

Casimir force on a piston at finite temperature in Randall-Sundrum models^{*}

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Abstract: The Casimir effect for a three-parallel-plate system at finite temperature within the framework of five-dimensional Randall-Sundrum models is studied. In the case of the Randall-Sundrum model involving two branes we find that the Casimir force depends on the plate distance and temperature after one outer plate has been moved to a distant place. Further we discover that the sign of the reduced force is negative if the plate and piston are located close together, but the nature of reduced force becomes repulsive when the plate distance is not very small and finally the repulsive force vanishes with extremely large plate separation. A higher temperature causes a greater repulsive Casimir force. Within the framework of a one-brane scenario the reduced Casimir force between the piston and one plate remains attractive no matter how high the temperature is. It is interesting that a stronger thermal effect leads to a greater attractive Casimir force instead of changing the nature of the force.

Key words: Casimir effect, Randall-Sundrum models, high-dimensional spacetime

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1 Introduction

More than 80 years ago the high-dimensional spacetime theory suggesting that our observable four-dimensional world is a subspace of a higher dimensional spacetime that has a long tradition was started by Kaluza and Klein [1, 2]. The high-dimensional spacetime models including dimensionality, topology and the geometric characteristics of extra dimensions are necessary. The main motivations for such approaches are to unify all of the fundamental interactions in nature. The issues with additional dimensions are also invoked for providing a breakthrough of cosmological constant and the hierarchy problems [3–8]. These models of high-dimensional spacetimes have their own compactification and properties of extra dimensions. More theories need to be developed and realized within the framework of extra dimensions. In the Kaluza-Klein theory, one extra dimension in our universe was introduced to be compactified to unify gravity and classical electrodynamics. Quantum gravity such as string theories or the braneworld scenario are developed to reconcile quantum mechan-

ics and gravity with the help of introducing seven extra spatial dimensions. The new approaches propose that the strong curvature of the extra spatial dimensions be responsible for the hierarchy problem. At first, the large extra dimensions (LED) were put forward [6]. In this model the additional dimensions are flat and of equal size and the radius of a toroid is limited to overcome the large gap between the scales of gravity and electroweak interaction while the size of extra space cannot be too small, or the hierarchy problem remains. Another model with warped extra dimensions was introduced [7, 8]. A five-dimensional theory compactified on a S^1/Z_2 manifold, named the Randall-Sundrum (RS) model, suggested a compact extra dimension with large curvatures to explain the reason why the large gap between the Planck and the electroweak scales exists. Here we choose the RS model as a five-dimensional theory compactified on a S^1/Z_2 manifold with bulk and boundary cosmological constants leading to a stable four-dimensional low-energy effective theory. In RSI, one of the RS models, there are two 3-branes with equal and opposite tensions and they are localized at $y=0$ and

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$y = L_0$ respectively, with Z_2 symmetry $y \longleftrightarrow -y$, $L_0 + y \longleftrightarrow L_0 - y$. The Randall-Sundrum model becomes RSII when one brane is located at infinity, $L_0 \longrightarrow \infty$. The standard model field and gauge fields live on the negative tension brane which is visible, while the positive tension brane with a fundamental scale M_{RS} is hidden.

The Casimir effect depends on the dimensionality and topology of spacetime [9–18] and has received a great deal of attention within spacetime models including additional spatial dimensions. There exists a strong influence from the size and the geometry of extra dimensions on the Casimir effect, the evaluation of the vacuum zero-point energy. The precision of the measurements of the attractive force between parallel plates as well as other geometries has been greatly improved practically [19–22], leading the Casimir effect to be a remarkable observable and trustworthy consequence of the existence of quantum fluctuations. The experimental results clearly show that the attractive Casimir force between the parallel plates vanishes when the plates move apart from each other to the very distant place. In particular it must be pointed out that no repulsive force appears. Therefore the Casimir effect for parallel plates can become a window to probe the high-dimensional Universe and can be used to research on a large class of related topics on the various models of spacetimes with more than four dimensions. More efforts have been made on the studies. Within the framework of several kinds of spacetimes with high dimensionality the Casimir effect for various systems has been discussed. The electromagnetic Casimir effect for parallel plates in a high-dimensional spacetime has been studied and the subtraction of the divergences in the Casimir energy at the boundaries is realized [23, 24]. Some topics were studied in high-dimensional spacetime described by Kaluza-Klein theory. It is shown analytically that the extra-dimension corrections to the Casimir effect for a rectangular cavity in the presence of compactified universal extra dimensions are strongly manifest [25]. More attention has been paid to the Casimir effect for the parallel-plate system in the background governed by Kaluza-Klein theory [25–37]. It is also proved rigorously that there will appear a repulsive Casimir force between two parallel plates when the plate distance is sufficiently large in the spacetime with compactified additional dimensions, and the higher the dimensionality, the greater the repulsive force, unless the Casimir energy outside the system consisting of two parallel plates is considered. It should be pointed out that the Casimir force is

modified by the compactified dimensions and the repulsive part of the modifications has nothing to do with the positions of the plates, so the repulsive parts of the Casimir force on the plates must be cancelled. In the case of a piston in the same environment, the Casimir force keeps attractive and more extra compactified dimensions cause a greater attractive force. The research on the Casimir energy within the framework of Kaluza-Klein theory to explain dark energy has been performed and is also fundamental, and a lot of progress has been made [38]. In the context of string theory the Casimir effect was also investigated [39–42]. Also in the Randall-Sundrum model, the Casimir effect has been investigated to stabilize the distance between branes [43–47]. In particular the evaluation of the Casimir force between two parallel plates under Dirichlet conditions has been performed in Randall-Sundrum models with one extra dimension [48–51]. We declare that the nature of the Casimir force between the piston and its closest plate becomes repulsive in the RSI model as the plate distance is larger than the separation between two branes [51]. In the case of RSII, the Casimir force between the piston and its nearest plate remains attractive while the influence from the warped dimension on the Casimir force between the two parallel plates is so small that it can be neglected.

The quantum field theory shares many of the effects at finite temperature. Thermal influence on the Casimir effect is manifest in many cases [10, 18, 27, 52–57]. The influence of a sufficiently high temperature can even change conclusions completely. The stronger thermal influence can lead the Casimir energy to be positive and the Casimir force to be repulsive in a system consisting of two parallel plates in two backgrounds with or without extra dimensions. The conclusions about the Casimir effect for a device with a piston in the Randall-Sundrum models mentioned above are drawn when the temperature is zero. It is necessary to investigate the Casimir effect for parallel plates in the Randall-Sundrum models under a nonzero temperature environment. We must confirm how the thermal influence modifies the results.

