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Medium modified fragmentation function: equivalence of the ACSX and QW formalisms in the high-Q^2 region*

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Abstract: We study the medium modified fragmentation function in high-energy heavy-ion collisions. We show that the ACSX and QW formalisms are equivalent to each other in the high-Q^2 limit in both theoretical and numerical aspects.

Key words: quark gluon plasma, parton energy loss, heavy ion collision physics

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1 Introduction

The strong suppression of high-p_{T} hadrons has been observed experimentally at the Relativistic Heavy Ion Collider (RHIC) [1–5]. The dominant dynamical mechanism of this suppression is commonly understood as that hard partons produced in A-A collisions at RHIC propagate through highly excited matter (quark gluon plasma, QGP) and lose virtuality, which causes p_{T} degradation by interaction with the medium before hadronization in the vacuum [6–9]. The interaction of partons with the medium affects high transverse momentum hadron production, thus it can be reflected by the parton fragmentation function. In this paper, we study the equivalence of the medium-evolved fragmentation function (ACSX) [10, 11] and the quenching weight fragmentation function (QW) [12], which can provide useful information on the properties of the QGP.

One motivation of this paper is the imminence of the Large Hadron Collider (LHC) heavy-ion program, in which real high p_{T} jets will be measured for the first time. This will provides a good method of inspecting the medium modified fragmentation function. It is known that the ACSX provides insight to the parton energy loss at RHIC energies and the ACSX fragmentation function is widely used at LHC energies, especially in the LHC/ALICE heavy-ion collision simulation program (such as Q-Pythia). We will show that the quenching weight fragmentation functions are a limiting case of the medium-evolved fragmentation functions and they are equivalent to each other in the high-Q^2 limit. We present a detailed derivation of the equivalence of the ACSX and QW. We wish to note that, although our ultimate target is to study LHC physics, our work can also find applications in RHIC physics.

2 Quenching weights

When a hard parton passes through a hot and dense QCD medium, it loses energy via the emission of gluons due to medium-induced effect. The medium effect can modify the parton fragmentation process [13–15]. One of the most popular methods to take into account the medium effect is the quenching weight fragmentation function in which the emission parton energy distribution is a Poisson distribution

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$$P(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int \mathrm{d}\omega_i \frac{\mathrm{d}I(\omega_i)}{\mathrm{d}\omega} \right] \delta \left(\epsilon - \sum_{i=1}^{n} \omega_i \right) \times \exp \left[- \int_0^{\infty} \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \right]. \tag{1}$$

We know that the probability distribution $P(\epsilon)$ has two components, a discrete part and a continuous part, with the form [6]

$$P(\epsilon) = p_0 \delta(\epsilon) + p(\epsilon), \tag{2}$$

where [12, 14]

$$p_0 = \exp \left[- \int \mathrm{d}\omega \int \mathrm{d}\boldsymbol{k}_\perp \frac{\mathrm{d}I^{\mathrm{med}}}{\mathrm{d}\omega \mathrm{d}\boldsymbol{k}_\perp} \right], \tag{3}$$

$$p(\epsilon) = p_0 \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int \mathrm{d}\omega_i \int \mathrm{d}\boldsymbol{k}_{\perp i} \frac{\mathrm{d}I^{\mathrm{med}}}{\mathrm{d}\omega_i \mathrm{d}\boldsymbol{k}_{\perp i}} \delta \left(\epsilon - \sum_{j=1}^{n} \frac{\omega_i}{E} \right). \tag{4}$$

Based on the above formulae, one can get the quenching weight fragmentation function

$$D^{\mathrm{med}}(x,t) \simeq p_0(t) D^{\mathrm{vac}}(x,t) + \int \frac{\mathrm{d}\epsilon}{1-\epsilon} p(\epsilon) D^{\mathrm{vac}} \left(\frac{x}{1-\epsilon}, t \right). \tag{5}$$

