Three-body hadronic molecules

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Abstract In this talk, I discuss our recent studies of three-hadron systems and the resonances found therein. The studies consist of solving Faddeev equations with the input two body interactions obtained from the chiral Lagrangians. The systems which we study are either made of two mesons and a baryon or of three mesons. The motivation for these studies comes from the data on many baryon resonances, especially the ones with $J^\pi = 1/2^+$, which show a large branching ratio to the two meson-one baryon decay channels. In addition to this, several new studies at BES, BELLE, BABAR etc., claim the existence of new meson resonances which seem to couple strongly to three-meson systems, where mostly two out of the three mesons appear as a known resonance. Hitherto, we have studied two meson-one baryon systems with strangeness = −1, 0 and 1 and three-meson systems made of two-pseudoscalars and a vector meson. As we will show in this manuscript, we find many resonances which couple to three-hadrons.

Key words hadron resonances, few-body systems.

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1 Introduction

Traditionally, three-baryon systems have been studied with great details. However, an equivalent attention has not been received by the systems made of three mesons or those of two mesons and a baryon. Indeed in the baryon sector, several resonances cannot be explained in the two-body dynamics, and there are evidences which suggest that they possess more complex structures. For example, the particle data book (PDB) [1] shows a large (40%–90%) branching ratio for the $N^*(1710)$ resonance to the ππN channel. The Roper resonance is another controversial case which does not seem to get reproduced in the quark models [2–4]. Further, the experimental studies of the reactions $K^- p \to \pi^0\pi^0\Lambda$ [5] and $K^- p \to \pi^0\pi^0\Sigma$ [6] indicate the presence of the Λ(1600) and Σ(1660) respectively in these processes.

In case of mesons, several new resonances are being found at BES, BELLE, BABAR, CLEO etc., facilities. These resonances do not seem to fit into the known quarkonium spectra and coincidently several of them seem to appear in reactions with three mesons in the final state where two out of the three form a known resonance. In other words, a new resonance seems to develop in a system made of a meson and a meson resonance. It does sound quite convincing that a resonance gets generated if a meson is added to a resonating two meson system. The situation gets even more promising if the interaction of each of the two meson sub-systems is attractive in nature. Some of the examples of such cases are:

1) X(2175) found in the $e^+e^- \to \phi f_0$ reaction by the BABAR and BES collaborations [7].

2) Y(4260) found in the $e^+e^- \to J\psi f_0$ reaction, with exceptionally strong coupling to the $J\psi f_0$ channel, by the BABAR, BES and CLEO collaborations [8].

3) Y(4660) found in the $J\psi(2s)f_0$ system by the BELLE collaboration [9].

4) X(1576) in the $K^*K\pi$ system by the BES collaboration [10], etc.

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There are many more resonances which couple to three-hadron systems and our idea is to find them in those systems, which are built by adding a meson/baryon to a meson-baryon/meson-meson system, which has earlier been studied and which interacts strongly to generate resonances dynamically. For this, we solve Faddeev equations in the formalism briefly explained in the next section.

2 Formalism

We start by calculating the potentials for the respective coupled channels of the three pairs constituting the three-body system. These coupled channels are chosen in such a way that the dynamics in the two body systems can generate the resonances necessary for the reproduction of the respective experimental data. These two body potentials are obtained from the chiral Lagrangians [11–14] and then the Bethe Salpeter equations are solved to obtain the corresponding t-matrices which are then used an input in the equations [15–20]

$$T_{\rm R}^{ij} = t^i g^{ij} t^2 + t^i \Big[G^{iji}\, T_{\rm R}^{ji} + G^{ijk}\, T_{\rm R}^{jk} \Big], \qquad (1)$$

where $i \neq j$, $j \neq k = 1, 2, 3$. These six coupled equations, which can be related to the Faddeev partitions T^i as

$$T^i = t^i \delta^3(\vec{k}_i^{\,\prime} - \vec{k}_i) + T_{\rm R}^{ij} + T_{\rm R}^{ik} \;, \qquad (2)$$

are summed to get the full three-body t-matrix

$$T_{\rm R} = T_{\rm R}^{12} + T_{\rm R}^{13} + T_{\rm R}^{21} + T_{\rm R}^{23} + T_{\rm R}^{31} + T_{\rm R}^{32} \;. \qquad (3)$$

