Distributions and correlations of constituent quarks in jets*

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Abstract: In the frame of the quark recombination model, we study the momentum distributions and correlations of constituent quarks in jets by analyzing the final state hadrons generated by PYTHIA for the hard parton fragmentation processes in vacuum. Parameterizations for the distributions are tabulated.

Key words: quark recombination, constituent quark, distribution function, jet fragmentation, PYTHIA

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1 Introduction

In high-energy collisions, the production of final state particles is an important process. There are many models to describe the particle production processes, such as the string fragmentation model [1] and the parton fragmentation model [2]. However, the string fragmentation model can only describe the processes of particle production at low $p_{\rm T}$, and the fragmentation model is valid for particle production at high $p_{\rm T}$. Furthermore, both have trouble in describing data in Au+Au collisions at the RHIC [3–5], such as the anomalistic p/π ratio being around 1 at $p_{\rm T} \approx 3$ GeV/c [6] and the scaling law of elliptic flow in the number of constituent quarks [7, 8]. In recent years, the quark recombination model [9–11] has been formulated to describe the processes of particle production. All final state hadrons are produced, in this model, via the combination of two or three constituent quarks in the final stage of evolution of the partonic system. As is well known, for a 2→1 or 3→ 1 process, the energy is not conserved when initial and final state particles are on the mass-shell if momentum is required to be conserved. However, as pointed out in Ref. [12], this effect is small on average and can be compensated for by the emission or absorption of very soft gluons.

In Ref. [13], the authors proposed a new recombination model based on pQCD. They defined constituent quark distributions in a jet as overlapping matrices of the parton field operators and the constituent quark states, similar to those for the parton fragmentation functions in vacuum. The single-hadron fragmentation function can then be cast as a convolution of the diquark (triquark) distribution functions and the recombination probabilities, which are determined by the hadrons' wavefunctions. In this way, the hard parton fragmentation is regarded as a two-stage process: firstly, the initial hard parton evolves into a shower of constituent quarks; secondly, those constituent quarks will combine with each other to form the final state hadrons. At finite temperature, the effective single-hadron fragmentation functions can be calculated from the recombination of constituent quarks. The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [14–16] evolution equations can be derived for constituent quark distribution functions.

In principle, once the constituent quark distribution functions in a jet in vacuum are given, their distribution functions in a thermal medium can be obtained from the interactions between the jets and the thermal medium. However, the distribution functions of the constituent quarks are not calculable in pQCD. If the partons suffer no energy loss, the distribution functions of constituent quarks in a jet should be independent of the process for the hard parton production but depend on the virtuality of the initially

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produced hard parton. With these distributions at hand, one can obtain lots of information about the final particles for different collision systems. In this paper, we attempt to determine the distributions of constituent quarks in jets.

As is well known, PYTHIA [17] is an event generator for a large number of physics processes. In this paper, we simulate the fragmentation process of hard partons in the vacuum using PYTHIA 6.4, then get the constituent quark distribution functions. Since the generator could describe various data well in many physical processes, including the spectra and correlations of the final particles, we are convinced that the generator takes the QCD effects, especially for the pQCD effect, into account properly for the physical processes. Thus we expect that the constituent quark distributions obtained from PYTHIA are close to the real ones.

2 Hadron spectra and constituent quark distributions

In the quark model, each hadron is composed of two or three constituent quarks. In the quark recombination model, the constituent quarks could be considered as the partons just before hadronization. By using PYTHIA to simulate jet fragmentation in the vacuum, one can obtain lots of information about all kinds of final state hadrons, such as their species and momenta. We are mainly concerned with the distributions of these constituent quarks here, so we try to get them from hadron momenta, because the momentum fraction of the constituent quark in a given hadron is determined by the hadron's wavefunction.

For a high energy π meson, the probability of the momentum fractions of two constituent quarks being z_1 and z_2 is [18]

$$P_{\pi}(z_1,z_2)=\delta(z_1+z_2-1). \quad (1)$$

Because the strange constituent quark is about 1.5 times heavier than that of the light constituent quarks, the mean momentum of the strange quark is 1.5 times that of the light quark. So, for the K meson, the corresponding probability distribution is [18]

$$P_{\rm K}(z_1,z_2)=12z_1z_2^2\delta(z_1+z_2-1), \quad (2)$$

where z_1 is the light constituent quark's momentum fraction and z_2 is for the strange quark.

