Influence of super-strong magnetic field on the electron chemical potential and β decay in the stellar surroundings*

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Abstract In this paper, considering the quantum effect of electrons in a super-strong magnetic field, the influence of a super-strong magnetic field on the chemical potential of a non-zero temperature electron is analyzed, the rates of β decay under the super-strong magnetic field are studied, and then we compare them with the case without a magnetic field. Here, the nucleus ^{63}Co is investigated in detail as an example. The results show that a magnetic field that is less than 10^{10} T has little effect on the electron chemical potential and β decay rates, but the super-strong magnetic field that is greater than 10^{10} T depresses the electron chemical potential and improves the β decay rates clearly.

Key words super-strong magnetic field, β decay, electron chemical potential

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1 Introduction

β decay and other weak interactions have a significant impact on the late stages of stellar evolution and the stellar core's evolution. For many years, therefore, researchers have paid much attention to the problem of nuclear β decay in the stellar surroundings [1–3]. In 1980, Fuller, Flower and Newman (FFN) computed the β decay rates inside the stars for nuclei with mass numbers 21 to 60 accurately in high-density environment. Because many nuclei lack detailed experimental data on excited levels, they adopted the nuclear shell model to deal with the energy and intensity Gamow-Teller (G-T) resonance transition [4]. In 1994, Aufderheide et al considered the stellar evolution generated cores, which are composed of some very neutron-rich nuclei, and analyzed the β decay rates of iron group nuclei with mass numbers greater than 60 without a magnetic field [5]. Subsequently, Langanke et al calculated a large number of nuclei to enrich the data of β decay rates in the stellar environment [6, 7].

The existence of a super-strong magnetic field in neutron stars and pulsars is well proven. The surface magnetic fields of many supernova and neutron stars are 10^8–10^{10} T, and the surface magnetic field of magnetars may reach 10^{11}–10^{12} T [8]. What's more, Shapiro et al pointed out that the internal magnetic field of neutron stars may be even stronger perhaps as large as 10^{10}–10^{14} T [9]. The existence of a super-strong magnetic field will affect the β decay and other weak interactions, and the β decay was studied in the strong magnetic field soon after the discovery of pulsars [10]. Recently, Zhang Jie et al discussed the influence of a strong magnetic field (less than 10^9 T) on the β^- decay and β^+ decay in the crusts of neutron stars [11, 12]. However, they ignored the impact of a magnetic field on the electron chemical potential and characteristic energy level. When the electron temperature is a constant, the stronger the magnetic field, the more significant the quantum effect of an electron, and the greater the influence of a magnetic field on the electron chemical potential and characteristic energy level. So the impact of a super-strong

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magnetic field that is greater than 10^{10} T on the electron chemical potential and characteristic energy level are so strong that they cannot be ignored. Up until now, nobody has studied the β decay rates in such a super-strong magnetic field (greater than 10^{10} T), or considered the effect of a super-strong magnetic field on the electronic chemical potential and characteristic energy level. In this paper, considering the influence of a super-strong magnetic field on the electron chemical potential and characteristic energy level, we research the effect of a super-strong magnetic field on the β decay rates and the electron chemical potential. In addition, more precise β decay rates as a function of magnetic field are obtained by a numerical method. Finally, the β decay rate of nucleus ^{63}Co in a super-strong magnetic field is investigated as an example. This may have a significant impact on the evolution of the stellar core [11, 12].

2 Electron chemical potential in a super-strong magnetic field

The electron chemical potential without a magnetic field is given by [4]

$$\rho/\mu_{\rm e}=\frac{1}{\pi^2 N_{\rm A}}\left(\frac{m_{\rm e}c}{\hbar}\right)^3\int_0^\infty(f_{-\rm e}-f_{+\rm e})p^2{\rm d}p, \qquad (1)$$

where ρ is the density in units of g/cm^3, $\mu_{\rm e}$ is the electron mean molecular weight, $m_{\rm e}$ is the electron rest mass, c is the velocity of light, $\hbar$ is Planck's constant and $N_{\rm A}$ is Avogadro's number.

$$f_{-\rm e}=\{1+\exp[(\varepsilon_n-1-U_{\rm F})/k_{\rm B}T]\}^{-1}$$

and

$$f_{+\rm e}=\{1+\exp[(\varepsilon_n+1+U_{\rm F})/k_{\rm B}T]\}^{-1}$$

are the Fermi-Dirac distribution functions of electrons and positrons, respectively, $k_{\rm B}$ is the Bolmnann' s constant, T is the electron temperature, $U_{\rm F}$ is the electron chemical potential, ε_n is the electron energy in units of $m_{\rm e}c^2$, which can be expressed as $\varepsilon_n=(p^2+1)^{1/2}$, and p is the electron momentum in units of $m_{\rm e}c$.