Before explaining these equations, we would like to mention one of the most important finding of our work, which is the finding of an explicit, analytic cancellations between the contribution of the off-shell parts of the two body t-matrices to the Faddeev equations and the three-body forces coming directly from the same chiral Lagrangian which we use to get the two-body interactions (see Appendices of Refs. [17, 18]). This finding makes our formalism free from uncertainties in the results due to the presence of unphysical off-shell parts of the two-body t-matrices.

Thus, in Eq. (1), the t^i's are the two-body t-matrices, which depend on the invariant mass of the interacting pair, and the g_{ij}'s are the three-body Green's function defined as

$$g^{ij}(\vec{k}_i^{\,\prime}, \vec{k}_j) = N_l \frac{1}{\sqrt{s} - E_i(\vec{k}_i^{\,\prime}) - E_l(\vec{k}_i^{\,\prime} + \vec{k}_j) - E_j(\vec{k}_j)}, \qquad (4)$$

where $l \neq i$, $l \neq j, = 1, 2, 3$ and $N_l = 1/E_l$ or M_l/E_l in case of a meson or a baryon propagator present in a diagram.

In order to define the G^{ijk} functions in Eq. (1), we write the equation for the $T_{\rm R}^{12}$ partition in the Faddeev formalism for one channel as

$$\begin{aligned} T_{\rm R}^{12} = {} & t^1(\sqrt{s_{23}})g^{12}(\vec{k}_1^{\,\prime}, \vec{k}_2)t^2(\sqrt{s_{13}}) + t^1(\sqrt{s_{23}}\,) \times \\ & \left(\int \frac{{\rm d}\vec{q}}{(2\pi)^3} g^{12}(\vec{k}_1^{\,\prime}, \vec{q}) t^2(\sqrt{s_{31}}(q)) g^{21}(\vec{q}, \vec{k}_1) \right) \times \\ & t^1(\sqrt{s_{23}}\,) + t^1(\sqrt{s_{23}}\,) \left(\int \frac{{\rm d}\vec{q}}{(2\pi)^3} g^{12}(\vec{k}_1^{\,\prime}, \vec{q}) \times \right. \\ & \left. t^2(\sqrt{s_{31}}(q)) g^{23}(\vec{q}, \vec{k}_1) \right) t^3(\sqrt{s_{12}}). \end{aligned} \qquad (5)$$

Let us consider the second term of the above equation and re-write it as

$$\begin{aligned} & t^1(\sqrt{s_{23}}) \left(\int \frac{{\rm d}\vec{q}}{(2\pi)^3} g^{12}(\vec{k}_1^{\,\prime}, \vec{q}) t^2(\sqrt{s_{31}}(q)) g^{21}(\vec{q}, \vec{k}_1) \times \right. \\ & \left. \underline{[g^{21}(\vec{k}_2^{\,\prime}, \vec{k}_1)]^{-1} \times [t^2(\sqrt{s_{31}})]^{-1}} \right) \times \\ & \underline{t^1(\sqrt{s_{31}}) g^{21}(\vec{k}_2^{\,\prime}, \vec{k}_1)} t^1(\sqrt{s_{23}}). \end{aligned} \qquad (6)$$

The loop dependent term in the bracket is called as the G^{121} function which makes the Eq. (6) equal to $t^1 G^{121} t^2 g^{21} t^1$, where all the terms except G^{121} are on-shell. Note that in Eq. (6), only an identity (the underlined part of the equation) has been multiplied which leaves the Eq. (5) unaltered and at the same time simplified for numerical calculations since once the G^{ijk} functions are calculated, solving Eq. (1) gets trivial. However, we should also mention that this formulation which is accurate for diagrams with three t-matrices, turns into an approximation for higher order diagrams, which has been shown to be reasonably good in Ref. [17].

