Controllable entanglement sudden birth of Heisenberg spins*

ZHENG Qiang(郑强)[1;1)] ZHI Qi-Jun(支启军)[2]
ZHANG Xiao-Ping(张小平)[3] REN Zhong-Zhou(任中洲)[3;2)]

[1] School of Mathematics and Computer Science, Guizhou Normal University, Guiyang 550001, China
[2] School of Physics and Electronics, Guizhou Normal University, Guiyang 550001, China
[3] School of Physics, Nanjing University, Nanjing 210093, China

Abstract: We investigate the Entanglement Sudden Birth (ESB) of two Heisenberg spins A and B. The third controller, qutrit C is introduced, which only has the Dzyaloshinskii-Moriya (DM) spin-orbit interaction with qubit B. We find that the DM interaction is necessary to induce the Entanglement Sudden Birth of the system qubits A and B, and the initial states of the system qubits and the qurit C are also important to control its Entanglement Sudden Birth.

Key words: entanglement sudden birth, Dzyaloshinskii-Moriya interaction, qutrit

PACS: 03.67-a, 03.67.Mn **DOI:** 10.1088/1674-1137/35/2/005

1 Introduction

Quantum entanglement is a key resource in quantum information processing (QIP) [1]. Recently, entanglement dynamics becomes an active subject. Yu and Eberly firstly found an interesting phenomenon, the so-called Entanglement Sudden Death (ESD) [2, 3]. Contrary to the currently extensively discussed sudden death of entanglement, Entanglement Sudden Birth (ESB) was introduced in Ref. [4]. As it is named, ESB means entanglement can be created nonsmoothly for a finite time [5], i.e., the initially separable state becomes the entangled state at later time.

Quantum simulation [6] of spin chains with many-body interactions is another focus. And there has been a lot of work on the thermal entanglement of spin chains [7–16], which is a kind of static correlation. Specifically, Refs. [15, 16] investigated the thermal entanglement of the (1/2, 1) mixed spins systems under the effect of the spin-orbit Dzyaloshinskii-Moriya (DM) interaction [17, 18].

These works arouse our interest in the ESB of spin systems. Similar to Ref. [19], in this paper we investigate the entanglement dynamics of the system composed of two Heisenberg spins A and B. However, here we concentrate on its Entanglement Sudden Birth. Moreover, we introduce the third controller, qutrit C, which only has DM interaction with the qubit B. Note in Ref. [19], the controller C is a qubit. A qudit, with its higher dimension than a qubit, provides certain benefits in QIP [20]. We find that the ESB of the system depends on the DM interaction, the initial states of the system and qutrit C.

2 Entanglement dynamics of system

For a bipartite system, it is convenient to adopt the Wootters concurrence [21] measuring the degree of entanglement. For two qubits, the concurrence can be calculated by

$$C(\rho)=\max(0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}), \qquad (1)$$

Received 7 April 2010, Revised 12 May 2010

* Supported by National Fundamental Research Programme of China (2010CB327803, 2007CB815004), National Natural Science Foundation of China (11065005, 10865004, 10775123), Governor's Foundation for Science and Education Elites of Guizhou Province QSZHZ (2010)20, Natural Science and Technology Foundation of Guizhou Province ([2009]2267) and Doctor Funding of Guizhou Normal University

1) E-mail: qzhengnju@gmail.com

2) E-mail: zren@nju.edu.cn

where λ_i $(i=1,2,3,4)$ are the eigenvalues in decreasing order of the matrix

$$R=\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y). \tag{2}$$

Here ρ^* are the complex conjugation of ρ in the standard basis and σ_y is the Pauli matrix

$$\sigma_y=\begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix}. \tag{3}$$

The concurrence is monotone, $C=0$ corresponds to the separable state, and $C=1$ to the maximal entanglement one.