It is fundamental and significant to study the Casimir force on the piston at finite temperature in the Randall-Sundrum models. Now we choose a piston device depicted in Fig. 1. One plate, called a piston, is inserted into a two-parallel-plate system and is parallel to the plates, dividing the system into two parts labelled by A and B respectively. In part A the distance between the left plate and the piston is a , the remains of the separation of the two original

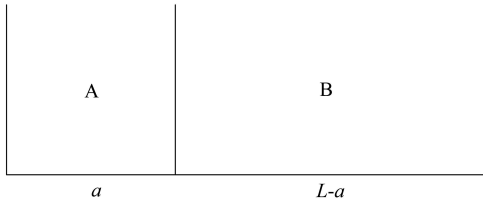


Fig. 1. Casimir piston.

plates is certainly $L - a$, which means that L denotes the total plate separation. The total vacuum energy density of the massless scalar fields obeying Dirichlet boundary conditions within the region involving a piston shown in Fig. 1 can be written as the sum of three terms,

$$\varepsilon = \varepsilon^A(a, T) + \varepsilon^B(L - a, T) + \varepsilon^{\text{out}}(T), \quad (1)$$

where $\varepsilon^A(a, T)$ and $\varepsilon^B(L - a, T)$ mean the energy density of Part A and B respectively, and the two terms depend on the temperature and their own size in these two parts. The term $\varepsilon^{\text{out}}(T)$ represents the vacuum energy density outside the system under thermal influence and is independent of characteristics inside the system. Having regularized the total vacuum energy density, we obtain the Casimir energy density,

$$\varepsilon_C = \varepsilon_R^A(a, T) + \varepsilon_R^B(L - a, T) + \varepsilon_R^{\text{out}}(T), \quad (2)$$

where $\varepsilon_R^A(a, T)$, $\varepsilon_R^B(L - a, T)$ and $\varepsilon_R^{\text{out}}(T)$ denote the finite parts of terms $\varepsilon^A(a, T)$, $\varepsilon^B(L - a, T)$ and $\varepsilon^{\text{out}}(T)$ in Eq. (1) respectively. It should be pointed out that $\varepsilon^{\text{out}}(T)$ is not a function of the position of the piston although it depends on the environment temperature. Further, the Casimir force per unit area on the piston is given with the help of a derivative of the Casimir energy density with respect to plate distance like $f'_C = -\frac{\partial \varepsilon_C}{\partial a}$ and can be written as,

$$f'_C = -\frac{\partial}{\partial a} [\varepsilon_R^A(a, T) + \varepsilon_R^B(L - a, T)]. \quad (3)$$

showing that the contribution of vacuum energy from the exterior region does not modify the Casimir force on the piston. According to previous studies, we should point out that the piston analysis is a correct way to perform the parallel-plate calculation because we cannot neglect the contribution to the vacuum energy from the area outside the confined region. Further, we wonder how the thermal influence modifies the Casimir effect for parallel plates in the models. This problem, to our knowledge, has not been examined. The main purpose of this paper is to research on the Casimir force between two parallel plates when the environment temperature does not vanish in the Randall-Sundrum models. We obtain

the Casimir force on a piston in the system consisting of three parallel plates with a nonzero temperature by means of the zeta-function regularization in the RSI and RSII models respectively. We also compute the Casimir force in the limit that one outer plate is moved to the extremely distant place. We discuss the dependence of the reduced force on temperature and compare our results with those with vanishing temperature and measurements. Our discussions and conclusions are listed at the end.

2 The Casimir effect for a piston at finite temperature in the RSI models

Here we discuss a massless scalar field living in the bulk at nonzero temperature in the RS models. Within the framework the spacetime metric is chosen as,

$$ds^2 = e^{-2k|y|} g_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (4)$$

where k is assumed to be of the order of the Planck scale which governs the degree of curvature of the AdS_5 with a constant negative curvature. That the extra dimension is compactified on an orbifold gives rise to the generation of the absolute value of y in the metric. The imaginary time formalism can be used to describe the scalar fields in thermal equilibrium [27, 51–54]. In the five-dimensional RS models we introduce a partition function for a system,

$$Z = N \int_{\text{periodic}} D\Phi \exp \left[\int_0^\beta d\tau \int d^3x dy \mathcal{L}(\Phi, \partial_\varepsilon \Phi) \right], \quad (5)$$

where \mathcal{L} is the Lagrangian density for the system under consideration, N a constant and “periodic” means $\Phi(0, x^\mu, y) = \Phi(\beta, x^\mu, y)$. Here $\beta = \frac{1}{T}$ is the inverse of the temperature. In five-dimensional spacetime with the background metric denoted in Eq. (4), the equation of motion for a massless bulk scalar field Φ is,

$$g^{\mu\nu} \partial_\mu \partial_\nu \Phi - e^{2ky} \partial_y (e^{-4ky} \partial_y \Phi) = 0, \quad (6)$$

where $g^{\mu\nu}$ is the usual four-dimensional flat metric with signature -2 . The field confining between the two parallel plates satisfies the Dirichlet boundary conditions $\Phi(x^\mu, y)|_{\partial\Omega} = 0$, $\partial\Omega$ positions of the plates in coordinates x^μ . Following Ref. [50], we can choose the y -dependent part of the field $\Phi(x^\mu, y)$ as $\chi^{(N)}(y)$ in virtue of separation of variables.

The general expression for nonzero modes can be obtained in terms of Bessel functions of the first and

second kind as,

$$\chi^{(N \neq 0)}(y) = e^{2ky} \left(a_1 J_2 \left(\frac{m_N e^{ky}}{k} \right) + a_2 Y_2 \left(\frac{m_N e^{ky}}{k} \right) \right), \tag{7}$$

where a_1 and a_2 are arbitrary constants. The effective mass term for the scalar field m_N can be obtained by means of integration of the fifth dimension y . In the case of the RSI model, the hidden and visible 3-branes are located at $y = 0$ and $y = \pi R$ respectively. Here we choose $L_0 = \pi R$. According to the modified Neumann boundary conditions

$$\left. \frac{\partial \chi^{(N)}}{\partial y} \right|_{y=0} = \left. \frac{\partial \chi^{(N)}}{\partial y} \right|_{y=\pi R} = 0,$$

a general reduced equation reads,

$$m = m_N \approx \kappa \left(N + \frac{1}{4} \right), \tag{8}$$

where

$$\kappa = \pi k e^{-\pi k R}, \tag{9}$$

here we assume $N \gg 1$ or equivalently $\pi k R \gg 1$ throughout our work.

