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Numerical optimization and multi-particle dynamics simulation of the radial matching section of the RFQ *

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Abstract: The ABC code is an optimization program for the development of matching channels and dynamical matchers in radio frequency quadrupole (RFQ) structures, and a new approach to this code to define the geometry of the radial matching section of the RFQ has been developed. This approach is based on the application of the numerical optimization step by step. This optimization is intended to search for the initial matching condition of a beam, the optimization of parameters of a cell of the channel on given characteristic parameters and traces of a beam in linear channels in both forward and backward directions. To further verify the results of the optimization, multi-particle beam dynamics simulations have been carried out using the BEAMPATH and TRACK codes. The result of the beam dynamics simulation shows that the optimization result of the ABC code is reasonable and this approach provides an opportunity to redesign the structure of the radial matching section of the RFQ.

Key words: radial matching section, optimization, ABC code, BEAMPATH code, TRACK code

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1 Introduction

In regular RFQ cells, the local Twiss parameters depend on the time or radio frequency phase, therefore it is necessary to match a time-dependant beam at the entrance of the main part of RFQ. However, the injection beam has constant Twiss parameters and these parameters do not vary with time. Therefore, the matching of a continuous beam into regular accelerating cells causes a special problem. It has been suggested that providing the transverse matching of the beam with the regular accelerating channel using the radial matching section will solve the problem, but there are some particular laws applicable to the changing of focusing strength in the existing procedures. Therefore, users sometimes have difficulty in matching the low energy beam transport (LEBT) with the RFQ.

To provide more freedom for radial matching section design, the ABC code [1] has been developed at the Institute for Theoretical and Experimental Physics. The code has been made available to the Institute of Modern Physics by its authors for trial use only. The code aids the design of the vanes profile along the radial matching section to minimize the mismatching of the injected beam with the structure and provides the desired Twiss parameters of the ellipses at the entrance of the RFQ.

2 The matched beam envelope in an unmodulated channel

The equation of the transverse oscillation in the RFQ can be simplified in the following form [2]:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-\left(\frac{\omega}{\pi}\right)^2\left[K^2\cos(\omega t)-\frac{1}{2}\gamma_0\sin\varphi\right]x=0. \qquad (1)$$

The first term in the square brackets determines the spatially uniformed quadrupole focusing. In an

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uniform line, the modulation coefficient is 1 and the synchronous phase is chosen to be −90°, there is no acceleration or modulation. In a particular case, the solution of the set of the envelope equations is periodic, which corresponds to the envelopes of the matched beam.

The beam envelope equations can be solved numerically in the period of the structure by varying the initial conditions: $R_x(0)$, $\mathrm{d}R_x(0)/\mathrm{d}z$ and $R_y(0)$, $\mathrm{d}R_y(0)/\mathrm{d}z$ unless the solution in the end of the period coincides with the initial conditions: $R_x(0)=R_x(L)$, $\mathrm{d}R_x(0)/\mathrm{d}z=\mathrm{d}R_x(L)/\mathrm{d}z$ and $R_y(0)=R_y(L)$, $\mathrm{d}R_y(0)/\mathrm{d}z=\mathrm{d}R_y(L)/\mathrm{d}z$, ($L=\beta\lambda$), as in the case of a beam with negligible current, this beam is called the matched beam [3] and it occupies the smallest fraction of the channel aperture. Fig. 1 shows the beam envelopes of a matched beam along an unmodulated channel including 20 cells (10 $\beta\lambda$).

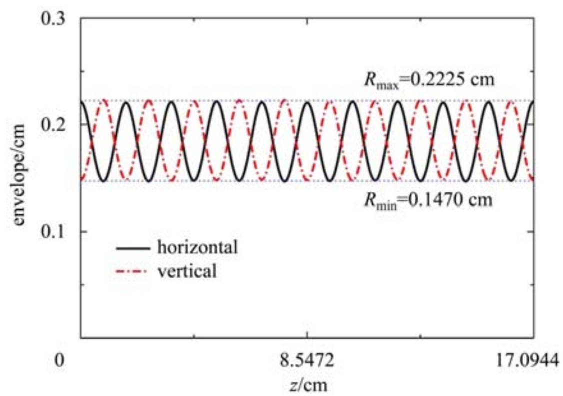

Fig. 1. (color online) The matched beam envelopes of the horizontal and the vertical directions along the unmodulated channel.