One can get the momentum fractions of constituent quarks in the baryon in the same way [19]. Here, it is supposed that the hadron moves with high speed and the constituent quarks move in the same direction as the hadron does. In other words, the intrinsic transverse momenta of the constituent quarks are neglected. Below, we will denote z as the longitudinal momentum fraction of a constituent quark relative to the initially produced hard parton instead of that relative to the hadron to which the quark belongs.

3 Inclusive momentum distributions of constituent quarks in jets

The jets produced in high energy collisions are mostly from gluons and light quark pairs $\rm u\bar{u}$ and $\rm d\bar{d}$. Thus, we only consider light parton jets, including those from hard $\rm u,\bar{u},d,\bar{d}$ and gluon. Nevertheless, it is obvious that we can extend the study to include heavy quark jets. With the assumption of isospin and charge conjugate symmetry, only u and gluon jets are independent in all parton fragmentation processes. So, we only need to simulate u quark and gluon jet fragmentations in a vacuum. The simulating process is as follows. First, by using PYHTIA, we generate 10 million $\rm u\bar{u}$ 2-jet events at $\sqrt{s}=200$ GeV and shut down all decay channels of the final hadrons. Second, we get the momentum information of constituent quarks in the way mentioned in the last section, including the longitudinal momentum fraction and the transverse momentum. The gg 2-jet fragmentation is studied in the same way as for $\rm u\bar{u}$.

Without taking into account charm and heavier quarks, the final state constituent quarks can only be u, d, s and their anti-quarks. We study the yield and momentum distributions of constituent quarks in u and gluon jets. The longitudinal momentum fraction z distribution of single constituent quarks is defined as $\rho_1=\frac{{\rm d}N}{{\rm d}z}$, which is normalized to the mean multiplicities of constituent quarks $\int{\rm d}z\rho_1(z)=\langle N\rangle$.

The mean multiplicities $\langle N\rangle$ of all constituent quarks in a jet are shown in Table 1. Although s and its anti-quark are produced in pairs, some of the s quarks from a u jet may combine with $\rm\bar{u}$ quarks from the $\rm\bar{u}$ jet to form hadrons, which are counted into the $\rm\bar{u}$ jet. At the same time, there are more u quarks in the u jet, which are easier to combine with the $\rm\bar{s}$ quarks. Consequently, there are more $\rm\bar{s}$ quarks than s quarks in the u jet. Then it follows that the mean multiplicity of the $\rm\bar{s}$ quark is larger than that for the s quark in the u jet. For the same reason, the mean multiplicity of the $\rm\bar{d}$ quark is larger than the d quark in the u jet.

Table 1. Mean multiplicities $\langle N \rangle$ of all constituent quarks in u and g jets at $\sqrt{s} = 200$ GeV.

	$\langle N \rangle_{\mathrm{u}}$	$\langle N \rangle_{\bar{\mathrm{u}}}$	$\langle N \rangle_{\mathrm{d}}$	$\langle N \rangle_{\bar{\mathrm{d}}}$	$\langle N \rangle_{\mathrm{s}}$	$\langle N \rangle_{\bar{\mathrm{s}}}$
u jet	3.3925	2.7127	2.7697	3.0689	0.9007	1.0227
g jet	5.2764	5.2762	5.2775	5.2776	1.7887	1.7888

The longitudinal momentum fraction distributions ρ_1 of different constituent quarks in a u jet are shown in Fig. 1. What deserves attention is that the distributions of s and $\bar{\mathrm{s}}$ are not the same. Similar distributions are shown in Fig. 2 for gluon jets. Now the light constituent quarks have the same distributions and the distribution of the strange constituent quark is lower than for the non-strange ones. For convenient application of later work, the distributions in Figs. 1 and 2 should be parameterized. We can parameterize the curves as

$$F_{\mathrm{i}}^{\mathrm{q}}(z) = \rho_1(z) = Az^a(1-z)^b \quad (3)$$

for constituent quark q distribution in a jet that originates from hard parton i, where A, a, b are parameters, whose fitted values are shown in Table 2. The fitting curves are also shown in Figs. 1 and 2. From the figures, we know that the curves fit all of the data very well except when z is close to 1 where the statistics are low. That is to say that the above function could be viewed as the expression of the distributions.