The modes of the vacuum for parallel plates under the Dirichlet and modified Neumann boundary conditions for plate positions and brane locations respectively as mentioned above in the RSI model at finite temperature can be expressed as,

$$\omega_{nNl} = \sqrt{p^2 + \left(\frac{n\pi}{D} \right)^2 + m_N^2 + \left(\frac{2\pi l}{\beta} \right)^2}, \tag{10}$$

where

$$p^2 = p_1^2 + p_2^2, \tag{11}$$

where p_1 and p_2 are the wave vectors in the directions of the unbound space coordinates parallel to the plate surface and d is the distance between the plates. Here n and N represent positive integers and l stands for an integer. The generalized zeta function reads,

$$\begin{aligned} \zeta_I(s; -\partial_E) &= \text{Tr}(-\partial_E)^{-s} \\ &= \int d^2 k \sum_{n=1}^{\infty} \sum_{N=1}^{\infty} \sum_{l=-\infty}^{\infty} \left[k^2 + \frac{n^2 \pi^2}{D^2} \right. \\ &\quad \left. + \kappa^2 \left(N + \frac{1}{4} \right)^2 + \left(\frac{2l\pi}{\beta} \right)^2 \right]^{-s}, \end{aligned} \tag{12}$$

where

$$\partial_E = \frac{\partial^2}{\partial \tau^2} + \nabla^2$$

with $\tau = it$. Furthermore, Eq. (12) can also be expressed in terms of the zeta functions of Epstein-Hurwitz type,

$$\begin{aligned} &\zeta_I(s; -\partial_E) \\ &= \frac{2\pi\Gamma(s-1)}{\Gamma(s)} E_3 \left(s-1; \frac{\pi^2}{D^2}, \kappa^2, \frac{4\pi^2}{\beta^2}; 0, \frac{1}{4}, 0 \right) \\ &\quad - \frac{2\pi\Gamma(s-1)}{\Gamma(s)} E_2^{\frac{\kappa^2}{16}} \left(s-1; \frac{\pi^2}{D^2}, \frac{4\pi^2}{\beta^2}; 0, 0 \right) \\ &\quad - \frac{\pi\Gamma(s-1)}{\Gamma(s)} E_2 \left(s-1; \frac{\pi^2}{D^2}, \kappa^2; 0, \frac{1}{4} \right) \\ &\quad + \frac{\pi\Gamma(s-1)}{\Gamma(s)} E_1^{\frac{\kappa^2}{16}} \left(s-1; \frac{\pi^2}{D^2}; 0 \right) \\ &\quad - \frac{2\pi\Gamma(s-1)}{\Gamma(s)} E_2 \left(s-1; \kappa^2, \frac{4\pi^2}{\beta^2}; \frac{1}{4}, 0 \right) \\ &\quad + \frac{2\pi\Gamma(s-1)}{\Gamma(s)} E_1^{\frac{\kappa^2}{16}} \left(s-1; \frac{4\pi^2}{\beta^2}; 0 \right) \\ &\quad + \frac{\pi\Gamma(s-1)}{\Gamma(s)} \kappa^{2-2s} \zeta_H \left(s-1, \frac{1}{4} \right), \end{aligned} \tag{13}$$

where the zeta functions of Epstein-Hurwitz type are defined as,

$$\begin{aligned} &E_p^{b_1 b_2 \dots b_p}(s; a_1, a_2, \dots, a_p; c_1, c_2, \dots, c_p) \\ &= \sum_{\{n_j\}=0}^{\infty} \left\{ \sum_{j=1}^p [a_j(n_j + c_j)^2 + b_j] \right\}^{-s}, \end{aligned} \tag{14}$$

$$\begin{aligned} &E_p(s; a_1, a_2, \dots, a_p; c_1, c_2, \dots, c_p) \\ &= \sum_{\{n_j\}=0}^{\infty} \left(\sum_{j=1}^p a_j(n_j + c_j)^2 \right)^{-s}, \end{aligned} \tag{15}$$

and

$$\zeta_H(s, q) = \sum_{n=0}^{\infty} (n+q)^{-s}$$

is the Hurwitz zeta function. The energy density of the two-parallel-plate system with thermal corrections is,

$$\varepsilon_I(D, T) = -\frac{1}{2} \frac{\partial}{\partial \beta} \left(\frac{\partial \zeta_I(s; -\partial_E)}{\partial s} \Big|_{s=0} \right). \tag{16}$$

We regularize the expression of the vacuum energy density of the system containing parallel plates to obtain its finite part at a nonzero temperature in the RSI model as follows,

$$\begin{aligned}
\varepsilon_{\text{IR}}(D, T) = & -\frac{\Gamma(4)}{64\pi^{\frac{5}{2}}\Gamma\left(\frac{5}{2}\right)}\kappa^3\sum_{n=1}^{\infty}\frac{\cos n\pi}{(2n)^4}-\frac{\pi^{\frac{3}{2}}}{512\Gamma\left(\frac{5}{2}\right)}\kappa^3 \\
& -\frac{\kappa^2}{2D}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty}n_1^{-2}\left(n_2+\frac{1}{4}\right)^2K_2\left(2n_1\kappa D\left(n_2+\frac{1}{4}\right)\right)+\frac{\pi}{32}\frac{\kappa^2}{D}\sum_{n=1}^{\infty}n^{-2}K_2\left(\frac{\kappa D}{2\sqrt{\pi}}n\right) \\
& +2^{\frac{1}{2}}\pi^{\frac{1}{2}}\beta^{-\frac{3}{2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}n_1^{-\frac{3}{2}}\left[\frac{n_2^2\pi^2}{D^2}+\kappa^2\left(n_3+\frac{1}{4}\right)^2\right]^{\frac{3}{4}}K_{\frac{3}{2}}\left[n_1\beta\sqrt{\frac{\pi^2n_2^2}{D^2}+\kappa^2\left(n_3+\frac{1}{4}\right)^2}\right] \\
& -2^{\frac{1}{2}}\pi^2\beta^{-\frac{3}{2}}\sum_{n_1, n_2=1}^{\infty}\left(\frac{\pi^2n_1^2+\kappa^2}{D^2+\frac{\kappa^2}{16}}\right)^{\frac{3}{4}}K_{\frac{3}{2}}\left(n_1\beta\sqrt{\frac{\pi^2n_2^2}{D^2}+\frac{\kappa^2}{16}}\right) \\
& +2^{\frac{1}{2}}\pi^{\frac{1}{2}}\beta^{-\frac{1}{2}}\sum_{n_1=1}^{\infty}\sum_{n_2, n_3=0}^{\infty}n_1^{-\frac{1}{2}}\left[\frac{\pi^2n_2^2}{D^2}+\kappa^2\left(n_3+\frac{1}{4}\right)^2\right]^{\frac{5}{4}} \\
& \times\left[K_{\frac{1}{2}}\left(n_1\beta\sqrt{\frac{\pi^2n_2^2}{D^2}+\kappa^2\left(n_3+\frac{1}{4}\right)^2}\right)+K_{\frac{5}{2}}\left(n_1\beta\sqrt{\frac{\pi^2n_2^2}{D^2}+\kappa^2\left(n_3+\frac{1}{4}\right)^2}\right)\right] \\
& -2^{\frac{1}{2}}\pi^{\frac{1}{2}}\beta^{-\frac{1}{2}}\sum_{n_1, n_2=1}^{\infty}n_1^{-\frac{1}{2}}\left(\frac{\pi^2n_2^2+\kappa^2}{D^2+\frac{\kappa^2}{16}}\right)^{\frac{5}{4}}\left[K_{\frac{1}{2}}\left(n_1\beta\sqrt{\frac{\pi^2n_2^2}{D^2}+\frac{\kappa^2}{16}}\right)+K_{\frac{5}{2}}\left(n_1\beta\sqrt{\frac{\pi^2n_2^2}{D^2}+\frac{\kappa^2}{16}}\right)\right] \\
& -8\pi^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)\zeta(3)\beta^{-3}\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi}{2}}{n}-12\pi\beta^{-4}\Gamma(2)\zeta(4), \tag{17}
\end{aligned}$$