In this example, a 40 keV proton beam is injected into the unmodulated quadrupole channel. The frequency of the channel is 162.5 MHz. The value of the normalized transverse emittance is selected to be 0.15 π·cm·mrad and the initial particle distribution is chosen to be a water bag. In the simulation, 10000 macro-particles are fixed at z=0 cm plane at t=0 and the initial phase of the channel is chosen to be 0°. The matched initial transverse condition is calculated to be $R_x(0)$=0.2225 cm, $\mathrm{d}R_x(0)/\mathrm{d}z$=0 rad and $R_y(0)$=0.1470 cm, $\mathrm{d}R_y(0)/\mathrm{d}z$=0 rad.

There is a 180° phase shift between the beam envelope oscillations in the horizontal and the vertical directions. Therefore, a symmetric beam (the same Twiss parameters in the horizontal and the vertical directions) is required for the RFQ injection.

Figure 2 illustrates the transverse particle distribution at z=0, $L/4$, $L/2$ and $3L/4$ planes, respectively.

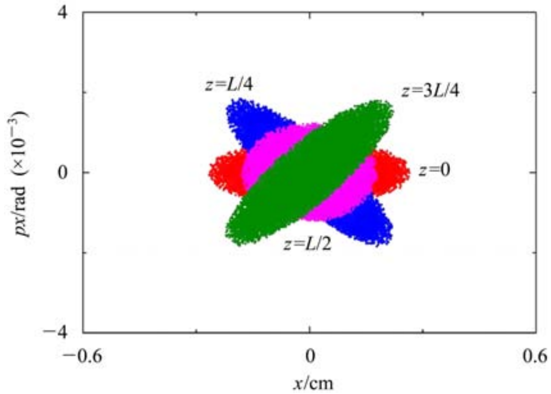

Fig. 2. (color online) The matched ellipses at the z=0, $L/4$, $L/2$, $3L/4$ planes in the x-px phase space.

The matched ellipse parameters vary with the RF phase or time and are periodical along the unmodulated channel. Therefore, it is necessary to provide a transition from a beam with time-independent feature to a beam that has the proper variations with time. The solution is to taper unmodulated vanes at the RFQ input so that the radius decreases and the focusing strength increases from near zero to its full value over a distance of a few cells. In this section, the quadrupole symmetry is maintained and this section is called the radial matching section.

3 Optimization using ABC code

The matched ellipse parameters are found in the interior cells of the RFQ in different phases that are 90° apart. They look very different and are shown in Fig. 2. The ellipses at different phases are tracked backwards and are very similar with a high degree of overlap. To obtain the best approximate match, the average of these ellipses is taken to be the matched ellipses at the RFQ input. It is the process of generating the radial matching section in the DESRFQ code. In the ABC code, the first regular cell is used to be the interior cell. The transverse phase advance of the cells in the radial matching section is optimized to match the desired input Twiss parameters. The smooth approximation of the transverse phase advance is given by the expression:

$$\mu^2 = \frac{2}{\pi^2}K^4+2\gamma_0\sin\varphi_{\mathrm{s}}, \tag{2}$$

$$K^2 = \left(\frac{\lambda}{2R_0}\right)^2\frac{ZU_{\mathrm{L}}}{A\varepsilon_0}, \quad \gamma_0=\pi^2\frac{ZU_{\mathrm{L}}T}{A\pi\beta^2\varepsilon_0}, \tag{3}$$

$$T = \frac{\pi}{4}\frac{\mathrm{m}^2-1}{m^2I_0\left(\dfrac{2\pi}{\beta\lambda}\cdot\dfrac{2R_0}{m+1}\right)+I_0\left(m\cdot\dfrac{2\pi}{\beta\lambda}\cdot\dfrac{2R_0}{m+1}\right)}. \tag{4}$$

Here, the parameter K represents the rigidity of the focusing channel, γ_0 is the defocusing factor and φ_s is the synchronous phase, λ is the wave length, R_0 is the average radius of the cell, Z is the charge number, A is the mass number, U_L is the vane voltage, and T is the acceleration efficiency. Since the vane voltage is fixed, the different radius produces a different focusing strength. In the optimization process, the focusing strength of every cell is varied so that a high degree of overlap for the desired input ellipse could be found. Fig. 3 illustrates the initial and optimized electrode shape of the radial matching section and Table 1 lists the matched injection Twiss parameters for the initial and optimized situations.