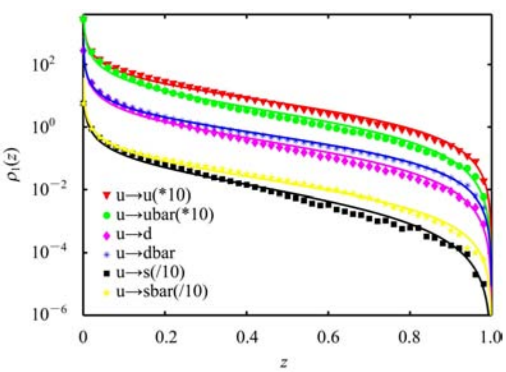

Fig. 1. The longitudinal momentum fraction distributions of all constituent quarks from the u jet at $\sqrt{s} = 200$ GeV.

Similarly, the transverse momentum distributions in a u and gluon jet can be obtained and are shown in Figs. 3 and 4 at $\sqrt{s} = 200$ GeV, respectively. The fitting function is

$$F_{\mathrm{i}}^{\mathrm{q}}(p_{\mathrm{T}}) = \frac{\mathrm{d}N}{p_{\mathrm{T}}\mathrm{d}p_{\mathrm{T}}} = A_{\mathrm{T}} p_{\mathrm{T}}^c \mathrm{e}^{-\mathrm{d}p_{\mathrm{T}}}, \quad (4)$$

where A_{T}, c, d are parameters whose fitted values are shown in Table 2. The fitting curves are also shown in Figs. 3 and 4.

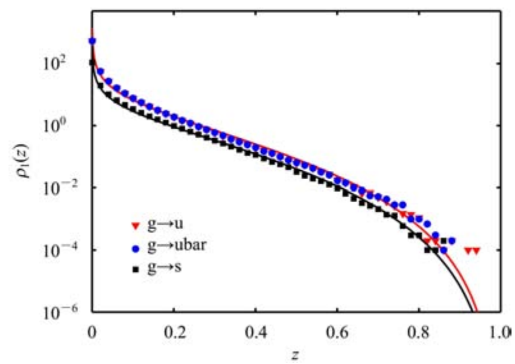

Fig. 2. The longitudinal momentum faction distributions of constituent quarks $\mathrm{u}, \bar{\mathrm{u}}, \mathrm{s}$ from a gluon jet at $\sqrt{s} = 200$ GeV.

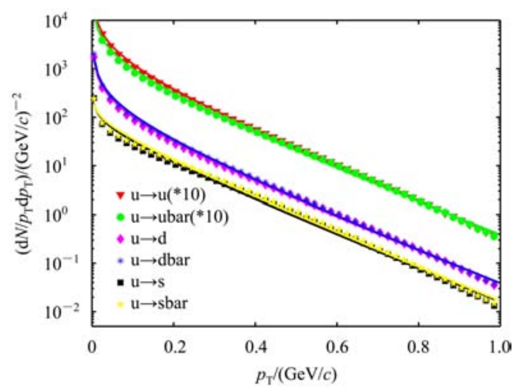

Fig. 3. The transverse momentum distributions of all constituent quarks from a u jet at $\sqrt{s} = 200$ GeV.

Table 2. The parameters of single constituent quark distributions. The parameters for the longitudinal momentum fraction distributions are shown on the left, and those for the transverse momentum distributions are shown on the right.

$F_{\mathrm{i}}^{\mathrm{q}}$	A	a	b	A_{T}	c	d
$F_{\mathrm{u}}^{\mathrm{u}}$	0.8414	−0.8761	1.5206	54.6631	−0.6780	7.2824
$F_{\mathrm{u}}^{\bar{\mathrm{u}}}$	0.4210	−0.9941	1.7174	43.8571	−0.6622	7.0328
$F_{\mathrm{u}}^{\mathrm{d}}$	0.4284	−0.9945	1.6784	44.3154	−0.6686	7.0527
$F_{\mathrm{u}}^{\bar{\mathrm{d}}}$	0.6543	−0.9193	1.4794	49.3196	−0.6763	7.1484
$F_{\mathrm{u}}^{\mathrm{s}}$	0.1621	−0.9877	1.9811	30.9736	−0.3457	7.5679
$F_{\mathrm{u}}^{\bar{\mathrm{s}}}$	0.2435	−0.9669	1.5591	39.7550	−0.2961	7.7702
$F_{\mathrm{g}}^{\mathrm{u}}$	1.3447	−0.9204	4.9407	81.5851	−0.6629	6.8281
$F_{\mathrm{g}}^{\mathrm{s}}$	0.7604	−0.8163	5.0712	62.9887	−0.3118	7.4335