where $K_\nu(z)$ is the modified Bessel function of the second kind. We replace the variable D in Eq. (17) with a and $L-a$ to obtain the Casimir energy densities of Part A and Part B like,

$$\varepsilon_{\text{IR}}^{\text{A}}(a, T) = \varepsilon_{\text{IR}}(a, T), \quad \varepsilon_{\text{IR}}^{\text{B}}(L-a, T) = \varepsilon_{\text{IR}}(L-a, T). \tag{18}$$

We obtain the Casimir force per unit area on the piston at finite temperature in the cosmological background like the RSI model as follows,

$$\begin{aligned}
f'_{\text{IC}} = & -\frac{\partial}{\partial a}[\varepsilon_{\text{IR}}^{\text{A}}(a, T) + \varepsilon_{\text{IR}}^{\text{B}}(L-a, T)] = -\frac{\kappa^2}{2a^2}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty}n_1^{-2}\left(n_2+\frac{1}{4}\right)^2\left[2n_1\kappa a\left(n_2+\frac{1}{4}\right)\right] \\
& -\frac{\kappa^3}{2a}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty}n_1^{-1}\left(n_2+\frac{1}{4}\right)^3\left[K_1\left(2n_1\kappa a\left(n_2+\frac{1}{4}\right)\right)+K_3\left(2n_1\kappa a\left(n_2+\frac{1}{4}\right)\right)\right] \\
& +\frac{1}{32}\frac{\kappa^3}{a}\sum_{n=1}^{\infty}n^{-2}K_2\left(\frac{\kappa a}{2\sqrt{\pi}}n\right)+\frac{\pi^{\frac{1}{2}}}{128}\frac{\kappa^3}{a}\sum_{n=1}^{\infty}n^{-1}\left[K_1\left(\frac{\kappa a}{2\sqrt{\pi}}n\right)+K_3\left(\frac{\kappa a}{2\sqrt{\pi}}n\right)\right] \\
& +\frac{3\sqrt{2}\pi^{\frac{5}{2}}}{2}\frac{1}{a^{\frac{5}{2}}\beta^{\frac{3}{2}}}\sum_{n_1=1}^{\infty}\sum_{n_2, n_3=0}^{\infty}n_1^{-\frac{3}{2}}n_2^2\left[\pi^2n_2^2+\kappa^2a^2\left(n_3+\frac{1}{4}\right)^2\right]^{-\frac{1}{4}}\times K_{\frac{3}{2}}\left[n_1\frac{\beta}{a}\sqrt{\pi^2n_2^2+\kappa^2a^2\left(n_3+\frac{1}{4}\right)^2}\right] \\
& +2\sqrt{2}\pi^{\frac{5}{2}}\frac{1}{a^{\frac{7}{2}}\beta^{\frac{1}{2}}}\sum_{n_1=1}^{\infty}\sum_{n_2, n_3=0}^{\infty}n_1^{-\frac{1}{2}}n_2^2\left[\pi^2n_2^2+\kappa^2a^2\left(n_3+\frac{1}{4}\right)^2\right]^{\frac{1}{4}} \\
& \times\left(K_{\frac{1}{2}}\left[n_1\frac{\beta}{a}\sqrt{\pi^2n_2^2+\kappa^2a^2\left(n_3+\frac{1}{4}\right)^2}\right]+K_{\frac{5}{2}}\left[n_1\frac{\beta}{a}\sqrt{\pi^2n_2^2+\kappa^2a^2\left(n_3+\frac{1}{4}\right)^2}\right]\right)
\end{aligned}$$