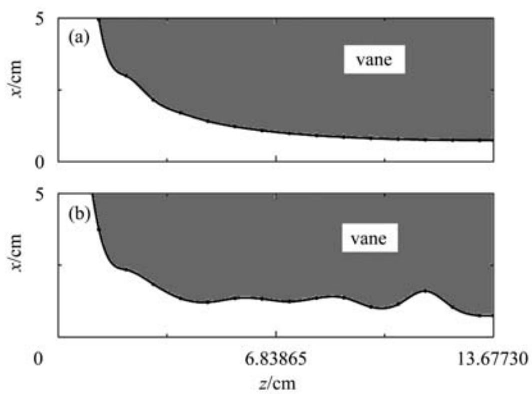

Fig. 3. (a) The original radial matching section. (b) The optimized radial matching section.

Table 1. The matched Twiss parameters for the original and optimized situations

parameter	α	β/(cm/rad)	mismatch
initial	1.488	14.50	1.014
optimized	0.117	15.02	1.055

After optimization, the convergent angle of the matching beam has been significantly decreased. An approximately parallel beam could meet the requirement of the injection. It makes the matching between the LEBT and the RFQ much easier. More detailed beam dynamics simulation has been done to verify the feasibility of the optimized solutions.

4 Simulation using the BEAMPATH code

The three different injections of the radial matching section have been simulated for comparison: (1) the input Twiss parameters are α=1.488, β=14.50 cm/rad, and the original shape of the electrode is selected. (2) The input Twiss parameters are α=0.117, β=15.02 cm/rad and the original shape of the electrode is selected. (3) The input Twiss parameters are α=0.117, β=15.02 cm/rad and the optimized shape of the electrode is selected. There is a 180° phase shift between the horizontal direction and the vertical direction. For a symmetric beam, if the horizontal beam could match the RFQ regular cells, the vertical beam could match the regular cells at the same time. In the following simulation only the transverse phase space is studied. Fig. 4 depicts the distributions at the end of the radial matching section.

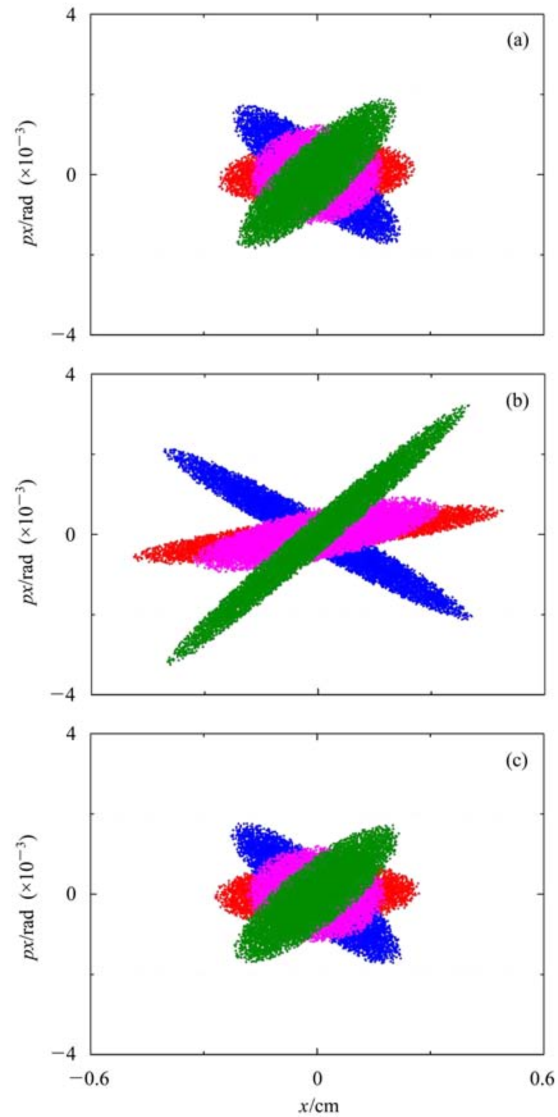

Fig. 4. (color online) The pictures of (a), (b) and (c) are the particle distributions at the end of the radial matching section for the three different injection conditions mentioned above.