In a super-strong magnetic field, Eq. (1) can be rewritten as [13]

$$\rho/\mu_{\rm e}=\frac{\Theta}{2\pi^2 N_{\rm A}}\left(\frac{m_{\rm e}c}{\hbar}\right)^3\sum_{n=0}^{\infty}g_{n0}\int_0^\infty(f_{-\rm e}-f_{+\rm e}){\rm d}p, \qquad (2)$$

where $\Theta=\hbar eB/m_{\rm e}^2c^3=2.27B_6$, e is the electron charge, B is the magnetic field strength, and B_6 is the magnetic field strength in units of 10^6 T. In a super-strong magnetic field, the expression of electron energy becomes $\varepsilon_n=(p^2+1+2n\Theta)^{1/2}$, $n=0,1,2\cdots$, corresponds to the Landau levels and $g_{n0}=2-\delta_{n0}$ is the spin degeneracy of electron.

Fig. 1 shows the electron chemical potential without a magnetic field as a function of temperature in different densities ($\rho=10^8$ g/cm^3, $\rho=3.2\times10^7$ g/cm^3) and electron fractions ($Y_{\rm e}=0.47$, $Y_{\rm e}=0.42$). Fig. 2 shows the electron chemical potential as a function of magnetic field in the same densities and electron fractions while the temperature is 4×10^9 K.

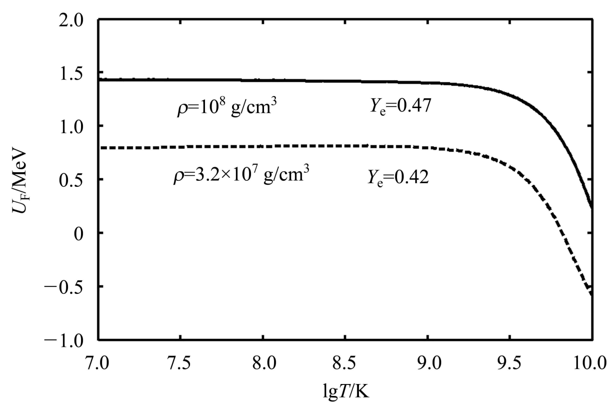

Fig. 1. The electron chemical potential $U_{\rm F}$ as a function of temperature T in different densities and electron fractions.

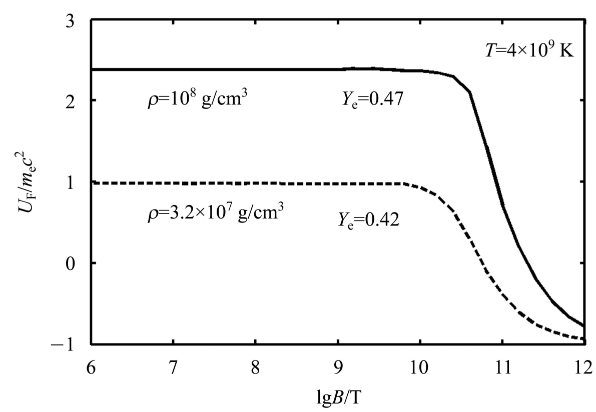

Fig. 2. The electron chemical potential $U_{\rm F}$ as a function of magnetic field B in different densities and electron fractions.

Figure 1 shows that the lower temperature without a magnetic field has little effect on the electron chemical potential, but the chemical potential decreases gradually when the temperature increases. Fig. 2 shows that the chemical potential is a constant when the magnetic field is less than 10^{10} T, but it begins to decrease rapidly when the magnetic field is larger than 10^{10} T. The stronger the magnetic field, the more obvious its decrease. Then by comparing Fig. 1 with Fig. 2, it can be concluded that, when the magnetic field is weak, the electron chemical potential returns to the situation where was to magnetic field.