$$\begin{aligned}
 & -\frac{3\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{1}{a^{\frac{5}{2}}\beta^{\frac{3}{2}}} \sum_{n_1, n_2=1}^{\infty} n_1^{-\frac{3}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16} \right)^{-\frac{1}{4}} K_{\frac{3}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16}} \right) \\
 & -2\sqrt{2}\pi^{\frac{5}{2}} \frac{1}{a^{\frac{7}{2}}\beta^{\frac{1}{2}}} \sum_{n_1, n_2=1}^{\infty} n_1^{-\frac{1}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16} \right)^{\frac{1}{4}} \times \left[K_{\frac{1}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16}} \right) + K_{\frac{5}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16}} \right) \right] \\
 & -\frac{\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{\beta^{\frac{1}{2}}}{\kappa^{\frac{1}{2}} a^5} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{\frac{1}{2}} n_2^2 \left[\pi^2 n_2^2 + \kappa^2 a^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{\frac{3}{4}} \\
 & \times \left[K_{-\frac{1}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \kappa^2 a^2 \left(n_3 + \frac{1}{4} \right)^2} \right) + 2K_{\frac{3}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \kappa^2 a^2 \left(n_3 + \frac{1}{4} \right)^2} \right) \right] \\
 & + K_{\frac{7}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \kappa^2 a^2 \left(n_3 + \frac{1}{4} \right)^2} \right) \left. \right] + \frac{\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{\beta^{\frac{1}{2}}}{\kappa^{\frac{1}{2}} a^5} \sum_{n_1, n_2=1}^{\infty} n_1^{\frac{1}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16} \right)^{\frac{3}{4}} \\
 & \times \left[K_{-\frac{1}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16}} \right) + 2K_{\frac{3}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16}} \right) + K_{\frac{7}{2}} \left(n_1 \frac{\beta}{a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 a^2}{16}} \right) \right] \\
 & + \frac{\kappa^2}{2(L-a)^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-2} \left(n_2 + \frac{1}{4} \right)^2 K_2 \left[2n_1 \kappa (L-a) \left(n_2 + \frac{1}{4} \right) \right] \\
 & + \frac{\kappa^3}{2(L-a)} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-1} \left(n_2 + \frac{1}{4} \right)^3 \left[K_1 \left(2n_1 \kappa (L-a) \left(n_2 + \frac{1}{4} \right) \right) + K_3 \left(2n_1 \kappa (L-a) \left(n_2 + \frac{1}{4} \right) \right) \right] \\
 & - \frac{1}{32} \frac{\kappa^3}{L-a} \sum_{n=1}^{\infty} n^{-2} K_2 \left(\frac{\kappa (L-a)}{2\sqrt{\pi}} n \right) - \frac{\pi^{\frac{1}{2}}}{128} \frac{\kappa^3}{L-a} \sum_{n=1}^{\infty} n^{-1} \left[K_1 \left(\frac{\kappa (L-a)}{2\sqrt{\pi}} n \right) + K_3 \left(\frac{\kappa (L-a)}{2\sqrt{\pi}} n \right) \right] \\
 & - \frac{3\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{1}{(L-a)^{\frac{5}{2}}\beta^{\frac{3}{2}}} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{-\frac{3}{2}} n_2^2 \left[\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{-\frac{1}{4}} \\
 & \times K_{\frac{3}{2}} \left[n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2} \right] \\
 & - 2\sqrt{2}\pi^{\frac{5}{2}} \frac{1}{(L-a)^{\frac{7}{2}}\beta^{\frac{1}{2}}} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{-\frac{1}{2}} n_2^2 \left[\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{\frac{1}{4}} \\
 & \times \left(K_{\frac{1}{2}} \left[n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2} \right] \right. \\
 & \left. + K_{\frac{5}{2}} \left[n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2} \right] \right) \\
 & + \frac{3\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{1}{(L-a)^{\frac{5}{2}}\beta^{\frac{3}{2}}} \sum_{n_1, n_2=1}^{\infty} n_1^{-\frac{3}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16} \right)^{-\frac{1}{4}} \\
 & \times K_{\frac{3}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16}} \right) + 2\sqrt{2}\pi^{\frac{5}{2}} \frac{1}{(L-a)^{\frac{7}{2}}\beta^{\frac{1}{2}}} \sum_{n_1, n_2=1}^{\infty} n_1^{-\frac{1}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16} \right)^{\frac{1}{4}}
 \end{aligned}$$

$$\begin{aligned}
& \times \left[K_{\frac{1}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16}} \right) + K_{\frac{3}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16}} \right) \right] \\
& + \frac{\pi^{\frac{5}{2}}}{\sqrt{2}} \frac{\beta^{\frac{1}{2}}}{\kappa^{\frac{1}{2}} (L-a)^5} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{\frac{1}{2}} n_2^2 \left[\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{\frac{3}{4}} \\
& \times \left[K_{-\frac{1}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2} \right) + 2K_{\frac{3}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2} \right) \right] \\
& + K_{\frac{7}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \kappa^2 (L-a)^2 \left(n_3 + \frac{1}{4} \right)^2} \right) \left[-\frac{\pi^{\frac{5}{2}}}{\sqrt{2}} \frac{\beta^{\frac{1}{2}}}{\kappa^{\frac{1}{2}} (L-a)^5} \sum_{n_1, n_2=1}^{\infty} n_1^{\frac{1}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16} \right)^{\frac{3}{4}} \right. \\
& \times \left[K_{-\frac{1}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16}} \right) + 2K_{\frac{3}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16}} \right) \right. \\
& \left. \left. + K_{\frac{7}{2}} \left(n_1 \frac{\beta}{L-a} \sqrt{\pi^2 n_2^2 + \frac{\kappa^2 (L-a)^2}{16}} \right) \right] \right]. \tag{19}
\end{aligned}$$

This expression represents the Casimir pressure on the piston before the right plate of the system depicted in Fig. 1 has been moved to a remote place. Further, we take the limit $L \rightarrow \infty$ which means that the right plate in part B is moved to a very distant place, then we obtain the following expression for the Casimir force per unit area on the piston within the framework of a two-brane Randall-Sundrum issue,

$$\begin{aligned}
f_{IC} &= \lim_{L \rightarrow \infty} f'_{IC} = -\frac{\kappa^4}{2\mu^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-2} \left(n_2 + \frac{1}{4} \right)^2 \left[2n_1 \mu \left(n_2 + \frac{1}{4} \right) \right] \\
& - \frac{\kappa^4}{2\mu} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-1} \left(n_2 + \frac{1}{4} \right)^3 \left[K_1 \left(2n_1 \mu \left(n_2 + \frac{1}{4} \right) \right) + K_3 \left(2n_1 \mu \left(n_2 + \frac{1}{4} \right) \right) \right] \\
& + \frac{\pi}{32} \frac{\kappa^4}{\mu^2} \sum_{n=1}^{\infty} n^{-2} K_2 \left(\frac{\mu}{2\sqrt{\pi}} n \right) + \frac{\pi^{\frac{1}{2}} \kappa^4}{128 \mu} \sum_{n=1}^{\infty} n^{-1} \left[K_1 \left(\frac{\mu}{2\sqrt{\pi}} n \right) + K_3 \left(\frac{\mu}{2\sqrt{\pi}} n \right) \right] \\
& + \frac{3\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{\kappa^4}{\mu^{\frac{5}{2}} \xi^{\frac{3}{2}}} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{-\frac{3}{2}} n_2^2 \left[\pi^2 n_2^2 + \mu^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{-\frac{1}{4}} \\
& \times K_{\frac{3}{2}} \left[n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \mu^2 \left(n_3 + \frac{1}{4} \right)^2} \right] + 2\sqrt{2}\pi^{\frac{5}{2}} \frac{\kappa^4}{\mu^{\frac{7}{2}} \xi^{\frac{1}{2}}} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{-\frac{1}{2}} n_2^2 \left[\pi^2 n_2^2 + \mu^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{\frac{1}{4}} \\
& \times \left(K_{\frac{1}{2}} \left[n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \mu^2 \left(n_3 + \frac{1}{4} \right)^2} \right] + K_{\frac{5}{2}} \left[n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \mu^2 \left(n_3 + \frac{1}{4} \right)^2} \right] \right) \\
& - \frac{3\sqrt{2}\pi^{\frac{5}{2}}}{2} \frac{\kappa^4}{\mu^{\frac{5}{2}} \xi^{\frac{3}{2}}} \sum_{n_1, n_2=1}^{\infty} n_1^{-\frac{3}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\mu^2}{16} \right)^{-\frac{1}{4}} K_{\frac{3}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \frac{\mu^2}{16}} \right) \\
& - 2\sqrt{2}\pi^{\frac{5}{2}} \frac{\kappa^4}{\mu^{\frac{7}{2}} \xi^{\frac{1}{2}}} \sum_{n_1, n_2=1}^{\infty} n_1^{-\frac{1}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\mu^2}{16} \right)^{\frac{1}{4}} \times \left[K_{\frac{1}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \frac{\mu^2}{16}} \right) \right. \\
& \left. + K_{\frac{5}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \frac{\mu^2}{16}} \right) \right] - \frac{\pi^{\frac{5}{2}} \kappa^4 \xi^{\frac{1}{2}}}{\sqrt{2} \mu^5} \sum_{n_1=1}^{\infty} \sum_{n_2, n_3=0}^{\infty} n_1^{\frac{1}{2}} n_2^2 \left[\pi^2 n_2^2 + \mu^2 \left(n_3 + \frac{1}{4} \right)^2 \right]^{\frac{3}{4}}
\end{aligned}$$

