L})\cdots S_{\rm a1}(p,q_1), \tag{22}$$

$$\hat{T}_{\rm a1\cdots L}(p\ ;\{q_i\})\ =\ S_{1a}(q_1,-p)\cdots S_{La}(q_{\rm L},-p), \tag{23}$$

which obey the following relations

$$S_{\rm ab}(p_{\rm a},p_{\rm b})\ T_{\rm a1\cdots L}(p_{\rm a};\{\ q_i\}\ )T_{\rm b1\cdots L}(p_{\rm b};\{\ q_i\}\ )\ =\ T_{\rm b1\cdots L}(p_{\rm b};\{\ q_i\}\ )\ T_{\rm a1\cdots L}(p_{\rm a};\{\ q_i\}\ )S_{\rm ab}(p_{\rm a},p_{\rm b}),\quad (24)$$

$$S_{\rm ba}(-p_{\rm b},-p_{\rm a})\ \hat{T}_{\rm a1\cdots L}(p_{\rm a};\{\ q_i\}\ )\hat{T}_{\rm b1\cdots L}(p_{\rm b};\{\ q_i\}\ )\ =\ \hat{T}_{\rm b1\cdots L}(p_{\rm b};\{\ q_i\}\ )\ \hat{T}_{\rm a1\cdots L}(p_{\rm a};\{\ q_i\}\ )S_{\rm ba}(-p_{\rm b},\ -p_{\rm a}),\quad (25)$$

$$\hat{T}_{\rm a1\cdots L}(p_{\rm a};\{\ q_i\}\ )S_{\rm ba}(p_{\rm b},\ -p_{\rm a})T_{\rm b1\cdots L}(p_{\rm b};\{\ q_i\}\ )\ =\ T_{\rm b1\cdots L}(p_{\rm b};\{\ q_i\}\ )\ S_{\rm ba}(p_{\rm b},\ -p_{\rm a})\hat{T}_{\rm a1\cdots L}(p_{\rm a};\{\ q_i\}\ ).\quad (26)$$

In the following we will show the integrability for the spin chain model defined by the transfer matrix Eq. (19). During the calculation the scalar functions $f(p_{\rm b},-p_{\rm a})$, $f(p_{\rm a},-p_{\rm b})$ are omitted. At first, we write $t(p_{\rm a};\{q_i\})t(p_{\rm b};\{q_i\})$ as

$$\begin{aligned}&t(p_{\rm a};\{q_i\})t(p_{\rm b};\{q_i\})=\\&{\rm tr}_{\rm a}R^{\rm L}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\}){\rm tr}_{\rm b}R^{\rm L}_{0{\rm b}}(p_{\rm b})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})=\\&{\rm tr}_{\rm a}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm Rt_a}_{\rm a}(p_{\rm a};\{q_i\}){\rm tr}_{\rm b}R^{\rm L}_{0{\rm b}}(p_{\rm b})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})=\\&{\rm tr}_{\rm ab}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})R^{\rm L}_{0{\rm b}}(p_{\rm b})\mathcal{T}^{\rm Rt_a}_{\rm a}(p_{\rm a};\{q_i\})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\}).\end{aligned}$$

Inserting the crossing property Eq. (11) into the above equation, we have

$$\begin{aligned}\cdots=\;&{\rm tr}_{\rm ab}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})R^{\rm L}_{0{\rm b}}(p_{\rm b})C_{\rm a}(-p_{\rm a})S_{\rm ba}(p_{\rm b},\overline{-p_{\rm a}})C_{\rm a}(-p_{\rm a})^{-1}S_{\rm ba}(p_{\rm b},-p_{\rm a})^{\rm t_a}\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})=\\&{\rm tr}_{\rm ab}\left(R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})C_{\rm a}(-p_{\rm a})S^{\rm t_b}_{\rm ba}(p_{\rm b},\overline{-p_{\rm a}})C_{\rm a}(-p_{\rm a})^{-1}R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})\right)^{\rm t_b}\left(\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})S_{\rm ba}(p_{\rm b},-p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\right)^{\rm t_a}=\\&{\rm tr}_{\rm ab}\left(R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})C_{\rm a}(-p_{\rm a})S^{\rm t_b}_{\rm ba}(p_{\rm b},\overline{-p_{\rm a}})C_{\rm a}(-p_{\rm a})^{-1}R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})\right)^{\rm t_at_b}\left(\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})S_{\rm ba}(p_{\rm b},-p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\right),\end{aligned}$$

where $\cdots$ denotes $t(p_{\rm a};\{q_i\})t(p_{\rm b};\{q_i\})$ for the sake of simplicity. Inserting the unitarity property Eq. (2) to the above result, we get

$$\begin{aligned}\cdots=\;&{\rm tr}_{\rm ab}\left(R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})C_{\rm a}(-p_{\rm a})S^{\rm t_b}_{\rm ba}(p_{\rm b},\overline{-p_{\rm a}})C_{\rm a}(-p_{\rm a})^{-1}R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})\right)^{\rm t_at_b}S_{\rm ba}(p_{\rm b},p_{\rm a})S_{\rm ab}(p_{\rm a},p_{\rm b})\times\\&\left(\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})S_{\rm ba}(p_{\rm b},-p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\right)=\\&{\rm tr}_{\rm ab}\left(S^{\rm t_at_b}_{\rm ba}(p_{\rm b},p_{\rm a})R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})C_{\rm a}(-p_{\rm a})S^{\rm t_b}_{\rm ba}(p_{\rm b},\overline{-p_{\rm a}})C_{\rm a}(-p_{\rm a})^{-1}R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})\right)^{\rm t_at_b}\times\\&\left(S_{\rm ab}(p_{\rm a},p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})S_{\rm ba}(p_{\rm b},-p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\right).\end{aligned}$$