3 β decay in a super-strong magnetic field

In the super-strong magnetic field, calculations of the total weak interactions with a sum over the initial states i and final states j, and then the β decay rates for the k-th nucleus (Z, A) by using the shell model of nuclei and considering the transition rules of decay, can be written as [3, 11]

$$\lambda_k^{\rm bd}(\rho,T,Y_{\rm e}) = \ln 2\sum_i \frac{(2J_i+1)\exp[-E_i/k_{\rm B}T]}{G(Z,A,T)}\times \sum_j \frac{\xi^{\rm B}(\rho,T,Y_{\rm e},Q_{ij})}{ft_{ij}}, \quad (3)$$

where J_i and E_i are the spin and excitation energy of the i-th state, respectively, Q_{ij} is the energy difference between the states i and j, and its expression is $Q_{ij}=(M_{\rm p}-M_{\rm d})c^2+E_i-E_j$. $M_{\rm p}$ and $M_{\rm d}$ are the masses of parent nucleus and daughter nucleus respectively, E_j is the excitation energy of the j-th state, and ft_{ij} is the comparative half-life. $G(Z,A,T)$ is the nuclear partition function (see Refs. [3,14] for details). $\xi^{\rm B}(\rho,\ T,\ Y_{\rm e},\ Q_{ij})$ is the β decay phase space integral at the magnetic field discussed below.

With the influence of a super-strong magnetic field, the state density of electron phase space, which can be valued by $d\nu_{\rm e}=\Theta/4\pi^2\lambda_{\rm e}^3{\rm d}p$, will change correspondingly, so it is necessary to modify the phase space integral. After neglecting the shape correction and screening correction, but considering the effect of temperature, the phase space integral in Eq. (3) can be expressed as [11, 13]

$$\xi^{\rm B}(\rho,T,Y_{\rm e},Q_{ij}) = \frac{\Theta}{2}\sum_{n=0}^{\infty}\frac{c^3}{(m_{\rm e}c^2)}g_{n0}\times \int_0^{\sqrt{Q_{ij}^2-Q_n^2}}(Q_{ij}-\varepsilon_n)^2\times \frac{F(Z+1,\varepsilon_n)}{1+\exp[(U_{\rm F}-\varepsilon_n)/k_{\rm B}T]}{\rm d}p, \quad (4)$$

where $Q_n=(m_{\rm e}^2c^4-2n\Theta)^{1/2}$, $F(Z+1,\varepsilon_n)$ is the Fermi function (see Ref. [3] for details).

According to the analysis of Aufderheide et al, the right side of Eq. (3) can be divided into two parts [3], namely

$$\lambda_k^{\rm bd} = \ln 2\frac{(2J_0+1)}{G(Z,A,T)}\exp(-E_{\rm peak}/k_{\rm B}T)\times \frac{\xi^{\rm B}(\rho,T,Y_{\rm e},E_{\rm peak}+Q_{00})}{ft_{\rm eff}}+ \ln 2\exp(-E_{\rm BGTR(0)}/k_{\rm B}T)\frac{G(Z+1,A,T)}{G(Z,A,T)}\times \frac{\xi^{\rm B}(\rho,T,Y_{\rm e},E_{\rm BGTR(0)}+Q_{00})}{ft_{0\to{\rm BGTR(0)}}}, \quad (5)$$

where the first part is the decay rate for the low-energy region close to the ground state, and the second part is the decay rate for the high-energy region dominated by the G-T resonance transition [3], $ft_{\rm eff}=6.06\times10^4$ [3], $f_{t_{0\to{\rm BGTR(0)}}}=10^{3.596}/|M_{\rm BGT(0)}|^2$ [3, 4], $|M_{\rm BGT(0)}|^2$ is the resonant G-T matrix element and $E_{\rm BGTR(0)}$ is the G-T resonance energy. Q_{00} is the nuclear Q-value of the ground state to ground state. If Q_{00} is negative, or not much more than the chemical potential, the function $\exp(-E_i/k_{\rm B}T)\times \xi^{\rm B}(\rho,T,Y_{\rm e},E_i+Q_{00})$ will peak somewhere above the ground state of the parent, and this peak is a characteristic energy level $E_{\rm peak}$. Now we calculate the $E_{\rm peak}$ in a super-strong magnetic field [1, 4, 14]:

$$E_{\rm peak}=5T-Q_{00}+U_{\rm F}+\delta(T,U_{\rm F}), \quad (6)$$

where

$$\delta(T,U_{\rm F}) = -0.6604+0.9429T-0.02119U_{\rm F}-0.9432TU_{\rm F}-0.0009524U_{\rm F}^2+0.06224TU_{\rm F}^2. \quad (7)$$