Let's consider the ESB of the system composed of two Heisenberg spins A and B. Compared with Ref. [19], here we introduce a qutrit C as the controller, which only has DM interaction with qubit B. The Hamiltonian of the whole system is

$$H=H^{\mathrm{AB}}+H^{\mathrm{BC}}_{\mathrm{DM}}, \tag{4}$$

with

$$\begin{aligned} H^{\mathrm{AB}} &= \frac{1}{2}w_1\sigma^x_{\mathrm{A}}\ \sigma^x_{\mathrm{B}}+\frac{1}{2}w_2\sigma^y_{\mathrm{A}}\ \sigma^y_{\mathrm{B}},\\ H^{\mathrm{BC}}_{\mathrm{DM}} &= \vec{D}\cdot(\vec{\sigma}_{\mathrm{B}}\times\vec{T}_{\mathrm{C}}). \end{aligned} \tag{5}$$

Choosing $\vec{D}=D\vec{x}$, $H^{\mathrm{BC}}_{\mathrm{DM}}$ reduces to

$$H^{\mathrm{BC}}_{\mathrm{DM}}=D(\sigma^y_{\mathrm{B}}\ T^z_{\mathrm{C}}-\sigma^z_{\mathrm{B}}\ T^y_{\mathrm{C}}), \tag{6}$$

where $T^{z,\ y}$ denote the spin-1 operators

$$T^y=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & -\mathrm{i} & 0\\ -\mathrm{i} & 0 & -\mathrm{i}\\ 0 & \mathrm{i} & 0\end{pmatrix},\quad T^z=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}, \tag{7}$$

and D is dimensionless.

We denote $|g\rangle$ ($|e\rangle$) as the ground (excited) state of a qubit, $|0\rangle$, $|1\rangle$ and $|2\rangle$ as the basis of a qutrit. The initial state of the whole system is chosen as

$$\rho_0=\rho^{\mathrm{AB}}(0)\otimes\rho^{\mathrm{C}}(0) \tag{8}$$

with

$$\begin{aligned} \rho^{\mathrm{AB}}(0) &= r|\varphi^{\mathrm{AB}}\rangle\langle\varphi^{\mathrm{AB}}|+\frac{1}{4}(1-r)I_{4\times4},\\ \rho^{\mathrm{C}}(0) &= |\varphi^{\mathrm{C}}\rangle\langle\varphi^{\mathrm{C}}|. \end{aligned} \tag{9}$$

Here

$$\begin{aligned} |\varphi^{\mathrm{AB}}\rangle &= \cos(\alpha)|ee\rangle+\sin(\alpha)|gg\rangle,\\ |\varphi^{\mathrm{C}}\rangle &= \cos(\beta)|0\rangle+\sqrt{p}\sin(\beta)|1\rangle\\ &\quad+\sqrt{1-p}\sin(\beta)|2\rangle, \end{aligned} \tag{10}$$

$I_{4\times4}$ is (4×4) square matrix. With the higher dimension, the qutrit provides two parameters β and p to control the ESB of the system. This is an advantage compared with the controller being a qubit, which has only one parameter [19].

The evolving density matrix of the whole system at arbitrary time t is

$$\rho(t)=U(t)\rho_0U(t)^{\dagger}, \tag{11}$$

where $U(t)=\exp(-\mathrm{i}Ht)$ $(\hbar=1)$.

Tracing over the state of the qutrit C, the reduced density matrix of system qubits A and B turns out to be

$$\rho^{\mathrm{AB}}=\mathrm{Tr}_{\mathrm{C}}[\rho(t)]. \tag{12}$$

For the general parameters, the analytical solutions of the system, even eigenvectors and eigenvalues of the Hamiltonian, are quite complicated. Hence we adopt the numerical simulations to study the effect of DM interaction and the initial state of the controller qubit on the ESB of the system. Our main results are summarized in the following figures. Note in all these figures the setting of the parameters α and r to ensure the initial state of the system is separable.

Figure 1 shows the evolutions of the concurrence of the system qubits ρ^{AB} with and without DM interaction. It is expected that when $D=0$ there is no ESB in the system qubits. Setting $w_{1,\ 2}=1$, one can obtain the evolution operator with $D=0$

$$U_1=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos t & -\mathrm{i}\sin t & 0\\ 0 & -\mathrm{i}\sin t & \cos t & 0\\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{13}$$

Applying U_1 to the state $|\varphi^{\mathrm{AB}}\rangle$, simple computation shows this state does not evolve. Note the initial state of the system is separable, so there is no entanglement all the time. This is consistent with the numerical results in Fig. 1. This figure shows when there is no DM interaction $D=0$, the concurrence of the system equals zero all the time. However, when DM interaction $D\neq0$, there is no entanglement at the early time, and it builds up later. Fig. 1 demonstrates that whether there is ESB in the system qubits depending on the DM interaction between qubit B and qutrit C.

