Fourth generation effects in $\bar{B}_s \to \pi^- K^+$ decay*

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Abstract: We study the effects of the fourth quark generation model in $\bar{B}_s \to \pi^- K^+$ decay. Combining with the up-to-date experimental measurement for $\mathcal{B}(\bar{B}_s \to \pi^- K^+)$ and $A_{cp}^{\rm dir}(\bar{B}_s \to \pi^- K^+)$ by the CDF Collaboration, we derive the new bound of weak phase $\phi_{\rm bd}$, which are $0^\circ < \phi_{\rm bd} < 44^\circ$, $321^\circ < \phi_{\rm bd} < 360^\circ$ and $0^\circ < \phi_{\rm bd} < 26^\circ$, $342^\circ < \phi_{\rm bd} < 360^\circ$ for $m_{t'} = 400$ and 600 GeV respectively. In these regions, $\mathcal{B}(\bar{B}_s \to \pi^- K^+)$ and $A_{cp}^{\rm dir}(\bar{B}_s \to \pi^- K^+)$ consist with the current experimental data within errors.

Key words: B_s meson, fourth generation, QCD factorization

PACS: 13.25.Hw, 12.60.-i ,12.38.Bx **DOI:** 10.1088/1674-1137/35/9/003

1 Introduction

Charmless two-body nonleptonic decays of B meson systems provide not only good opportunities to test the Standard Model (SM) but also powerful means to probe different NP scenarios beyond the SM. The two-body charmless B_s decays also play a similar role. Recently, some interesting measurements of B_s decays have been reported by CDF Collaboration at Fermilab Tevantron. B_s decays have attracted much more attention. In particular, CDF Collaboration has made the first measurement of $\bar{B}_s \to \pi^- K^+$ [1]

$$\begin{aligned}\mathcal{B}(\bar{B}_s \to \pi^- K^+) &= (5.0 \pm 0.7 \pm 0.8) \times 10^{-6},\\ A_{cp}^{\rm dir}(\bar{B}_s \to \pi^- K^+) &= (0.39 \pm 0.15 \pm 0.08).\end{aligned} \tag{1}$$

In the SM, the QCDF [2] and PQCD [3] results for $\mathcal{B}(\bar{B}_s \to \pi^- K^+)$ are significantly larger than the CDF measurement, but the SCET [4] ones agree with the CDF data. For $A_{cp}^{\rm dir}(\bar{B}_s \to \pi^- K^+)$, all of the theoretical predictions agree with the CDF data for large error. Many efforts for the decay have also been done within possible New Physics (NP) scenarios [5]. In this paper, using QCDF, we try to discuss the effects of the fourth quark generation model in this decay.

2 $\bar{B}_s \to \pi^- K^+$ decay

Before incorporating the effects of the extra fourth quark generation, we provide a quick survey of $\bar{B}_s \to \pi^- K^+$ decay in the SM within the QCDF framework. The effective Hamiltonian responsible for $b \to d$ transitions is given as [6]

$$\begin{aligned}\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\Bigg[& V_{ub}V_{ud}^*(C_1Q_1^u + C_2Q_2^u)\\ & + V_{cb}V_{cd}^*(C_1Q_1^c + C_2Q_2^c)\\ & - V_{tb}V_{td}^*\left(\sum_{i=3}^{10} C_iQ_i + C_{7\gamma}Q_{7\gamma} + C_{8g}Q_{8g}\right)\Bigg]\\ & + \text{h.c.},\end{aligned} \tag{2}$$

where $V_{qb}V_{qd}^*(q = u, c, t)$ are the CKM factors, C_i are the effective Wilson coefficients and Q_i the relevant four-quark operations.