$$\begin{aligned}
 & \times \left[K_{-\frac{1}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \mu^2} \left(n_3 + \frac{1}{4} \right)^2 \right) + 2K_{\frac{3}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \mu^2} \left(n_3 + \frac{1}{4} \right)^2 \right) \right. \\
 & \left. + K_{\frac{7}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \mu^2} \left(n_3 + \frac{1}{4} \right)^2 \right) \right] + \frac{\pi^{\frac{5}{2}} \kappa^4 \xi^{\frac{1}{2}}}{\sqrt{2} \mu^5} \sum_{n_1, n_2=1}^{\infty} n_1^{\frac{1}{2}} n_2^2 \left(\pi^2 n_2^2 + \frac{\mu^2}{16} \right)^{\frac{3}{4}} \\
 & \times \left[K_{-\frac{1}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \frac{\mu^2}{16}} \right) + 2K_{\frac{3}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \frac{\mu^2}{16}} \right) + K_{\frac{7}{2}} \left(n_1 \frac{\xi}{\mu} \sqrt{\pi^2 n_2^2 + \frac{\mu^2}{16}} \right) \right], \tag{20}
 \end{aligned}$$

while we introduce two dimensionless variables, the scaled temperature and the relationship between plate separation and the distance between two 3-branes respectively,

$$\xi = \kappa\beta = \pi k\beta e^{-\pi k R}, \tag{21}$$

$$\mu = \kappa a = \pi k a e^{-\pi k R}. \tag{22}$$

The terms with series in Eq. (20) converge very quickly and only the first several summands need to be taken into account for numerical calculation in further discussions. If the temperature approaches zero, the Casimir force will recover to be the result of Ref. [51]. We have to perform the burden and surprisingly difficult calculation on Eq. (20) in order to explore the Casimir force on the piston at finite temperature in the cosmological background governed by the RSI model. It is clear that the force expression depends on the plate–piston distance and temperature. For a definite temperature like $\xi = 1$, the numerical evaluations of the Casimir force per unit area on the piston from Eq. (20) lead to the data presented in Fig. 2. We find that the sign of the Casimir force is negative when the dimensionless variable μ defined in (22) is very tiny. When the distance between the plate and piston is larger than the branes separation R , meaning the value of μ is sufficiently large, the nature of the Casimir force becomes repulsive although the force vanishes as the plate separation approaches infinity like $\lim_{\mu \rightarrow \infty} f_{IC} = 0$. The curves of the dependence of the Casimir force per unit area for the piston on the plates distance for different temperatures are similar. They possess several general characters such as the attractive Casimir force with very small μ or a repulsive one with sufficiently large μ and asymptotic behaviour $f_{IC}(\mu \rightarrow \infty, T) = 0$. All of the expressions for the Casimir force with thermal corrections have positive maxima. The dependence of the top values of the curves on the scaled temperature ξ defined in Eq. (21) is shown in Fig. 3. The higher temperature or equivalently lower scaled value

leads to larger positive top magnitude, which means that the Casimir force between the plate and piston is an increasing function of temperature. The thermal influence has not cancelled the positive nature of the Casimir force but results in a stronger repulsive force. There also appears a repulsive Casimir force between two parallel plates inevitably under thermal influence in the RSI model, which is excluded by the experimental evidence. It should be emphasized that there appears a term like

$$\frac{\pi}{32} \frac{\kappa^2}{D} \sum_{n=1}^{\infty} n^{-2} K_2 \left(\frac{\kappa D}{2\sqrt{\pi}} n \right)$$

in expression (17) and it is the term that finally leads the Casimir force between two parallel plates in the RSI model to become repulsive when the plate separation is not extremely small. We also find that our results involving the term

$$\frac{\pi}{32} \frac{\kappa^2}{D} \sum_{n=1}^{\infty} n^{-2} K_2 \left(\frac{\kappa D}{2\sqrt{\pi}} n \right)$$

are subject to $m_{N=0} = 0$. The term

$$\frac{\pi}{32} \frac{\kappa^2}{D} \sum_{n=1}^{\infty} n^{-2} K_2 \left(\frac{\kappa D}{2\sqrt{\pi}} n \right)$$

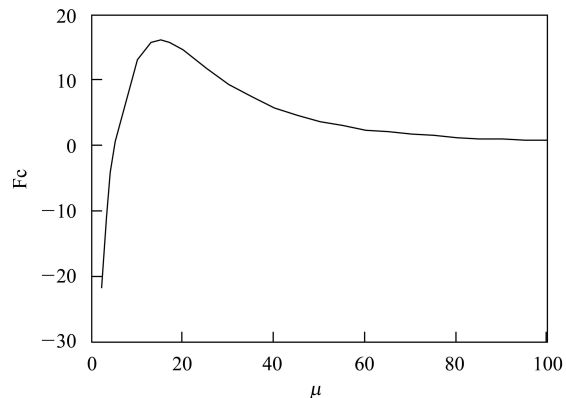


Fig. 2. The Casimir force per unit area in units of κ^4 between the plate and piston versus the dimensionless variable denoted as $\mu = \kappa a$ when $\xi = 1$.

will not appear if $m_{N=0} = \frac{\kappa}{4}$ is chosen, then the Casimir energy and Casimir force will be the same as Frank et al's [50, 53], and $m_{N=0} = \frac{\kappa}{4}$ is not acceptable here. It should be pointed out that the equation is valid asymptotically for $N \gg 1$ although the reduced equation (8) for the effective mass of the scalar bulk field is expressed as an approximation. The error is about 3% when $N = 1$ and the error is 0.3% and 0.1% for $N = 2$ and $N = 3$ respectively, etc., displaying that the error drops very quickly with increasing N [53]. The deviation from the approximation in the case of small N cannot change the above conclusion.