In Fig. 2, we investigate the evolutions of the concurrence of the system qubits ρ^{AB} with the different values of p, i.e., the state of controller qutrit C. It's easy to see that for the different values of p, the first ESB time is the same, about $t_0=2$. With the increase of p, the maximum value of the evolution concurrence $C_{\max}$ also increases.

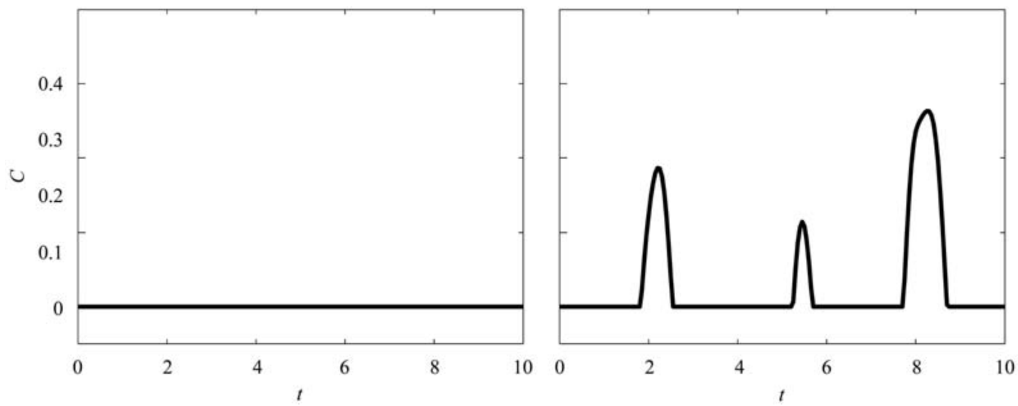

Fig. 1. The evolutions of concurrence of the system, corresponding to $D=0$(left), 2(right), respectively. The other parameters are $\alpha=\frac{\pi}{2}$, $r=0.5$, $\beta=\frac{\pi}{4}$, $p=0.5$, $w_1=w_2=1$.

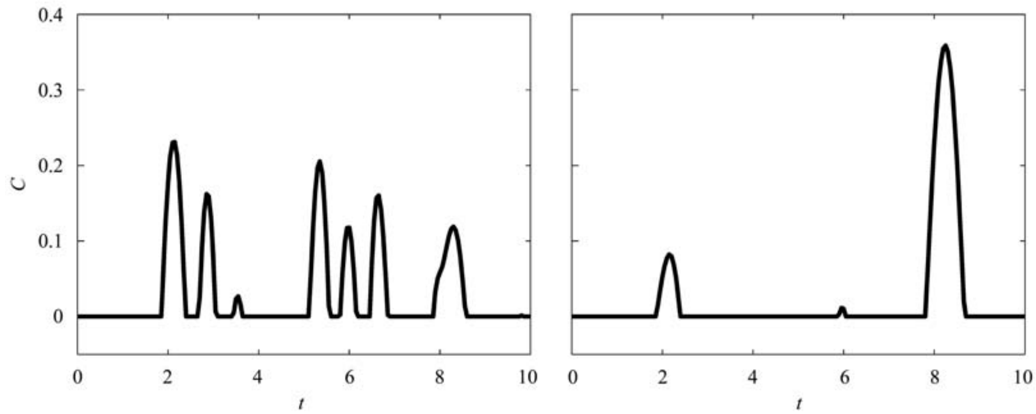

Fig. 2. The evolutions of the concurrence of the system, corresponding to $p=0$(left), 1(right), respectively. The other parameters are $\alpha=\frac{\pi}{2}$, $r=0.5$, $\beta=\frac{\pi}{4}$, $D=2$, $w_1=w_2=1$.