According to QCDF, when the final-state hadrons emitted from B-meson decays are both light ones, the matrix element of each operator in the effective Hamiltonian can be written as

$$\begin{aligned}&\langle M_1 M_2 | Q_i | B \rangle\\ &= \sum_j F_j^{B \to M_1} \int_0^1 dx T_{ij}^I(x) \Phi_{M_2}(x) + (M_1 \leftrightarrow M_2)\end{aligned}$$

Received 1 December 2010, Revised 30 December 2010

* Supported by National Natural Science Foundation of China (10805014, 10947020, 10979008) and Natural Science Foundation of He'nan Province (092300410205, 102300410210)

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$$+\int_0^1 \mathrm{d}\xi\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y T_i^{\mathrm{II}}(\xi,x,y)\Phi_{\mathrm{B}}(\xi)\Phi_{\mathrm{M}_1}(x)\Phi_{\mathrm{M}_2}(y) \quad (3)$$

where $F_j^{\mathrm{B}\to\mathrm{M}_1}$ is the transition form factor, the kernels T_{ij}^{I} and T_i^{II} denote the short-distance contributions and can be calculated perturbatively and $\Phi_{\mathrm{X}}(\mathrm{X}=\mathrm{B},\mathrm{M}_{1,2})$ are the universal nonperturbative light cone distribution amplitudes of the corresponding mesons. Since the weak annihilation contributions are suppressed by $1/m_{\mathrm{b}}$ in heavy quark limit, they are not included in Eq. (3).

With the effective Hamiltonian Eq. (2) and the QCD factorization formula Eq. (3), we can write out the decay amplitude for a general two-body charmless $\mathrm{B}\to\mathrm{M}_1\mathrm{M}_2$ decay as

$$\begin{aligned}&\mathcal{A}(\mathrm{B}\to\mathrm{M}_1\mathrm{M}_2)\\ =&\frac{G_{\mathrm{F}}}{\sqrt{2}}\sum_{\mathrm{p=u,c}}\sum_i V_{\mathrm{pb}}V_{\mathrm{pd}}^*\alpha_i^{\mathrm{p}}(\mu)\langle M_1M_2|Q_i|B\rangle_{\mathrm{F}}, \quad (4)\end{aligned}$$

in which $\langle M_1M_2|Q_i|B\rangle_{\mathrm{F}}$ is the factorized matrix element, the general form of the effective coefficients $\alpha_i^{\mathrm{p}}(M_1M_2)(i=1,\cdots,10)$ at next-leading order is given as

$$\begin{aligned}\alpha_i^{\mathrm{p}}(M_1M_2)=&\left(C_i+\frac{C_{i\pm1}}{N_{\mathrm{c}}}\right)N_i(M_2)\\ &+\frac{C_{i\pm1}}{N_{\mathrm{c}}}\frac{C_{\mathrm{F}}\alpha_{\mathrm{s}}}{4\pi}\left[V_i(M_2)+\frac{4\pi^2}{N_{\mathrm{c}}}H_i(M_1M_2)\right]\\ &+P_i^{\mathrm{p}}(M_2) \quad (5)\end{aligned}$$

where the upper (lower) signs "±" apply when i is odd (even), the quantities $V_i(M_2)$ account for one-loop vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions and $P_i^{\mathrm{p}}(M_2)$ for penguin contractions. The explicit expressions for these functions could be found in Ref. [2].

The weak annihilation contributions can be expressed as

$$\begin{aligned}\mathcal{A}^{\mathrm{ann}}(\mathrm{B}\to\mathrm{M}_1\mathrm{M}_2)\propto&\frac{G_{\mathrm{F}}}{\sqrt{2}}\sum_{\mathrm{p=u,c}}\sum_i\lambda'_{\mathrm{p}}f_{\mathrm{B}}f_{\mathrm{M}_1}\\ &\times f_{\mathrm{M}_2}b_i(M_1M_2) \quad (6)\end{aligned}$$

where f_{B} and f_{M} are the decay constants of the initial B and the final-state mesons, respectively and the parameters $b_i(M_1M_2)$ are defined as