Here, the electron chemical potential $U_{\rm F}$ is a function of the super-strong magnetic field. If $E_{\rm peak}$ is negative, the ground state to ground state transition is calculated, thus $E_{\rm peak}=0$ [11].

4 β decay rates of nucleus ^{63}Co in a super-strong magnetic field

The neutron-rich nucleus ^{63}Co is abundant during the process of stellar evolution, so the large-scale shell model calculations for the G-T strength distributions can be derived, too [7]. What's more, the β decay of nucleus ^{63}Co and other neutron-rich nuclei can prevent the decrease in the total electron fractions in the stellar core. For the reaction $^{63}{\rm Co}({\rm e}^-,\bar{\nu}_{\rm e})^{63}{\rm Ni}$, Q_{00}=3.674 MeV, $E_{\rm BGTR(0)}$= 4.0431 MeV, $|M_{\rm BGT(0)}|^2$ = 79/14, and J_0 = 3/2 [14]. Fig. 3 shows that, when the electron fraction is 0.47, the density is $10^8{\rm g/cm}^3$ and the temperature is 4×10^9 K, the β decay rate of nucleus ^{63}Co is a function of the super-strong magnetic field.

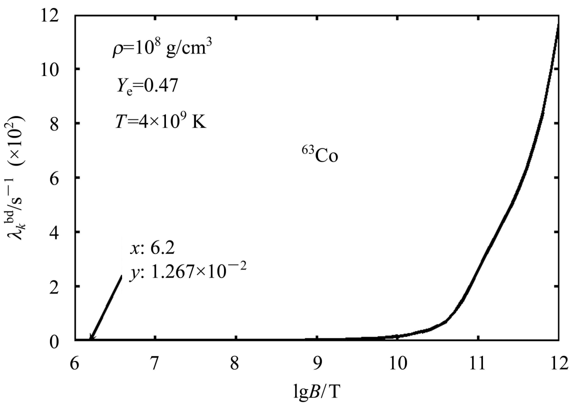

Fig. 3. The β decay rates λ_k of nucleus ^{63}Co as a function of magnetic field B.

Figure 3 shows that, when the magnetic field is less than 10^{10} T, the curve changes more slowly, and this indicates that a weak magnetic field has almost no effect on the β decay rates. But when the magnetic field is greater than 10^{10} T, the curve rises sharply, which indicates that the super-strong magnetic field clearly increases the rate of β decay. When the magnetic field is weak, the β decay rate returns to the situation where there was to magnetic field. By comparing Fig. 2 with Fig. 3, it can be seen that the electron chemical potential is a constant in a weak magnetic field, and the rates of β decay are also changeless. However when the magnetic field is stronger, the electron chemical potential decreases gradually with the magnetic field, while the β decay rates increase 2 to 3 orders of magnitude with the magnetic field, indicating that when we research the process of β decay, the effect of a super-strong magnetic field on the electronic chemical potential should be considered.

5 Conclusions

A super-strong magnetic field changes the distribution of electron phase space in the stellar surroundings. Therefore, in a high-density and strong magnetic field environment, the electrons generated in the process of β decay tend to concentrate on the Landau levels. Also, the unoccupied states outside the nucleus increase, which serves to increase the electron outgoing rates. Moreover, the effect of a super-strong magnetic field makes the electron chemical potential decrease, and this in turn reduces the Fermi energy, which serves to increase the electron outgoing rates, too. Thus a super-strong magnetic field has a significant influence on the rates of β decay. Finally in this paper, we take the nucleus ^{63}Co as an example for investigating the β decay rates in a super-strong magnetic field, in which the effect of a super-strong magnetic field on the chemical potential is also considered specially. The conclusion derived has astrophysical significance for the further study of the late stages of stellar evolution and the evolution of the stellar core.

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