We shall now review the results of the different studies, which have been carried out using this formalism.

3 Results

As mentioned earlier, we have studied systems made of two-mesons and a baryon and those of three mesons. We have considered all the interactions in our work in the S-wave, which means, eg., that the total spin parity of the three-body systems of two pseudoscalars and a baryon is $1/2^+$. Eq. (1) depen on two variables: the total energy of the system, which is denoted by $\sqrt{s}$, and the invariant mass of the particles 2 and 3, denoted by $\sqrt{s_{23}}$. We shall first discuss the results of our study of two-meson–one-baryon systems.

3.1 Baryon resonances in three-body systems

The first three-body system which we studied was the $\pi\bar{K}N$ system and its coupled channels [15, 16]. In this work we found that the three-body dynamics generated several hyperon resonances which could be related with the resonances listed in the PDB [1]. In fact the status of many of these known states is poor or controversial. The reason for such a status becomes clear by considering our finding that these resonances couple strongly to three-body decay channels. To be precise, we found evidence for the $\Sigma(1770)$, $\Sigma(1660)$, $\Sigma(1620)$, $\Sigma(1560)$, $\Lambda(1810)$ and $\Lambda(1600)$. Out of which, the spin parity of the $\Sigma(1560)$ is not known, which we predict as $1/2^+$. In the isospin zero case, we have actually found three resonances which seems to be expected from the listings of the PDB since under the headings of both the $\Lambda(1810)$ and $\Lambda(1600)$, it is mentioned that probably there are two resonances in that region. We show the squared amplitude of the full three-body T matrix in Fig. 1 for the $\pi\pi\Sigma$ channel in the isospin 1, as an example. In the figure one can see two resonances; one at 1630 and another at 1656 MeV.

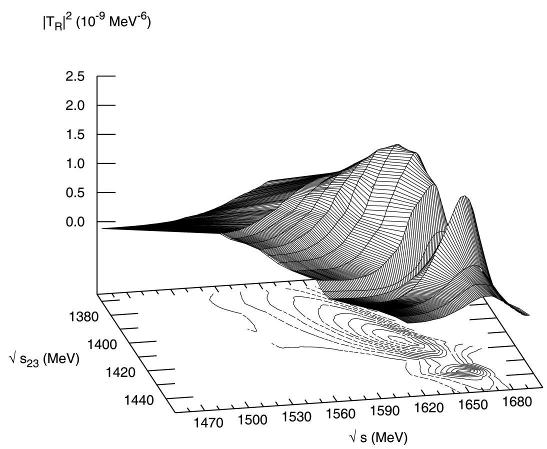

Fig. 1. The squared amplitude for the $\pi\pi\Sigma$ channel projected in the isospin 1.

Next, we studied the $S=0$ meson-meson-baryon systems, where the $N^*(1710)$ appeared neatly as a resonance of the $\pi\pi N$ with the a σN [17] structure. But there are other $J^P=1/2^+$ states, like the $N^*(2100)$ and the $\Delta(1910)$ which did not appear in the work reported in Ref. [17]. From the work of [21] we know that the chiral unitary study of the πN system made by using the lowest order Lagrangian provides a fair amplitude up to $\sqrt{s}=1600$ MeV but fails beyond this energy. For instance, the $N^*(1650)$ does not appear in the approach. As a consequence, any three body states which would choose to cluster a πN subsystem into this resonance would not be obtained in the approach of Ref. [17].

We then made a new study of the $S=0$ systems, now by using the experimental πN amplitudes as the input. However, at energies which are below the πN threshold, we used the theoretical amplitudes of [21]. In order to be consistent, we first calculated the t-matrix around 1700 MeV and found that the new three-body amplitude in this region remained almost unchanged. These new calculations [19] lead to generation of three more $1/2^+$ baryon resonances with $S=0$; one corresponding to the $\Delta(1910)$ and another to the $N^*(2100)$ and yet another around 1920 MeV with isospin 1/2. There is no known N^* resonance around 1920 MeV but there are many speculations of existence of one as we have discussed in Ref. [22].