The multi-particle dynamics simulation of the radial matching section is performed using the BEAMPATH code [4]. At t=0, 10000 macro-particles are fixed at the z=0 cm plane and the initial phase of the RFQ is chosen to be 0, 90, 180 and 270°, respectively. Since there are four initial phases, four different ellipses are formed. In the (1) and (3) situations, the injection beam still matches the RFQ well and the particle distributions are similar to the particle distributions in Fig. 2. This means that the particles from the radial matching section can be smoothly accepted by the following RFQ regular cells.

The distributions of the second situation are shown in Fig. 4(b), the beam from the radial matching section is mismatched with the following RFQ regular cells. The particle distributions are asymmetric and the region occupied by the particles is larger compared with the area of Fig. 2. Further beam dynamics simulation has been done in the channel composed of the radial matching section and the following pre-buncher section. The envelope oscillations for the four initial phases in the three injection situations are shown in Fig. 5. The part before the dotted line represents the radial matching section and the subsequent part the pre-buncher section.

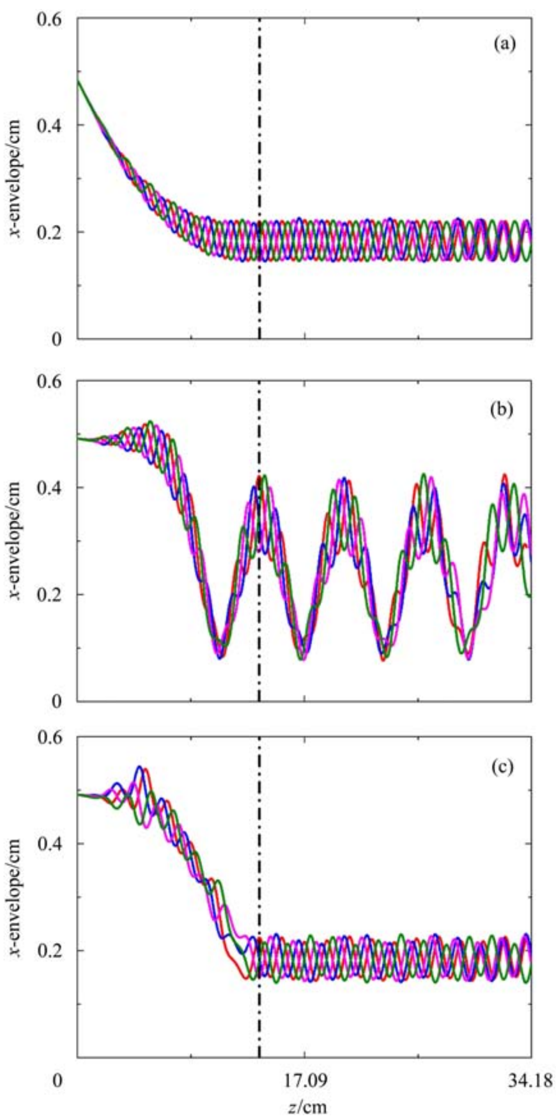

Fig. 5. (color online) The pictures of (a), (b) and (c) are the beam envelope oscillations for the three injection situations.

As shown in Fig. 5(a) and Fig. 5(c), the beam envelope in the regular cells is approximately a straight line. The maximum beam size of the following regular cells section is about 0.2256 cm and 0.2302 cm, respectively. In Fig. 5(b), the beam envelope is not a periodic oscillation in the regular cells, which corresponds to the mismatched beam. The maximum beam size of the regular cells is 0.4126 cm, much larger than the intrinsic matched beam size. The simulation results of the BEAMPATH code are consistent with the results of the ABC code.

5 Simulation using TRACK code

The multi-particle dynamics simulation of the whole RFQ is performed using the TRACK code [5]. The TRACK code tracks the particles through the whole RFQ in the three dimensional fields and the field in the regular section is presented by an 8-term Fourier-Bessel expression, which is the same as the PARMTEQ-M code [6]. The beam dynamic simulations in the whole RFQ for the two matched injections using the TRACK code are shown in Fig. 6.