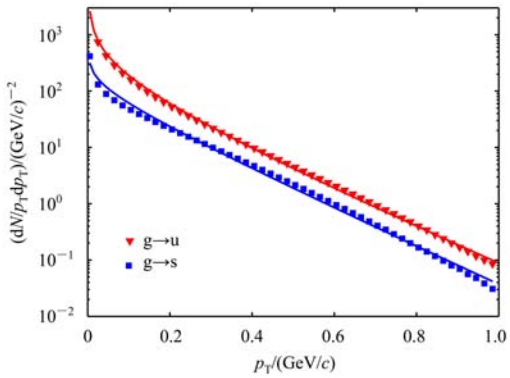

Fig. 4. The transverse momentum distributions of constituent quarks u, s from a gluon jet at $\sqrt{s}=200$ GeV.

4 The joint momentum distributions of double constituent quarks

In this section, we consider the joint momentum distributions for double constituent quarks. For u jets, we consider eight different constituent quark combinations: u and u, u and ū, u and d, u and d̄, u and s, u and s̄, ū and d, ū and d̄. For gluon jets, we only consider three kinds of quark combinations: u and u, u and d, u and s.

Here, the momentum distribution of double constituent quarks q_1q_2 in a jet originated from hard parton i is defined as

$$F_{\rm i}^{\rm q_1q_2}(z_1,z_2)=\rho_2(z_1,z_2)=\frac{{\rm d}N_{12}}{{\rm d}z_1{\rm d}z_2}. \tag{5}$$

It is normalized to $\int {\rm d}z_1{\rm d}z_2\rho_2(z_1,z_2)=\langle N_1N_2\rangle$ for $q_1\neq q_2$.

The longitudinal momentum fraction distribution of a u and u combination from a u jet is shown in Fig. 5, and that for a u and s combination is shown in Fig. 6 for a set of z_1s as a functions of z_2. The fitting function is

$$\rho_2(z_1,z_2)=Az_1^az_2^b(z_1+z_2)^c(1-z_1)^d(1-z_2)^e(1-z_1-z_2)^f, \tag{6}$$

where A,a,b,c,d,e,f are seven parameters. The fitting curves are also shown in Figs. 5 and 6. Other quark pair distributions in the u jet can also be described by this function with suitable parameters. For economy of space, other distributions are not shown here, but the fitted parameters are tabulated in Table 3.

Table 3. The fitted parameters for double constituent quarks' longitudinal momentum fraction distributions.

$F_{\rm i}^{\rm q_1q_2}(z_1,z_2)$	A	a	b	c	d	e	f
$F_{\rm u}^{\rm uu}(z_1,z_2)$	0.4741	−0.9365	−0.9365	−0.1525	−1.8084	−1.8084	3.2588
$F_{\rm u}^{\rm u\bar{u}}(z_1,z_2)$	0.2516	−0.8241	−0.9992	−0.5317	−0.8493	0.0337	1.2908
$F_{\rm u}^{\rm ud}(z_1,z_2)$	0.1783	−0.9111	−0.9391	−0.5723	−3.8915	−1.8150	3.2155
$F_{\rm u}^{\rm u\bar{d}}(z_1,z_2)$	0.3415	−0.9287	−0.9945	−0.1997	−1.063	−0.2556	1.5456
$F_{\rm u}^{\rm us}(z_1,z_2)$	0.0715	−0.8658	−0.7313	−0.8570	−5.2907	−2.4278	3.9110
$F_{\rm u}^{\rm u\bar{s}}(z_1,z_2)$	0.2101	−0.9949	−0.8650	−0.0080	0.8132	2.2253	−0.2210
$F_{\rm u}^{\rm \bar{u}d}(z_1,z_2)$	0.1182	−0.9946	−0.9503	−0.5185	−2.1823	−1.6642	3.1927
$F_{\rm u}^{\rm \bar{u}\bar{d}}(z_1,z_2)$	0.2710	−0.9932	−0.9429	−0.1795	−2.1897	−2.6535	3.9837
$F_{\rm g}^{\rm uu}(z_1,z_2)$	0.2742	−0.9875	−0.9875	0.80533	−1.4918	−1.4918	5.1567
$F_{\rm g}^{\rm ud}(z_1,z_2)$	0.2856	−0.9491	−0.9701	−0.9267	−1.4491	−1.3393	4.8158
$F_{\rm g}^{\rm us}(z_1,z_2)$	0.6444	−0.9202	−0.7800	−0.4052	−2.9455	−3.1653	7.7268