3 Medium-evolved fragmentation function

It is well known that the evolution equation of fragmentation function $D(x,t)$ is the DGLAP equation

$$t \frac{\partial}{\partial t} D(x,t) = \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_{\mathrm{s}}}{2\pi} P(z) D \left(\frac{x}{z}, t \right), \tag{6}$$

where $P(z)$ is a splitting function which describes the probability of a parton branching into two daughter partons with fractions of momenta z and $1-z$. To simplify the computation, we use the integral format of DGLAP equation (see Ref. [16])

$$D(x,t) = \Delta(t) D(x,T) + \Delta(t) \int_T^t \frac{\mathrm{d}t_1}{t_1} \frac{1}{\Delta(t_1)} \int \frac{\mathrm{d}z}{z} P(z) D \left(\frac{x}{z}, t_1 \right), \tag{7}$$

where $\Delta(t)$ is the Sudakov form factor

$$\Delta(t) = \exp \left[- \int_T^t \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \frac{\alpha_{\mathrm{s}}(t',z)}{2\pi} P(z,t') \right]. \tag{8}$$

The first term on the right-hand side of Eq. (7) corresponds to the contribution with no splittings between T and t while the second one gives the evolution when some finite amount of radiation is present.

3.1 Medium-modified splitting function and Sodakov factor

We suppose the medium modification to the total splitting function as [12]

$$P^{\mathrm{tot}}(z) = P^{\mathrm{vac}}(z) + \Delta P(z,t). \tag{9}$$

To get the medium modified splitting function, we first derive the vacuum splitting function, and then we use an analogous method to the vacuum case to get the medium modified splitting function. Let us look at the vacuum component of the energy distribution of emitted gluons [17]

$$\frac{\mathrm{d}I^{\mathrm{vac}}}{\mathrm{d}z \mathrm{d}\boldsymbol{k}_\perp^2} = \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{1}{k_\perp^2} P^{\mathrm{vac}}(z), \tag{10}$$

where

$$P^{\mathrm{vac}}(z) = \frac{1+z^2}{1-z} C_R \simeq \frac{2}{1-z} C_R. \tag{11}$$

Then we get

$$P^{\mathrm{vac}}(z, \boldsymbol{k}_\perp^2) = \frac{2\pi k_\perp^2}{\alpha_{\mathrm{s}}} \frac{\mathrm{d}I^{\mathrm{vac}}}{\mathrm{d}z \mathrm{d}\boldsymbol{k}_\perp^2}. \tag{12}$$

Similarly, we have

$$\Delta P(z,t) \simeq \frac{2\pi t}{\alpha_{\mathrm{s}}} \frac{\mathrm{d}I^{\mathrm{med}}}{\mathrm{d}z \mathrm{d}t}. \tag{13}$$

Substituting Eq. (9) into Eq. (8), we can get the medium modified Sudakov factor

$$\begin{aligned} &\Delta^{\mathrm{tot}}(t) \\ &= \exp \left[- \int_T^t \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \frac{\alpha_{\mathrm{s}}(t',z)}{2\pi} (P^{\mathrm{vac}}(z,t') + \Delta P(z,t')) \right] \\ &= \exp \left[- \int_T^t \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \frac{\alpha_{\mathrm{s}}(t',z)}{2\pi} P^{\mathrm{vac}}(z,t') \right] \\ &\quad \times \exp \left[- \int_T^t \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \frac{\alpha_{\mathrm{s}}(t',z)}{2\pi} \Delta P(z,t') \right] \\ &= \Delta^{\mathrm{vac}}(t) \Delta^{med}(t). \end{aligned} \tag{14}$$