The particle motion in a realistic RFQ field is nonlinear and most comprehensive beam dynamics studies can be carried out by numerical simulation. In the simulation, 10000 macro-particles are selected, an RF period scale of the continuous beam (the total phase width is 360 degrees in the frequency of 162.5 MHz) with the 0.1% energy spread is chosen. The input transverse emittance and the particle distribution are the same as the above simulation using the BEAMPATH code.

In Fig. 6, the pairs of lower curves are the beam rms sizes, the upper curves are the beam envelopes. The beam parameters at the exit of the RFQ are illustrated in Table 2. The unit of the normalized transverse emittance is π·cm·mrad.

Table 2. The beam parameters at the exit of the RFQ.

emittance	input	original	optimized
ε_{nx}(4rms)	0.10038	0.10052	0.10073
ε_{ny}(4rms)	0.10000	0.09976	0.10028
ε_{nx}(99.5%)	0.14065	0.14128	0.14383
ε_{ny}(99.5%)	0.14053	0.14093	0.14390

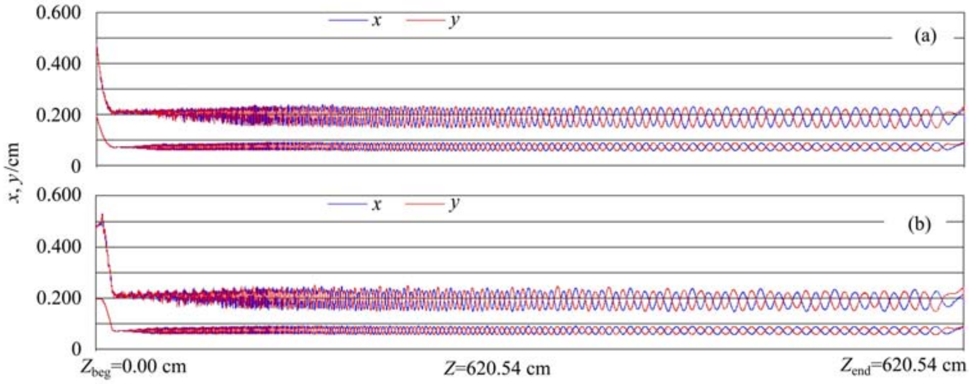

Fig. 6. (color online) (a) The beam envelopes of the original injection. (b) The beam envelopes of the optimized injection.

The beam dynamics simulation results show that both the convergent beam injection with the original shape of electrode and the approximately parallel beam injection with the optimized shape of electrodes can produce a low transverse emittance output and the acceleration efficiencies in two different injections are very high. The beam envelope of the main RFQ part in Fig. 6(b) is slightly larger than the beam envelope in Fig. 6(a). As a result, the approximately parallel beam injection could get a slightly higher transverse emittance growth than the convergent beam injection. The longitudinal emittance output is almost the same in the two injection situations. In Fig. 6(a) and Fig. 6(b), the rms beam envelopes in the main part of the RFQ are a straight line, the continuous time-independent beam could be matched perfectly with the RFQ regular cells by the radial matching section.

6 Conclusion

The matching problem between the LEBT and the RFQ is very important for suppressing the transverse emittance growth in the RFQ. A new approach has been developed for the optimization of the shape of the electrodes in the radial matching section of the RFQ. The beam dynamics simulation results of the BEAMPATH and the TRACK codes show that the optimized radial matching section can match the approximately parallel beam very well between the LEBT and the main part of RFQ. There is no rms transverse emittance growth and a high acceleration efficiency is obtained.

Approximately parallel beam injection has many advantages for the match. For instance, in the design of the LEBT, a solenoid in front of the RFQ is employed for the injection. If the drift between the solenoid and the RFQ is fixed, then the larger convergent angle implies a larger beam size in the solenoid. According to previous research [7], a large beam size in the solenoid would cause an impressive transverse emittance growth. The optimized radial matching section could transport the small convergent angle beam to the regular RFQ cells smoothly and the beam from the RFQ could still maintain a good quality, the same as the large convergent beam injection.

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