Table 4. The fitted parameters for double constituent quarks' transverse momentum distributions.

$F_{\rm i}^{\rm q_1q_2}(p_{\rm T_1},\ p_{\rm T_2})$	$A_{\rm T}$	$a_{\rm T}$	$b_{\rm T}$	$c_{\rm T}$	$d_{\rm T}$	$e_{\rm T}$
$F_{\rm u}^{\rm uu}(p_{\rm T_1},\ p_{\rm T_2})$	0.2069	0.2671	0.2671	0.0546	7.2778	7.2778
$F_{\rm u}^{\rm u\bar{u}}(p_{\rm T_1},\ p_{\rm T_2})$	0.1798	0.1679	0.3226	0.1113	5.9304	7.2779
$F_{\rm u}^{\rm ud}(p_{\rm T_1},\ p_{\rm T_2})$	0.1437	0.1859	0.3513	−0.0114	5.5780	7.2026
$F_{\rm u}^{\rm u\bar{d}}(p_{\rm T_1},\ p_{\rm T_2})$	0.1867	0.1548	0.2936	0.1678	6.1521	7.4256
$F_{\rm u}^{\rm us}(p_{\rm T_1},\ p_{\rm T_2})$	0.1323	0.2267	0.7140	0.0062	6.0814	7.9369
$F_{\rm u}^{\rm u\bar{s}}(p_{\rm T_1},\ p_{\rm T_2})$	0.1646	0.2363	0.7743	−0.0439	6.1126	8.0344
$F_{\rm u}^{\rm \bar{u}d}(p_{\rm T_1},\ p_{\rm T_2})$	0.1092	0.1312	0.2716	0.1882	5.5927	7.2266
$F_{\rm u}^{\rm \bar{u}\bar{d}}(p_{\rm T_1},\ p_{\rm T_2})$	0.1162	0.1649	0.3515	0.0020	4.9406	7.3913
$F_{\rm g}^{\rm uu}(p_{\rm T_1},\ p_{\rm T_2})$	0.4776	0.2634	0.2634	0.1060	6.8411	6.8411
$F_{\rm g}^{\rm ud}(p_{\rm T_1},\ p_{\rm T_2})$	0.3859	0.1903	0.3438	0.0257	5.2891	6.9601
$F_{\rm g}^{\rm us}(p_{\rm T_1},\ p_{\rm T_2})$	0.3637	0.2084	0.7186	0.0689	5.5755	7.7711

In the same way, we can get the transverse momentum distributions of double constituent quarks,

$$F_{\rm i}^{\rm q_1q_2}(p_{\rm T_1},\ p_{\rm T_2})=\frac{{\rm d}N_{12}}{{\rm d}^2p_{\rm T_1}{\rm d}^2p_{\rm T_2}}.$$

The transverse momentum distributions of uu and us pairs from u jets are shown in Figs. 7 and 8, respectively. And the fitting function is

$$\begin{aligned}&F_{\rm i}^{\rm q_1q_2}(p_{\rm T_1},\ p_{\rm T_2})\\&=A_{\rm T}p_{\rm T_1}^{a_{\rm T}}p_{\rm T_2}^{b_{\rm T}}(p_{\rm T_1}+p_{\rm T_2})^{c_{\rm T}}{\rm e}^{-(d_{\rm T}p_{\rm T_1}+e_{\rm T}p_{\rm T_2})},\end{aligned}\quad(7)$$

where $A_{\rm T}$, $a_{\rm T}$, $b_{\rm T}$, $c_{\rm T}$, $d_{\rm T}$, $e_{\rm T}$ are six parameters. The fitted parameters for the transverse momentum distributions of double constituent quarks are tabulated in Table 4.