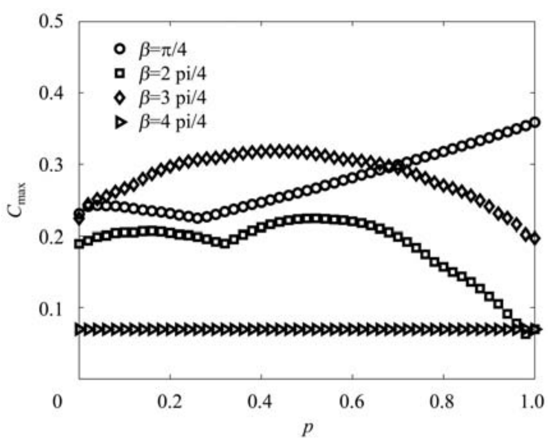

Fig. 3. The variation of $C_{\max}$ with respect to p for different β. The other parameters are $\alpha=\frac{\pi}{2}$, $r=0.5$, $D=2$, $w_1=w_2=1$.

We further study the relationship between $C_{\max}$ and p. Fig. 3 displays this relationship strongly correlating with the other parameter β of the qutrit. When $\beta=\frac{\pi}{4}$, $C_{\max}$ are the two linear functions of p. And $\beta=\pi/4$ and $p=1$ are optimal values, with which the system gets the maximum value of entanglement. However, if $\beta=\pi$, $C_{\max}$ is a constant. For the case $\beta=\frac{2\pi}{4},\frac{3\pi}{4}$, the relationship between $C_{\max}$ and p is complex. Generally speaking, $C_{\max}$ firstly increases and then decreases with the increase of p. Figs. 2 and 3 show that one can adjust the state of qutrit to effectively control the ESB of system. Extensive numerical results display the system concurrence is the periodic function of β with the period $T_0=\pi$.

We also consider another kind of initially separable state, i.e., the small purity r in Fig. 4 and Fig. 5. Fig. 4 shows that the ESB of the system relies on the state of qutrit, too. When $\beta=\frac{2\pi}{4}$, there is no ESB, whereas when $\beta=\frac{3\pi}{4}$, ESB exists.

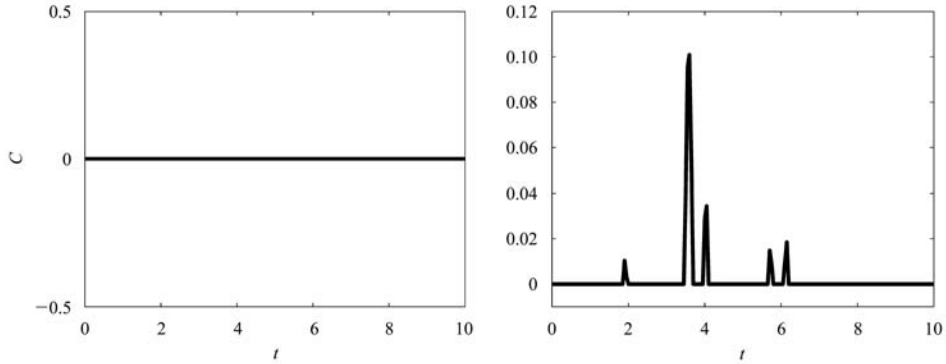

Fig. 4. The evolutions of the concurrence of the system, corresponding to $\beta=\frac{2\pi}{4}$ (left), $\frac{3\pi}{4}$ (right), respectively. The other parameters are $\alpha=\frac{\pi}{4}$, $r=0.2$, $p=0.3$, $D=3$, $w_1=w_2$=1.

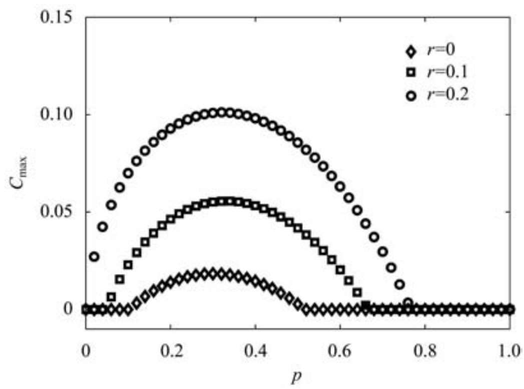

Fig. 5. The variation of $C_{\max}$ with respect to p for different r. The other parameters are $\alpha=\frac{\pi}{4}$, $\beta=\frac{3\pi}{4}$, $D=3$, $w_1=w_2$=1.