We have studied also two meson-one baryon systems for strangeness equal to one with the hope to find a resonance around 1542 MeV, i.e., an evidence for the Θ^+ [23]. Since the KN interaction obtained from chiral Lagrangians is basically repulsive in nature [24], it is not appealing to look for a narrow (long lived) resonance, as the one claimed in Ref. [23], in this system. This is why very early there were suggestions that if the peak represented a new state, it could be a bound state of three hadrons, $K\pi N$, with the pion acting as a glue between the nucleon and the K. However, investigations along this line, weakly concluded the difficulty to have this system as a bound state [25, 26].

The calculations with our formalism did not result in any structure in the energy region close to 1542 MeV. However, we did obtain a peak with a broad structure in the isospin zero amplitude (i.e., when the πK subsystem is in isospin 1/2) around 1720 MeV. The full width at half maximum of the peak is of the order of 200 MeV [20]. The value of $\sqrt{s_{23}}$, for which this bump is found, is around the mass of the $\kappa(800)$ resonance. Thus it can be interpreted as a $\kappa(800)N$ resonance.

3.2 Meson resonances in three-body systems

As mentioned in the introduction, a resonance with mass 2175 MeV has been found in different experimental studies [7] which seems to couple strongly to the $\phi f_0(980)$ system. In Ref. [27] the $e^+e^- \to \phi f_0(980)$ reaction, for which the data are available from BABAR [7], was studied using a loop mechanism involving pseudoscalar and vector meson loops. The authors of Ref. [27] could explain the background of the data from BABAR on the invariant mass of $\phi f_0(980)$ [7] but they could not explain the peak at an energy of 2175 MeV in the same data. In the chiral models, the $f_0(980)$ resonance is dynamically generated in the $K\bar{K}$ interaction [14]. Therefore, a study of the $\phi K\bar{K}$ system could explain the experimental results. In Ref. [18] we solved the Eq. (1) taking $\phi K\bar{K}$ and $\phi\pi\pi$ as coupled channels and found a peak in the $\phi K\bar{K}$ channel around 2150 MeV with a width of 20 MeV when the invariant mass of the $K\bar{K}$ system is close to 980 MeV (see Fig. 2), thus, confirming the experimental findings. Using the results of Ref. [27], we implemented the final state interaction for the $e^+e^- \to \phi f_0(980)$ reaction in terms of our three-body amplitude and calculated the cross sections. This resulted into a peak in the cross section around 2175 MeV in accordance with the experimental cross sections obtained by different experimental groups [7]. Similar findings have also been reported in Refs. [28, 29].

We have also studied the $J/\psi K\bar{K}$ and $J\psi\pi\pi$ systems in order to look for the $Y(4260)$ resonance, which as mentioned in Ref. [30] seems to be very similar to the X(2175) resonance and where we report the finding of a resonance which can be related to the Y(4260).

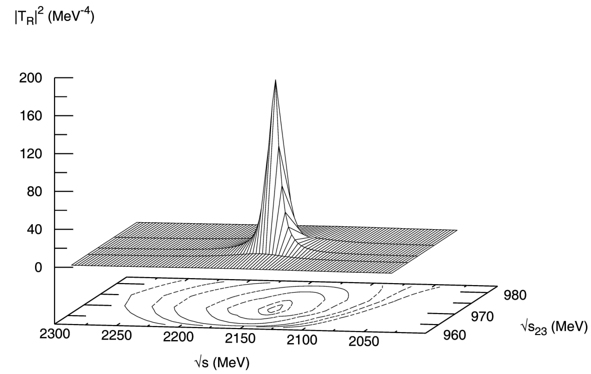

Fig. 2. The squared amplitude for the $\phi K\bar{K}$ channel.