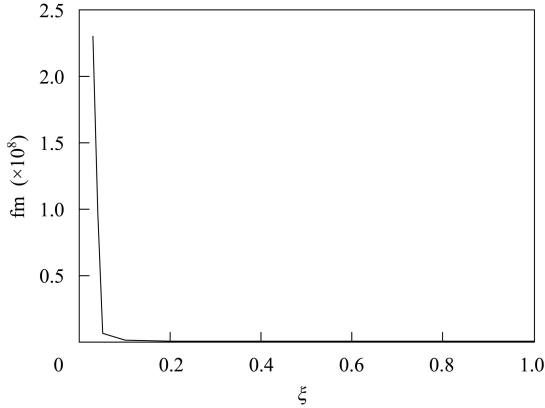


Fig. 3. The dependence of the top values of the Casimir force per unit area in units of κ^4 between the plate and piston on the scaled temperature $\xi = \kappa\beta$.

3 The Casimir force for a piston at finite temperature in the RSII models

In this section, we proceed with the same study on the Casimir effect in the RSII model, in which the 3-brane at $y = \pi R$ is at infinity. That the 3-brane is moved to infinity leads the spectrum of the Kaluza-Klein masses to be continuous and run all $m > 0$. The generalized zeta function becomes,

$$\begin{aligned} \zeta_{\text{II}}(s; \partial_E) &= \text{Tr}(-\partial_E)^{-s} \\ &= \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \int_0^{\infty} \frac{dm}{k} \int d^2 p \left[p^2 + \left(\frac{n\pi}{D} \right)^2 \right. \\ &\quad \left. + m^2 + \left(\frac{2l\pi}{\beta} \right)^2 \right]^{-s}, \end{aligned} \quad (23)$$

here the parameter k is the same as in metric (4) and is determined by the 5D Planck mass and bulk

cosmological constant. Similarly after integration the generalized zeta function for the RSII model can be expressed with the help of the Epstein zeta functions as,

$$\begin{aligned} \zeta_{\text{II}}(s; \partial_E) &= \frac{\pi^{\frac{3}{2}}}{k} \frac{\Gamma\left(s - \frac{3}{2}\right)}{\Gamma(s)} E_2\left(s - \frac{3}{2}; \frac{\pi^2}{D^2}, \frac{4\pi^2}{\beta^2}\right) \\ &\quad + \frac{\pi^{\frac{3}{2}}}{2k} \frac{\Gamma\left(s - \frac{3}{2}\right)}{\Gamma(s)} \left(\frac{\pi}{D}\right)^{3-2s} \zeta(2s-3), \end{aligned} \quad (24)$$

where

$$E_p(s; a_1, a_2, \dots, a_p) = \sum_{\{n_j\}=1}^{\infty} \left(\sum_{j=1}^p a_j n_j^2 \right)^{-s}, \quad (25)$$

and $\zeta(s)$ is the Riemann zeta function. Similarly the vacuum energy density of a device involving two parallel plates at finite temperature in the RSII scenario is,

$$\varepsilon_{\text{II}}(D, T) = -\frac{1}{2} \frac{\partial}{\partial \beta} \left(\frac{\partial \zeta_{\text{II}}(s; -\partial_E)}{\partial s} \Big|_{s=0} \right). \quad (26)$$

We regularize the expressions to obtain the finite parts of the vacuum energy density for parallel plates in the RSII model when the environmental temperature does not vanish,

$$\begin{aligned} \varepsilon_{\text{IIR}}(D, T) &= -\frac{\sqrt{\pi}}{8} \frac{1}{kD^4} \Gamma\left(\frac{5}{2}\right) \zeta(5) \\ &\quad + \frac{2\pi^3}{k\beta^2 D^2} \sum_{n_1, n_2=1}^{\infty} \binom{n_2}{n_1}^2 K_2\left(\frac{\pi\beta}{D} n_1 n_2\right) \\ &\quad + \frac{\pi^4}{k\beta D^3} \sum_{n_1, n_2=1}^{\infty} \frac{n_2^3}{n_1} \left[K_1\left(\frac{\pi\beta}{D} n_1 n_2\right) \right. \\ &\quad \left. + K_3\left(\frac{\pi\beta}{D} n_1 n_2\right) \right]. \end{aligned} \quad (27)$$

Now we choose the variable D in Eq. (27) as a and $L - a$ respectively to obtain the Casimir energy densities of Part A and Part B as follows,

$$\begin{aligned} \varepsilon_{\text{IIR}}^{\text{A}}(a, T) &= \varepsilon_{\text{IIR}}(a, T), \\ \varepsilon_{\text{IIR}}^{\text{B}}(L - a, T) &= \varepsilon_{\text{IIR}}(L - a, T). \end{aligned} \quad (28)$$