Making use of the maximum value of evolving concurrence $C_{\max}$, we try to find the condition of the ESB in the latter case. Fig. 5 plots $C_{\max}$ with respect to the parameter p of the qutrit. This figure shows for the fixed initial purity r of the system, not all the values of p can induce the ESB. For example, corresponding to $r=0$, ESB only exists when $p\in(0.1,\,0.5)$. With the increase of r, this ESB regions become wider. What is more interesting, with the increase of p, $C_{\max}$ firstly increases and then decreases. $C_{\max}$ gets the maximum value at about $p\approx0.3$, irrespective to the value of r. With the increase of r, $C_{\max}$ becomes bigger. These results indicate that the initial states of the system and the controller qutrit are also important to control the ESB of system.

3 Conclusions

Using the concurrence, this paper considers the system composed of two Heisenberg spin-$\frac{1}{2}$ qubits A and B. The third controller qubit C is introduced, which only has the DM interaction with the qubit B. We find that the DM interaction is necessary to induce the Entanglement Sudden Birth for the system qubits. We can also control the Entanglement Sudden Birth by adjusting the initial states of the system and the qutrit C.

References

1 NIELSEN M A, CHUANG I L. Quantum Computation and Quantum Information. Cambridge: Cambridge University Press, 2000
2 YU T, EBERLY J H. Phys. Rev. Lett., 2004, **93**: 140404
3 YU T, EBERLY J H. Science, 2009, **323**: 598
4 FICEK Z, TANAS R. Phys. Rev. A, 2008, **77**: 054301
5 López C E, Romero G, Lastra F, Solano E. Phys. Rev. Lett., 2008, **101**: 080503
6 LIU W Z, ZHANG J F, LONG G L. Chin. Sci. Bull., 2009, **54**(22): 4262–4265; LIU W Z, ZHANG J F, DENG Z W, LONG G L. Sci. Chin. Ser. G, 2008, **51**(8): 1089–1096
7 ZHANG J F, LONG G L, ZHANG W et al. Phys. Rev. A, 2005, **72**: 012331
8 ZHANG Y, LONG G L. Commun. Theor. Phys., 2007, **48**(2): 249–254
9 LIU D, ZHANG Y, LONG G L. Prog. Nat. Sci., 2007, **17**(10): 1147–1151
10 HU M L, TIAN D P. Chin. Phys. C, 2008, **32**(4): 303–307
11 QIN M, TIAN D P. Chin. Phys. C, 2009, **33**(4): 249–251
12 QIN M, LI Y B. Chin. Phys. C, 2010, **34**(4): 448–451
13 WANG X G. Phys. Lett. A, 2001, **281**: 101
14 ZHANG G F. Phys. Rev. A, 2007, **75**: 034304; ZHOU Y, ZHANG G F, LI S S et al. Eur. Phys. Lett., 2009, **86**: 50004
15 MA X S, ZHANG J Y, CONG H S, WANG A M. Sci. Chin. Ser. G, 2008, **51**(12): 1897–1904
16 WANG F, JIA H H, ZHANG H L, ZHANG X A, CHANG S L. Sci. Chin. Ser. G, 2009, **52**(12): 1919–1924
17 Dzyaloshinskii I. J. Phys. Chem. Solid, 1958, **4**: 241
18 MORIYA T. Phys. Rev. Lett., 1960, **4**: 228
19 ZHENG Q, SUN P, ZHANG X P, REN Z Z. Chin. Phys. C, 2010, **34**(10): 1583–1586
20 BARLETT S D, de GUISE H , SANDERS B C. Phys. Rev. A, 2002, **65**: 052316
21 WOOTERS W K. Phys. Rev. Lett., 1998, **80**: 2245