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References

1 YAO W M et al. J. Phys. G, 2006, **33**: 1
2 Isgur N, Karl G. Phys. Rev. D, 1978, **18**: 4187
3 Isgur N, Karl G. Phys. Rev. D, 1979, **19** : 2653; 1981, **23**: 817
4 Glozman L Y, Riska D O. Phys. Rept., 1996, **268** 263
5 Prakhov S et al. Phys. Rev. C, 2004, **69**: 042202
6 Prakhov S et al (Crystall Ball collaboration). Phys. Rev. C, 2004, **70**: 034605
7 Aubert B et al (BABAR collaboration). Phys. Rev. D, 2006, **74**: 091103; Phys. Rev. D, 2007, **76**: 012008; Ablikim M et al (BES collaboration). Phys. Rev. Lett. 2008, **100**: 102003
8 Aubert B et al (BABAR collaboration). Phys. Rev. Lett., 2005, **95**: 142001; Coan T E et al (CLEO collaboration). Phys. Rev. Lett., 2006, **96**: 162003; HE Q et al (CLEO collaboration). Phys. Rev. D, 2006, **74**: 091104; YUAN C Z et al (Belle collaboration). Phys. Rev. Lett., 2007, **99**: 182004
9 WANG X L et al (Belle collaboration). Phys. Rev. Lett., 2007, **99**: 142002
10 Ablikim M et al (BES collaboration). Phys. Rev. Lett., 2006, **97**: 142002
11 Weinberg S. Phys. Rev., 1968, **166**: 1568; Bernard V, Kaiser N, Meissner N G. Int. J. Mod. Phys. E, 1995, **4**: 193; Ecker G. Prog. Part. Nucl. Phys., 1995, **35**: 1
12 Oset E, Ramos A. Nucl. Phys. A, 1998, **635**: 99
13 Inoue T, Oset E, Vicente Vacas M J. Phys. Rev. C, 2002, **65**: 035204
14 Oller J A, Oset E. Nucl. Phys. A, 1997, **620**: 438; 1999, **652**: 407
15 Martinez Torres A, Khemchandani K P, Oset E. Phys. Rev. C, 2008, **77** : 042203
16 Martinez Torres A, Khemchandani K P, Oset E. Eur. Phys. J. A, 2008, **35**: 295; Khemchandani K P, Martinez Torres A, Oset E. Few Body Syst., 2008, **44** : 145
17 Khemchandani K P, Martinez Torres A, Oset E. Eur. Phys. J. A, 2008, **37**: 233
18 Martinez Torres A et al. Phys. Rev. D, 2008, **78**: 074031
19 Martinez Torres A, Khemchandani K P, Oset E. Phys. Rev. C, 2009, **79**: 065207
20 Khemchandani K P, Martinez Torres A, Oset E. Phys. Lett. B, 2009, **675**: 407
21 Inoue T, Oset E. Vicente Vacas M J. Phys. Rev. C, 2002, **65**: 035204
22 Martinez Torres A, Khemchandani K P, Meissner U G, Oset E. Eur. Phys. J. A, 2009, **41**: 361
23 Nakano T et al (LEPS collaboration). Phys. Rev. Lett., 2003, **91**: 012002
24 Bernard V, Kaiser N, Meissner U G. Int. J. Mod. Phys. E, 1995, **4**: 193
25 Bicudo P, a Marques G M. Phys. Rev. D, 2004, **69**: 011503
26 Llanes-Estrada F J, Oset E, Mateu V. Phys. Rev. C, 2004, **69**: 055203
27 Napsuciale M, Oset E, Sasaki K, Vaquera-Araujo C A. Phys. Rev. D, 2007, **76**: 074012; Gomez-Avila S, Napsuciale M, Oset E. Phys. Rev. D, 2009 , **79** : 034018
28 Coito S, Rupp G, everen E Van. Phys. Rev. D, 2009, **80**: 094011
29 Alvarez-Ruso L, Oller J A, Alarcon J M. Phys. Rev. D, 2009, **80** : 054011
30 Martinez Torres A, Khemchandani K P, Gamermann D, Oset E. Phys. Rev. D, 2009, **80**: 094012