3.2 Medium-evolved fragmentation function

Taking the medium modified splitting function and Sodakov factor into Eq. (7), one can get

$$\frac{D(x,t)}{\Delta(t)} = D(x,T) + \int_T^t \frac{\mathrm{d}t_1}{t_1} \int \frac{\mathrm{d}z_1}{z_1} P(z_1) \left[D \left(\frac{x}{z_1}, T \right) \right.$$

$$+\int_T^{t_1}\frac{\mathrm{d}t_2}{t_2}\int\frac{\mathrm{d}z_2}{z_2}P(z_2)\frac{D\left(\dfrac{x}{z_1z_2},T\right)}{\Delta(t_2)}\Bigg]$$

$$= D(x,T)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_T^{t_{i-1}}\frac{\mathrm{d}t_i}{t_i}\times\int\frac{\mathrm{d}z_i}{z_i}P(z_i)D\left(\frac{x}{z_1z_2\cdots z_n},T\right). \tag{15}$$

Here, we mark t as t_0 for convenience. Using the vacuum fragmentation function

$$D^{\mathrm{vac}}(x,T)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_T^{t_{i-1}}\frac{\mathrm{d}t_i}{t_i}\int\frac{\mathrm{d}z_i}{z_i}P(z_i)\times D^{\mathrm{vac}}\left(\frac{x}{z_1z_2\cdots z_n},T\right)=\frac{D^{\mathrm{vac}}(x,t)}{\Delta^{\mathrm{vac}}(t)} \tag{16}$$

and initial condition

$$D^{\mathrm{tot}}(x,T)=D^{\mathrm{vac}}(x,T), \tag{17}$$

Eq. (15) becomes

$$\frac{D^{\mathrm{tot}}(x,t)}{\Delta^{\mathrm{tot}}(t)} = D^{\mathrm{vac}}(x,T)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_T^{t_{i-1}}\frac{\mathrm{d}t_i}{t_i}\int\frac{\mathrm{d}z_i}{z_i}\times[P^{\mathrm{vac}}(z_i)+\Delta P(z_i)]\,D^{\mathrm{vac}}\left(\frac{x}{z_1z_2\cdots z_n},T\right). \tag{18}$$

After some algebra, we get a simplified medium-evolved fragmentation function as

$$D^{\mathrm{tot}}(x,t) \simeq \Delta^{\mathrm{med}}(t)D^{\mathrm{vac}}(x,t)+\Delta^{\mathrm{med}}(t)\int\frac{\mathrm{d}\epsilon}{1-\epsilon}\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\frac{\mathrm{d}t_i}{t_i}\int\mathrm{d}z_i\times\Delta P(z_i)\delta\left(\epsilon-\sum_{j=1}^{n}x_j\right)D^{\mathrm{vac}}\left(\frac{x}{1-\epsilon},t\right). \tag{19}$$

We note that to get Eq. (19) we use a trick

$$\prod_{i=1}^{n}\int_T^{t_{i-1}}\frac{\mathrm{d}t_i}{t_i} = \int_T^{t_0(=t)}\frac{\mathrm{d}t_1}{t_1}\int_T^{t_1}\frac{\mathrm{d}t_2}{t_2}\cdots\int_T^{t_{n-1}}\frac{\mathrm{d}t_n}{t_n} = \frac{1}{n!}\prod_{i=1}^{n}\int_T^{t}\frac{\mathrm{d}t_i}{t_i}. \tag{20}$$

4 Equivalence of two medium-modified fragmentation functions at high Q^2 limit

In the high-Q^2 limit, we show the equivalence of the quenching weight fragmentation function [18]

$$D^{\mathrm{med}}(x,t) \simeq p_0(t)D^{\mathrm{vac}}(x,t)+\int\frac{\mathrm{d}\epsilon}{1-\epsilon}p(\epsilon)D^{\mathrm{vac}}\left(\frac{x}{1-\epsilon},t\right), \tag{21}$$

where

$$p_0 = \exp\left(-\int\mathrm{d}\omega\int\mathrm{d}\boldsymbol{k}_\perp\frac{I^{\mathrm{med}}}{\mathrm{d}\omega\mathrm{d}\boldsymbol{k}_\perp}\right), \tag{22}$$