$$\begin{aligned}b_1(M_1M_2)=&\frac{G_{\mathrm{F}}}{N_{\mathrm{c}}^2}C_1A_1^i,\\ b_3^{\mathrm{p}}(M_1M_2)=&\frac{G_{\mathrm{F}}}{N_{\mathrm{c}}^2}\left[C_3A_1^i+C_5(A_3^i+A_3^{\mathrm{f}})+N_{\mathrm{c}}C_6A_3^{\mathrm{f}}\right],\\ b_2(M_1M_2)=&\frac{G_{\mathrm{F}}}{N_{\mathrm{c}}^2}C_2A_1^i,\\ b_{3,\mathrm{EW}}^{\mathrm{p}}(M_1M_2)=&\frac{G_{\mathrm{F}}}{N_{\mathrm{c}}^2}\left[C_9A_1^i+C_7(A_3^i+A_3^{\mathrm{f}})+N_{\mathrm{c}}C_8A_3^{\mathrm{f}}\right],\\ b_4^{\mathrm{p}}(M_1M_2)=&\frac{G_{\mathrm{F}}}{N_{\mathrm{c}}^2}\left[C_4A_1^i+C_6A_2^i\right],\\ b_{4,\mathrm{EW}}^{\mathrm{p}}(M_1M_2)=&\frac{G_{\mathrm{F}}}{N_{\mathrm{c}}^2}\left[C_{10}A_1^i+C_8A_2^i\right],\end{aligned} \quad (7)$$

where the explicit expressions for the basic building blocks $A_{\mathrm{k}}^{i,\mathrm{f}}$ could be found in Ref. [2].

Then the total branching ratio reads

$$\mathcal{A}(\mathrm{B_s}\to\mathrm{M}_1\mathrm{M}_2)=\frac{\tau_{\mathrm{B_s}}|p_{\mathrm{c}}|}{8\pi M_{\mathrm{B_s}}^2}\left|\mathcal{A}(\mathrm{B_s}\to\mathrm{M}_1\mathrm{M}_2)\right|^2 \quad (8)$$

where $\tau_{\mathrm{B_s}}$ is the $\mathrm{B_s}$ lifetime, $|p_{\mathrm{c}}|$ is the center of mass momentum in the center of mass frame of $\mathrm{B_s}$ meson.

The direct CP asymmetry is defined as

$$A_{cp}^{\mathrm{dir}}(\mathrm{B_s}\to\mathrm{f})=\frac{\mathcal{B}(\bar{\mathrm{B}}_{\mathrm{s}}\to\bar{\mathrm{f}})-\mathcal{B}(\mathrm{B_s}\to\mathrm{f})}{\mathcal{B}(\bar{\mathrm{B}}_{\mathrm{s}}\to\bar{\mathrm{f}})+\mathcal{B}(\mathrm{B_s}\to\mathrm{f})}. \quad (9)$$

In the SM, the amplitude of $\bar{\mathrm{B}}_{\mathrm{s}}\to\pi^-\mathrm{K}^+$ decay can be written as

$$\mathcal{A}^{\rm SM}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+) = \frac{G_{\rm F}}{\sqrt{2}}A_{{\rm K}\pi}\sum_{\rm p=u,c}V_{\rm pb}V^*_{\rm pd}\left[\delta_{\rm pu}\alpha_1+\alpha_4^{\rm p}+\alpha^{\rm p}_{\rm 4,EW}+\beta^{\rm p}_3-\frac{1}{2}\beta^{\rm p}_{\rm 3,EW}\right] \tag{10}$$

where

$$A_{{\rm K}\pi}=M^2_{{\rm B}_{\rm s}}F_0^{\bar{\rm B}_{\rm s}\to{\rm K}}(0)f_\pi. \tag{11}$$

Now we will consider the effect of a sequential fourth generation of quarks [7]. This model is an extension of the SM with the addition of a fourth quark generation. It retains all the features of the SM except that it brings in the new members denoted by (t′, b′). The fourth up-type quark (t′) like the u, c, t quarks contributes in the $\rm b\to s$ transition at the loop level, hence it will modify the SM result. The effects of the fourth quark generation in various B decays are extensively studied in the literature [8–10].