As can be seen from Figs. 5 to 8, we can see that the curves fit the data obtained from PYTHIA very well. Thus, the functions with given parameters can be viewed as the momentum distributions of double constituent quarks. For gluon jets, the results are similar and the corresponding fitted parameters are also given in Table 3 and Table 4.

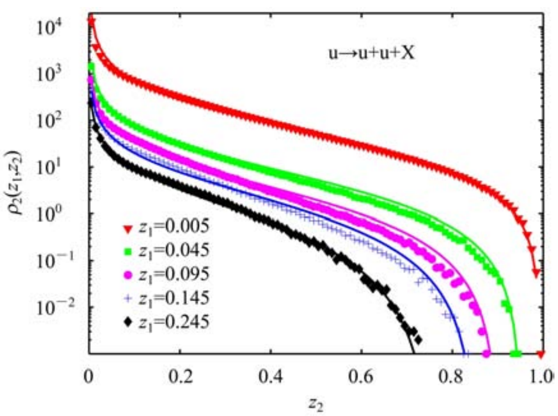

Fig. 5. The longitudinal momentum fraction distributions of a constituent quark uu combination from the u jet at $\sqrt{s}=200$ GeV.

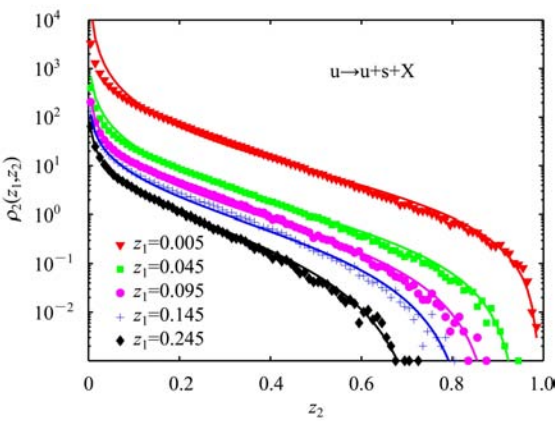

Fig. 6. The longitudinal momentum fraction distributions of a constituent quark us combination from the u jet at $\sqrt{s}=200$ GeV.

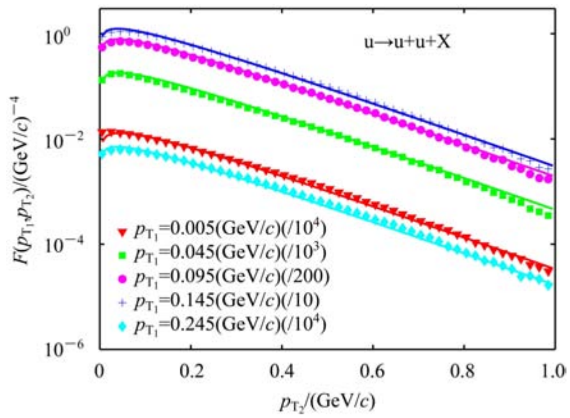

Fig. 7. The transverse momentum distributions of a constituent quark uu combination from the u jet at $\sqrt{s}=200$ GeV.

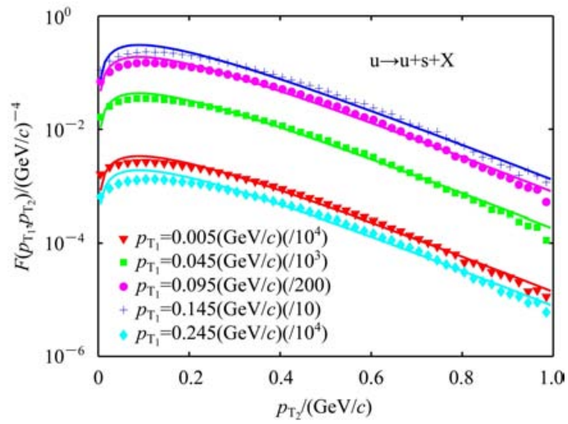

Fig. 8. The transverse momentum distributions of a constituent quark us combination from the u jet at $\sqrt{s}=200$ GeV.