$$p(\epsilon) = p_0\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\mathrm{d}\omega_i\int\mathrm{d}\boldsymbol{k}_{\perp i}\frac{I^{\mathrm{med}}}{\mathrm{d}\omega_i\mathrm{d}\boldsymbol{k}_{\perp i}}\times\delta\left(\epsilon-\sum_{j=1}^{n}\frac{\omega_i}{E}\right), \tag{23}$$

and the medium-evolved fragmentation function

$$D^{\mathrm{med}}(x,t) \simeq \Delta^{\mathrm{med}}(t)D^{\mathrm{vac}}(x,t)+\Delta^{\mathrm{med}}(t)\int\frac{\mathrm{d}\epsilon}{1-\epsilon}\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\frac{\mathrm{d}t_i}{t_i}\int\mathrm{d}z_i\times\Delta P(z_i)\delta\left(\epsilon-\sum_{j=1}^{n}x_j\right)D^{\mathrm{vac}}\left(\frac{x}{1-\epsilon},t\right). \tag{24}$$

In order to get the quenching weight fragmentation function from the medium-evolved fragmentation function in the high-Q^2 limit, let us look at the the modified Sodakov factor in Eq. (24); we have

$$\Delta^{\mathrm{med}}(t) = \exp\left(-\int_T^t\frac{\mathrm{d}t'}{t'}\int\mathrm{d}z\frac{\alpha_\mathrm{s}}{2\pi}\Delta P(z,t')\right) = \exp\left(-\int_T^t\frac{\mathrm{d}t'}{t'}\int\mathrm{d}z\frac{\alpha_\mathrm{s}}{2\pi}\frac{2\pi t'}{\alpha_\mathrm{s}}\frac{\mathrm{d}I^{\mathrm{med}}}{\mathrm{d}z\mathrm{d}t'}\right) = p_0. \tag{25}$$

Now we can see that the first term in Eq. (24) is the same as the first term in Eq. (21) quenching weight fragmentation function. To further simplify Eq. (24), we substitute ΔP from Eq. (13) and Δ^{med} from Eq. (25) into Eq. (24), then we obtain the quenching weight fragmentation function from the medium-evolved fragmentation function

$$
\begin{aligned}
D^{\mathrm{med}}(x,t) &\simeq p_0(t)D^{\mathrm{vac}}(x,t)+\int\frac{\mathrm{d}\epsilon}{1-\epsilon}p_0\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\mathrm{d}\omega_i\int\mathrm{d}\boldsymbol{k}_{\perp i}\frac{\mathrm{d}I^{\mathrm{med}}}{\mathrm{d}\omega_i\mathrm{d}\boldsymbol{k}_{\perp i}}\delta\left(\epsilon-\sum_{j=1}^{n}\frac{\omega_i}{E}\right)D^{\mathrm{vac}}\left(\frac{x}{1-\epsilon},t\right)\\
&\simeq p_0(t)D^{\mathrm{vac}}(x,t)+\int\frac{\mathrm{d}\epsilon}{1-\epsilon}p(\epsilon)D^{\mathrm{vac}}\left(\frac{x}{1-\epsilon},t\right).
\end{aligned}
\tag{26}
$$

The above equation is exactly the same as the quenching weight fragmentation function. Up to now we have shown the equivalence of the quenching weight fragmentation function and the medium-evolved fragmentation function from the theoretical point of view in the high-Q^2 limit. We also study the numerical computation of their equivalence, which is shown in Fig. 1 and Fig. 2. Fig. 1 and Fig. 2 show the ratio of the medium modified fragmentation function to the vacuum fragmentation function for gluons onto pions computed with $E_{\mathrm{jet}}=40$ GeV, $\hat{q}=1$ GeV^2/fm, $L=6$ fm and $E_{\mathrm{jet}}=100$ GeV, $\hat{q}=1$ GeV^2/fm, $L=6$ fm respectively. We can see the discrepancy between QW and ACSX decreasing with increasing Q^2. The ratio of the QW and ACSX is plotted in Fig. 3. It tells us that the QW and ACSX are equivalent to each other in the high-Q^2 limit.