Due to the additional fourth generation, there will be mixing among the new b′ quark and the three down type quarks of the SM and the resulting mixing matrix will be a 4×4 matrix. Accordingly, the unitarity condition becomes $\lambda_{\rm u}+\lambda_{\rm c}+\lambda_{\rm t}+\lambda_{\rm t'}=0$, thus the effective Hamiltonian will be modified as

$$\mathcal{H}_{\rm eff}=-\frac{G_{\rm F}}{\sqrt{2}}\left[V_{\rm tb}V^*_{\rm td}\sum C_iO_i+V_{\rm t'b}V^*_{\rm t'd}\sum C_i^{\rm t'}O_i\right], \tag{12}$$

where we assume $V_{\rm t'b}V^*_{\rm t'd}=r_{\rm bd}{\rm e}^{{\rm i}\phi_{\rm bd}}$, $C_i^{\rm t'}$ are the new Wilson coefficients arising due to the t′ quark in the loop. If we neglect the RG evolution of these coefficients from t′ mass scale to the weak scale $M_{\rm W}$, the values of these Wilson coefficients at the $M_{\rm W}$ scale can be obtained from the corresponding contributions of the t quark by replacing the mass of t quark in the Inami Lim functions [11] by t′ mass. These values can be evolved to the $m_{\rm b}$ scale using the renormalization group equation [6].

After obtaining the values of the new Wilson coefficients at the b quark mass scale, we can write the new amplitude $\mathcal{A}^{\rm NP}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ in a straight forward manner from Eq. (5) by replacing $\alpha(\pi^-{\rm K}^+)$ with the new effective coefficients $\alpha^{\rm t'}(\pi^-{\rm K}^+)$, which are new contributions from the t′ quark.

We obtain the total decay amplitude

$$\mathcal{A}=\mathcal{A}^{\rm SM}+\mathcal{A}^{\rm NP}. \tag{13}$$

3 Numerical results and conclusions

There are 3 real parameters $m_{\rm t'}$, $r_{\rm bd}$ and the weak phase $\phi_{\rm bd}$ in this model, $\mathcal{B}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ and $A^{\rm dir}_{cp}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ depend on all of them.

Taking $r_{\rm bd}=-0.0038$ ($m_{\rm t'}$=400 GeV), $r_{\rm bd}=-0.0027$ ($m_{\rm t'}$=600 GeV) and $r_{\rm bd}=-0.0021$ ($m_{\rm t'}$=800 GeV) respectively [9], we plot $\mathcal{B}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ versus the NP weak phase $\phi_{\rm bd}$ in Fig. 1. We obtain the limits on the NP weak phase $\phi_{\rm bd}$, which are

$$\begin{aligned}&0^\circ<\phi_{\rm bd}<44^\circ,\quad 321^\circ<\phi_{\rm bd}<360^\circ,\\ &\text{for } m_{\rm t'}=400\ \text{GeV},\\ &0^\circ<\phi_{\rm bd}<26^\circ,\quad 342^\circ<\phi_{\rm bd}<360^\circ,\\ &\text{for } m_{\rm t'}=600\ \text{GeV}.\end{aligned} \tag{14}$$

For $m_{\rm t'}=800$ GeV, varying the current experimental data of $\mathcal{B}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ within 2σ, we can not obtain the limit on the parameter $\phi_{\rm bd}$.

From Fig. 1, if we take the values of the parameter $\phi_{\rm bd}$ from these regions Eq. (14), the theoretical prediction within the SM for $\mathcal{B}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ agrees with the current experimental data within errors.