5 Correlations of different quark pairs in jets

From the above results, we can get the correlations of different quark pairs. The correlation between a quark pair is defined as

$$C(z_1,z_2)=\frac{\rho_2(z_1,z_2)}{\rho_1(z_1)\rho_1(z_2)}-1.\quad(8)$$

If there is no correlation between two quarks, $\rho_2(z_1,z_2)=\rho_1(z_1)\rho_1(z_2)$, then $C(z_1,z_2)=0$. Apparently, only in the region of $z_1+z_2<1$, $C(z_1,z_2)$ is meaningful. The correlations of constituent quark-pairs uu and u$\bar{\rm u}$ in a u jet are shown in Figs. 9 and 10, respectively.

For a uu quark-pair in the u jets, the correlation is symmetric because of the indistinguishability of identical particles. When the longitudinal momentum fraction of a constituent u quark is small, the correlation of the uu quark-pair is positive. However,

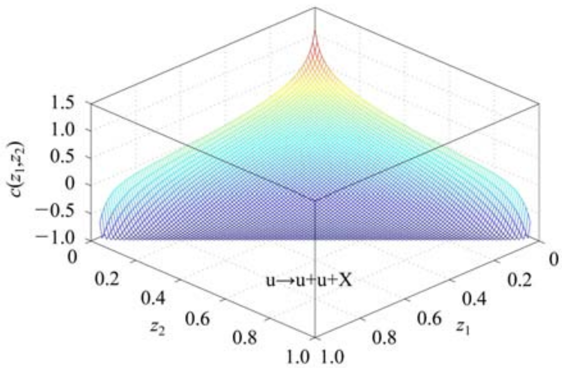

Fig. 9. The correlation of a constituent quark uu from the u jet at $\sqrt{s}=200$ GeV.

when one of the constituent u quark's longitudinal momentum fraction is small and the others is not too small, the correlation becomes almost zero. That is, there is almost no correlation between those two constituent u quark pairs for this case. When the sum of the two constituent quarks' longitudinal momentum fraction equals 1, the correlation of the uu quark-pair is close to −1. According to the definition of quark pair's correlation, when the correlation is −1, the joint distribution of quarks is 0. More explicitly, the sum of longitudinal momentum fractions of two u constituent quarks can not be 1. When the sum is close to one, the sum of longitudinal momentum fractions of all other constituent quarks approaches 0. Then the phase space for those quarks is extremely small and this leads to their joint distribution approaching 0.

Figure 10 is for the correlation of a uū quark-pair in the u jets. Fig. 10 shows that their correlation is quite large when the longitudinal momentum fraction of constituent quarks u and ū is very small. When the constituent u quark's longitudinal momentum fraction is not too small and that for ū's is small, their correlation is extremely large. In other words, there is strong correlation between u and ū when both longitudinal momentum fractions are small and when a u with large z_1 and a ū with small z_2. However, there is extremely weak correlation for other cases. The correlations of other quark-pairs in the u jets are similar to the case for uū. Similarly, the correlations between constituent quark-pairs in the gluon jet can be obtained.

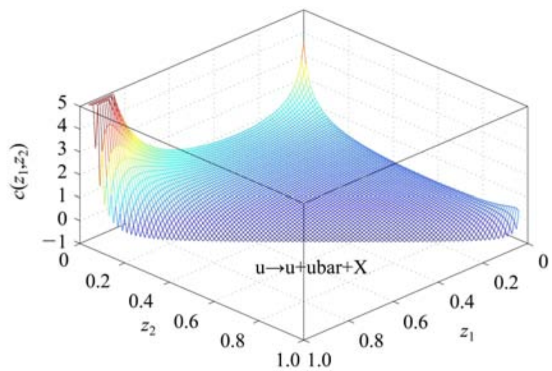

Fig. 10. The correlation of constituent quark uū from the u jet at $\sqrt{s}=200$ GeV.

6 Discussion

In this paper, we have investigated the distributions and correlations of constituent quarks in jets and parameterized them. These distribution functions and correlations, which are independent of the collision system for hard parton production, are universal and can be used for different particle production processes. Taking into account the energy loss in QGP, one will be able to study the particle production in high-energy heavy-ion collisions. These problems will be discussed later.

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