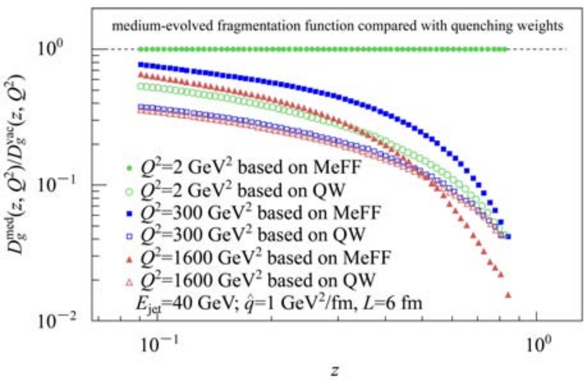

Fig. 1. Ratio of the medium modified fragmentation function to the vacuum fragmentation function with transport coefficient $\hat{q}$=1 GeV^2/fm, path length L =6 fm and jet energy E_{jet}=40 GeV, where z is the energy fraction, the ratio of the energy of the final particle to the energy of the jet (or parton). The subscript g is for gluon.

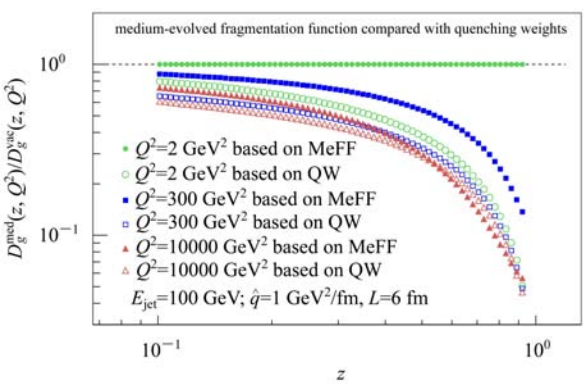

Fig. 2. Ratio of the medium modified fragmentation function to the vacuum fragmentation function with the same characteristic as for the plot in Fig. 1 except jet energy E_{jet}=100 GeV, where z is the energy fraction, the ratio of the energy of the final particle to the energy of the jet (or parton). The subscript g is for gluon.

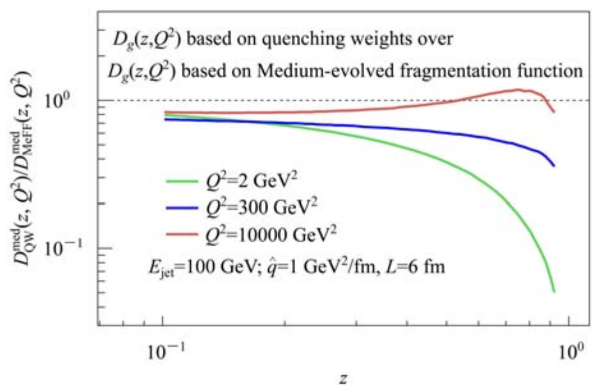

Fig. 3. Ratio of QW to ACSX fragmentation function for gluons onto pions computed with $E_{\mathrm{jet}}=100$ GeV, $\hat{q}=1$ GeV^2/fm and $L=6$ fm, where z is the energy fraction, the ratio of the energy of the final particle to the energy of the jet (or parton).

5 Discussion

It is known that the energy distribution of emitted gluons in the QW formalism is a Poisson distribution, which is computed in the eikonal approximation. In the QW formalism, the multiple gluon radiation is considered to be independent. The QW separately treats the vacuum and medium contribution to the parton shower evolution, the vacuum radiation occurring after the 'medium' radiation and taking place in time after the fast parton leaves the medium, which looks artificial. The ACSX formalism treats the medium and vacuum contribution to the shower development on the same footing, which ensures the parton energy momentum conservation during evolution and takes into account the virtuality evolution based on the DGLAP evolution equation. These advantages enable the ACSX to have wider applications.

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