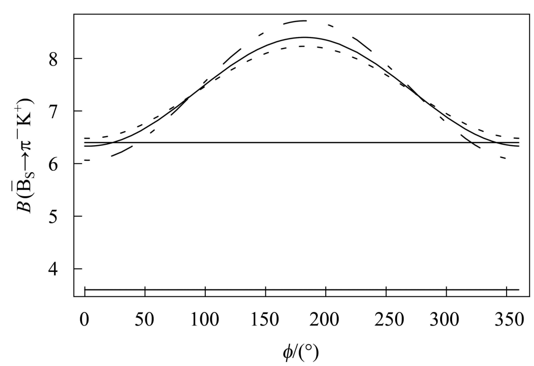

Fig. 1. $\mathcal{B}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ (in units of 10^{-6}) versus the weak phase $\phi_{\rm bd}$, where the dot-dashed, solid and dashed lines correspond to $m_{\rm t'}$=400, 600 and 800 GeV respectively. The horizontal lines are the current experimental data at the 2σ error level.

In Fig. 2, we plot $A^{\rm dir}_{cp}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ versus the NP weak phase $\phi_{\rm bd}$. Varying the current experimental data of $A^{\rm dir}_{cp}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ within 2σ, we also obtain the limits on the NP weak phase $\phi_{\rm bd}$, which are

$$\begin{aligned}&0^\circ<\phi_{\rm bd}<65^\circ,\quad 155^\circ<\phi_{\rm bd}<360^\circ,\\ &\text{for } m_{\rm t'}=400\ \text{GeV},\\ &0^\circ<\phi_{\rm bd}<84^\circ,\quad 146^\circ<\phi_{\rm bd}<360^\circ,\\ &\text{for } m_{\rm t'}=600\ \text{GeV},\\ &0^\circ<\phi_{\rm bd}<360^\circ,\\ &\text{for } m_{\rm t'}=800\ \text{GeV}.\end{aligned} \tag{15}$$

If the values of the NP weak phase $\phi_{\rm bd}$ fall into these regions Eq. (15), $A^{\rm dir}_{cp}(\bar{\rm B}_{\rm s}\to\pi^-{\rm K}^+)$ matches up to the current experimental data within errors.

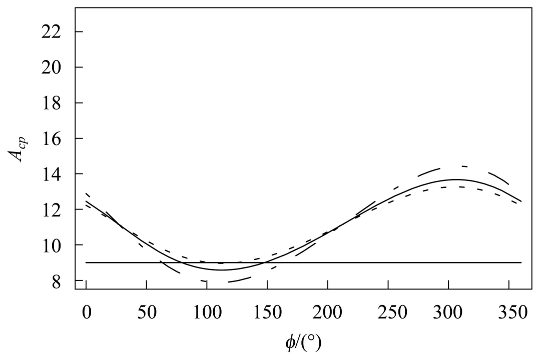

Fig. 2. $A_{cp}^{\rm dir}(\bar{B}_s \to \pi^- K^+)$ (in units of 10^{-2}) versus the weak phase $\phi_{\rm bd}$, where the dot-dashed, solid and dashed lines correspond to $m_{t'} = 400$, 600 and 800 GeV respectively. The horizontal line is the lower limit of the current experimental data at the 2σ error level.

Finally, we obtain the overlapped regions of the NP weak phase $\phi_{\rm bd}$, which are

$$\begin{aligned} &0^\circ < \phi_{\rm bd} < 44^\circ, \quad 321^\circ < \phi_{\rm bd} < 360^\circ, \\ &\text{for } m_{t'} = 400 \text{ GeV}, \\ &0^\circ < \phi_{\rm bd} < 26^\circ, \quad 342^\circ < \phi_{\rm bd} < 360^\circ, \\ &\text{for } m_{t'} = 600 \text{ GeV}. \end{aligned} \tag{16}$$

These limits we obtain on the NP weak phase $\phi_{\rm bd}$ are stronger than those coming from Refs. [9, 10]. In these regions Eq. (16), $\mathcal{B}(\bar{B}_s \to \pi^- K^+)$ and $A_{cp}^{\rm dir}(\bar{B}_s \to \pi^- K^+)$ consist with the current experimental data within errors.

In summary, using QCD factorization approach, we have studied the $\bar{B}_s \to \pi^- K^+$ decay in the fourth quark generation model. When setting $m_{t'}$, $r_{\rm bd}$ and using the current experimental data, we give out the bounds on the weak phase $\phi_{\rm bd}$.

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