Now using the left and right reflection equations Eqs. (14, 21), we obtain

$$\begin{aligned}\cdots=\;&{\rm tr}_{\rm ab}\left(R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})C_{\rm b}(-p_{\rm b})S^{\rm t_a}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C_{\rm b}(-p_{\rm b})^{-1}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})S^{\rm t_at_b}_{\rm ab}(-p_{\rm a},-p_{\rm b})\right)^{\rm t_at_b}\times\\&\left(\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})S_{\rm ab}(p_{\rm a},-p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})S_{\rm ba}(-p_{\rm b},-p_{\rm a})\right)=\\&{\rm tr}_{\rm ab}S_{\rm ab}(-p_{\rm a},-p_{\rm b})\left(R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})C_{\rm b}(-p_{\rm b})S^{\rm t_a}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C_{\rm b}(-p_{\rm b})^{-1}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\right)^{\rm t_at_b}\times\\&\left(\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})S_{\rm ab}(p_{\rm a},-p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})S_{\rm ba}(-p_{\rm b},-p_{\rm a})\right)=\\&{\rm tr}_{\rm ab}\left(R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})C_{\rm b}(-p_{\rm b})S^{\rm t_a}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C_{\rm b}(-p_{\rm b})^{-1}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\right)^{\rm t_at_b}\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\times\\&S_{\rm ab}(p_{\rm a},-p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_1;\{q_i\})\left(S_{\rm ba}(-p_{\rm b},-p_{\rm a})S_{\rm ab}(-p_{\rm a},-p_{\rm b})\right).\end{aligned}$$

Using the unitarity property (2) again, we achieve

$$\begin{aligned}\cdots=\;&{\rm tr}_{\rm ab}\left(R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})C_{\rm b}(-p_{\rm b})S^{\rm t_a}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C_{\rm b}(-p_{\rm b})^{-1}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\right)^{\rm t_at_b}\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})S_{\rm ab}(p_{\rm a},-p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})=\\&{\rm tr}_{\rm ab}\left(R^{\rm Lt_b}_{0{\rm b}}(p_{\rm b})C_{\rm b}(-p_{\rm b})S^{\rm t_a}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C_{\rm b}(-p_{\rm b})^{-1}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\right)^{\rm t_b}\left(\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})S_{\rm ab}(p_{\rm a},-p_{\rm b})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})\right)^{\rm t_a}=\\&{\rm tr}C^{\rm t_b}_{\rm b}(-p_{\rm b})^{-1}S^{\rm t_at_b}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C^{\rm t_b}_{\rm b}(-p_{\rm b})R^{\rm L}_{0{\rm b}}(p_{\rm b})R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\})S^{\rm t_a}_{\rm ab}(p_{\rm a},-p_{\rm b})=\\&{\rm tr}S^{\rm t_a}_{\rm ab}(p_{\rm a},-p_{\rm b})C^{\rm t_b}_{\rm b}(-p_{\rm b})^{-1}S^{\rm t_at_b}_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C^{\rm t_b}_{\rm b}(-p_{\rm b})R^{\rm L}_{0{\rm b}}(p_{\rm b})R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\})=\\&{\rm tr}\left(C_{\rm b}(-p_{\rm b})S_{\rm ab}(p_{\rm a},\overline{-p_{\rm b}})C_{\rm b}(-p_{\rm b})^{-1}S^{\rm t_b}_{\rm ab}(p_{\rm a},-p_{\rm b})\right)^{\rm t_at_b}R^{\rm L}_{0{\rm b}}(p_{\rm b})R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\}).\end{aligned}$$

At last, using the crossing property (3) again, we arrive at

$$\begin{aligned}\cdots=\;&{\rm tr}R^{\rm L}_{0{\rm b}}(p_{\rm b})R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\})={\rm tr}R^{\rm L}_{0{\rm b}}(p_{\rm b})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\})R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\})=\\&{\rm tr}_{\rm b}R^{\rm L}_{0{\rm b}}(p_{\rm b})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\}){\rm tr}_{\rm a}R^{\rm Lt_a}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{R\rm t_a}_{\rm a}(p_{\rm a};\{q_i\})={\rm tr}_{\rm b}R^{\rm L}_{0{\rm b}}(p_{\rm b})\mathcal{T}^{\rm R}_{\rm b}(p_{\rm b};\{q_i\}){\rm tr}_{\rm a}R^{\rm L}_{0{\rm a}}(p_{\rm a})\mathcal{T}^{\rm R}_{\rm a}(p_{\rm a};\{q_i\})=\\&t(p_{\rm b};\{q_i\})t(p_{\rm a};\{q_i\}).\end{aligned}$$

This means

$$[t(p_{\rm a};\{q_i\}),t(p_{\rm b};\{q_i\})]=0. \tag{27}$$

So the spin chain model is integrable.

4 Discussion

We constructed the transfer matrix for the open chain attached to the so called $Z=0$ giant gravition brane, and showed the integrability for the model defined by the transfer matrix. There are at least two things that we need to explore further. One is how to derive the Hamiltonian corresponding to the transfer matrix. The other is the exact solutions for the transfer matrix. For the $Y=0$ case, the Bethe equations for the transfer matrix have been obtained by Galleas [11] using the algebraic Bethe ansatz method [16, 17] and by Nepomechie [12] using the analytical Bethe ansatz method [18]. However, for the $Z=0$ case, the work will become more tough.

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