According to Eq. (3), the Casimir per unit area on the piston belonging to a three-parallel-plate system in the RSII model introduces,

$$\begin{aligned}
 f'_{\text{HC}} = & -\frac{\partial}{\partial a} [\varepsilon_{\text{IRR}}^{\text{A}}(a, T) + \varepsilon_{\text{IRR}}^{\text{B}}(L - a, T)] = -\frac{\sqrt{\pi}}{2ka^5} \Gamma\left(\frac{5}{2}\right) \zeta(5) + \frac{4\pi^3}{k\beta^2 a^3} \sum_{n_1, n_2=1}^{\infty} \left(\frac{n_2}{n_1}\right)^2 K_2\left(\frac{\pi\beta}{a} n_1 n_2\right) \\
 & + \frac{2\pi^4}{k\beta a^4} \sum_{n_1, n_2=1}^{\infty} \frac{n_2^3}{n_1} \left[K_1\left(\frac{\pi\beta}{a} n_1 n_2\right) + K_3\left(\frac{\pi\beta}{a} n_1 n_2\right) \right] - \frac{\pi^5}{2ka^5} \sum_{n_1, n_2=1}^{\infty} \left[K_0\left(\frac{\pi\beta}{a} n_1 n_2\right) + 2K_2\left(\frac{\pi\beta}{a} n_1 n_2\right) \right. \\
 & \left. + K_4\left(\frac{\pi\beta}{a} n_1 n_2\right) \right] + \frac{\sqrt{\pi}}{2k(L-a)^5} \Gamma\left(\frac{5}{2}\right) \zeta(5) - \frac{8\pi^3}{k\beta^2(L-a)^3} \sum_{n_1, n_2=1}^{\infty} \left(\frac{n_2}{n_1}\right)^2 K_2\left(\frac{\pi\beta}{L-a} n_1 n_2\right) \\
 & - \frac{2\pi^4}{k\beta(L-a)^4} \sum_{n_1, n_2=1}^{\infty} \frac{n_2^3}{n_1} \left[K_1\left(\frac{\pi\beta}{L-a} n_1 n_2\right) + K_3\left(\frac{\pi\beta}{L-a} n_1 n_2\right) \right] \\
 & + \frac{\pi^5}{2k(L-a)^5} \sum_{n_1, n_2=1}^{\infty} \left[K_0\left(\frac{\pi\beta}{L-a} n_1 n_2\right) + 2K_2\left(\frac{\pi\beta}{L-a} n_1 n_2\right) + K_4\left(\frac{\pi\beta}{L-a} n_1 n_2\right) \right]. \tag{29}
 \end{aligned}$$

In order to show the Casimir force between the piston and its closer plate and compare our conclusions with measurements, we let $L \rightarrow \infty$ to find,

$$\begin{aligned}
 f_{\text{HC}} = & -\frac{\sqrt{\pi}}{2ka^5} \Gamma\left(\frac{5}{2}\right) \zeta(5) + \frac{4\pi^3}{k\beta^2 a^3} \sum_{n_1, n_2=1}^{\infty} \left(\frac{n_2}{n_1}\right)^2 \\
 & \times K_2\left(\frac{\pi\beta}{a} n_1 n_2\right) + \frac{2\pi^4}{k\beta a^4} \sum_{n_1, n_2=1}^{\infty} \frac{n_2^3}{n_1} \\
 & \times \left[K_1\left(\frac{\pi\beta}{a} n_1 n_2\right) + K_3\left(\frac{\pi\beta}{a} n_1 n_2\right) \right] \\
 & - \frac{\pi^5}{2ka^5} \sum_{n_1, n_2=1}^{\infty} \left[K_0\left(\frac{\pi\beta}{a} n_1 n_2\right) \right. \\
 & \left. + 2K_2\left(\frac{\pi\beta}{a} n_1 n_2\right) + K_4\left(\frac{\pi\beta}{a} n_1 n_2\right) \right]. \tag{30}
 \end{aligned}$$

The dependence of the reduced Casimir force per unit area on plate-piston separation with some values of temperature is plotted in Fig. 4. If the thermal influence is omitted, the above expression of the reduced Casimir pressure will be recovered to give the findings in Ref. [51], just containing a deviation from the results of the conventional parallel-plate system. As the temperature is high enough, i.e. $\beta \rightarrow 0$, then

$$f_{\text{HC}}(\beta \rightarrow 0) = -\frac{16\sqrt{\pi}}{k\beta^5} \Gamma\left(\frac{5}{2}\right) \zeta(5). \tag{31}$$

It is clear that the magnitude of the Casimir force on the piston increases with the fifth power of temperature. It also indicated that the sign of the reduced force remains negative, which means that the plate and piston still attract each other, while higher temperature certainly gives rise to greater attractive Casimir force instead of causing the reduced force to be repulsive. In four-dimensional flat spacetime the sign of the Casimir force will change to positive when the temperature is high enough. Our results about the nature of the Casimir force between the piston and the remaining plate at finite temperature in the

context of the RSII model are different from those in the background whose dimensionality is four, which are not disfavoured by the measurements.

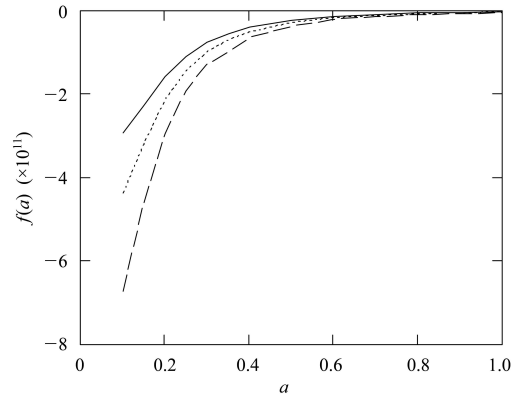


Fig. 4. The dashed, dotted and solid curves of the Casimir force per unit area on the piston as functions of plate-piston distance in a 5-dimensional RSII model for $\beta = 0.01, 0.011, 0.012$ respectively.

4 Conclusions

The Casimir force between two parallel plates involving the contribution from an exterior vacuum energy with thermal corrections is studied in the presence of one warped extra dimension of the models proposed by Randall and Sundrum. In the two-brane scenario called the RSI model we derive the Casimir force at finite temperature for the three-parallel-plate system where the middle plate is called a piston. We get the exact form of the reduced Casimir force per unit area between one plate and the piston as one outer plate is moved away. In this limiting case we find that the sign of the reduced force depending on the temperature and distance between the plate and the piston will become positive when the plate-piston gap is not extremely small although the force will disappear as the piston and plate move far from each

other. The stronger thermal influence brings on a greater repulsive Casimir force between the plates. In the case of the RSI model at finite temperature a repulsive Casimir force is produced due to warps between the parallel plates and the repulsive force is associated with plate distance, so the repulsive parts of the Casimir force on the piston can not be cancelled although the repulsive parts will vanish when the two parallel plates move away from each other. The appearance of the repulsive Casimir force between one plate and the piston conflicts with the experimental results. It is obvious that the RSI model cannot be reliable according to our analysis even

when we consider the thermal influence during our research.

In the case of one brane called the RSII model we perform the same study and procedure to find the reduced Casimir force per unit area on the piston. We find that the reduced force with thermal corrections is great when the piston and plate are located very close to each other, or vanishes with very large plate-piston distance while the force always remains attractive, no matter how high the temperature. It is interesting that a stronger thermal influence gives rise to a greater attractive Casimir force, instead of changing the force to be repulsive.

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