Event generators for η/η' rare decays into $\pi^+\pi^-l^+l^-$ and $e^+e^-\mu^+\mu^-$ *

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Abstract: Study of the rare and forbidden decays of η/η' offers a sensitive probe to test fundamental symmetries of quantum chromodynamics and search for new physics beyond the Standard Model. To study the rare decays of η/η' to $\pi^+\pi^-e^+e^-$, $\pi^+\pi^-\mu^+\mu^-$ and $e^+e^-\mu^+\mu^-$ at the BESIII detector, we developed several event generators based on the vector meson dominant model with finite-width corrections and the pseudoscalar mesons mixing theory. The various distributions from event generators are in good agreement with the theoretical predictions, which indicates that the event generators work very well after implemention in the BESIII Monte Carlo simulation package. In the BESIII physics analysis, the performance of the event generators will be improved in accordance with the distributions of different variables of η/η' from data and the improvement on the theoretical calculations.

Key words: event generators, rare decays of η/η', the BESIII detector

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1 Introduction

The rare and forbidden decays of η/η' have received considerable attention both theoretically and experimentally in recent years because they could be used to test the Standard Model (SM), such as quantum electrodynamics, quantum chromodynamics, and even more physics beyond the SM, such as light dark matter [1, 2] and CP violation [3, 4].

In the η sector, experimental data are made by η-factories such as KLOE [4] , CELSIUS/WASA [5], CBELSA/TAPS [6] and CMD-2 [7], for which the decays of η meson have become an important area of modern hadron physics. The study of η' meson rare decays are very scarce, making it difficult for the theoretical predictions to be tested by experiments due to the lack of a huge data sample. Recently, the branching fraction of the decay $\eta'\to\pi^+\pi^-e^+e^-$ was measured to be $(25^{+12}_{-9}\pm5)\times10^{-4}$ by CLEO collaboration [8] using $\Psi(2S)\to\pi^+\pi^-J/\psi$, $J/\psi\to\gamma\eta'$ events acquired with the CLEO-c detector at the CESR e^+e^- collider. For $\eta'\to\pi^+\pi^-\mu^+\mu^-$, no evident signal was observed, and the CLEO collaboration just presented the branching fraction upper limit, which is less than 2.4×10^{-4} at 90% confidence level.

At present, BESIII [9, 10] has collected a sample of 2.25×10^8 J/ψ events [11] and 1.06×10^8 Ψ' events [12], and about 1 billion J/ψ and Ψ' events will be collected soon, which offers a great opportunity to study the rare and forbidden decays of η/η'. To study these processes, reliable event generators are very important in order to provide detection efficiency and suppress background events.

In this paper, we developed event generators for generating the rare decays of $\eta/\eta'\to\pi^+\pi^-e^+e^-$, $\pi^+\pi^-\mu^+\mu^-$ and $e^+e^-\mu^+\mu^-$ at BESIII. In the next section the theoretical formulas of the invariant decay amplitudes are discussed. The vector meson dominant (VMD) model is used to obtain the form factors in which the width of the vector meson is introduced to eliminate the singularities on the vector meson mass. In the third section the various distri-

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butions are derived from both event generations and theoretical calculations, and the results are consistent with each other.

2 Theoretical formulas for the η/η' rare decays

In this section we will discuss the theoretical formulas we used for the pseudoscalar mesons: η/η' rare decays which are induced by the chiral anomaly. The η/η' rare decays into $\pi^+\pi^-l^+l^-$ are governed by the box anomaly, while the decays into $e^+e^-\mu^+\mu^-$ are governed by the triangle anomaly.

$\eta/\eta' \to \pi^+\pi^-l^+l^-$ processes are similar to the $\eta/\eta' \to \pi^+\pi^-\gamma$ ones in which the photon can be replaced by an off-shell one that decays into a lepton pair. The four-momenta for the decay $P(P) \to \pi^+(p_+)\pi^-(p_-)l^+(k_+)l^-(k_-)$ are defined as $P = p_+ + p_- + k_+ + k_-$, $p = p_+ + p_-$ and $k = k_+ + k_-$ [3]. The relevant variables can be written as

$$\begin{aligned} s_{p_+p_-} &= s_{\pi\pi} = (p_+ + p_-)^2, \\ s_{k_+k_-} &= s_{ll} = (k_+ + k_-)^2, \\ \beta_p &= \beta_\pi = \sqrt{1 - \frac{4m^2_{p_+p_-}}{s_{p_+p_-}}}, \\ \beta_k &= \beta_l = \sqrt{1 - \frac{4m^2_{k_+k_-}}{s_{k_+k_-}}}, \end{aligned} \tag{1}$$

where $m_{p_+p_-}$ is the mass of the outgoing particle pair p_+ and p_-. The invariant decay amplitude can be given as [3]:

$$\begin{aligned} &A(P \to \pi^+\pi^-l^+l^-) \\ &= \frac{1}{k^2} e\bar{u}(k_-, s_-)\gamma^\mu v(k_+, s_+) \\ &\quad \times (M\epsilon_{\mu\nu\alpha\beta}k^\nu p_+^\alpha p_-^\beta + E_+ p_+^\mu + E_- p_-^\mu), \end{aligned} \tag{2}$$

where $\bar{u}$ and v are the Dirac spinors which denote the lepton and anti-lepton separately and satisfy the orthogonal, normalized and complete relations

$$\begin{aligned} \bar{u}^{(1)}u^{(2)} &= 0, \quad \bar{u}u = 2m_l, \\ \bar{v}^{(1)}v^{(2)} &= 0, \quad \bar{v}v = -2m_l, \\ \sum_{s=1,2} u^{(s)}\bar{u}^{(s)} &= (\gamma_\mu k_-^\mu + m_l), \\ \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} &= (\gamma_\mu k_+^\mu - m_l). \end{aligned}$$

M is the usual magnetic form factor and E_+ and E_- are the CP violating electric form factors which can be zero to the leading order. After the Lorentz indices are contracted, the spins of the final states are added and the initial ones are averaged, the squared matrix element for the decay $P \to \pi^+\pi^-l^+l^-$ can be written as

$$\begin{aligned} &\overline{|\mathcal{A}_{P\to\pi^+\pi^-l^+l^-}|}^2(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_l, \phi) \\ &= \frac{e^2}{8k^2}|M(s_{\pi\pi}, s_{ll})|^2 \times \lambda(m_P^2, s_{\pi\pi}, s_{ll}) \\ &\quad \times [1 - \beta_l^2 \sin^2\theta_l \sin^2\phi] s_{\pi\pi}\beta_\pi^2 \sin^2\theta_\pi, \end{aligned} \tag{3}$$

to the leading order with only the magnetic term. Here $\theta_\pi = \theta_p$ is the polar angle of the p_+ in the p_+p_- rest frame with respect to the direction of flight of the p_+p_- in the P rest frame, $\theta_l = \theta_k$ is the polar angle of the k_- in the k_+k_- rest frame with respect to the direction of flight of the k_+k_- in the P rest frame, and ϕ is the azimuthal angle between the plane formed by p_+p_- in the P rest frame and the corresponding plane formed by the k_+k_- [13, 14]. The high order corrections with the electric term and the mixed term are visibly smaller than the leading one [15]. $\lambda(a,b,c) \equiv a^2 + b^2 + c^2 - 2(ab + bc + ca)$. The magnetic form factor $M(s_{\pi\pi}, s_{ll})$ can be given by

$$M(s_{\pi\pi}, s_{ll}) = \mathcal{M} \times \mathrm{VMD}(s_{\pi\pi}, s_{ll}), \tag{4}$$

where $\mathcal{M}$ is given in the pseudoscalar mesons mixing theory. For the $SU(3)$ flavor symmetry breaking, η and η' are the mixing states in the flavor octet and singlet pseudoscalar states (η_8 and η_0) with an η-η' mixing angle $\theta_{\mathrm{mix}} \approx -20°$ [16]. The mixing is described in the flavor states according to

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} -\sin\theta_{\mathrm{mix}} & \cos\theta_{\mathrm{mix}} \\ \cos\theta_{\mathrm{mix}} & \sin\theta_{\mathrm{mix}} \end{pmatrix} \cdot \begin{pmatrix} \eta_0 \\ \eta_8 \end{pmatrix}.$$

With this theory, the pseudoscalar mesons mixing parameter $\mathcal{M}$ is given as

$$\mathcal{M} = \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}\cos\theta_{\mathrm{mix}} - 2\sqrt{2}\frac{f_\pi}{f_0}\sin\theta_{\mathrm{mix}}\right) \quad \text{if} \quad P = \eta; \tag{5}$$

$$\mathcal{M} = \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}\sin\theta_{\mathrm{mix}} + 2\sqrt{2}\frac{f_\pi}{f_0}\cos\theta_{\mathrm{mix}}\right) \quad \text{if} \quad P = \eta', \tag{6}$$

with the pion decay constant $f_\pi \approx 92.4$ MeV, the octet pseudoscalar decay constant $f_8 \approx 1.3 f_\pi$, the singlet pseudoscalar decay constant $f_0 \approx 1.04 f_\pi$ and the $\mathrm{VMD}(s_{\pi\pi}, s_{ll})$ is the VMD factor [15, 17] and derived from the VMD model with finited-width corrections [18] as

$$\begin{aligned}\mathrm{VMD}(s_{\pi\pi},s_{ll}) = &\ 1-\frac{3}{4}(c_1-c_2+c_3)+\frac{3}{4}(c_1-c_2-c_3)\\ &\times\frac{m_V^2}{m_V^2-s_{ll}-\mathrm{i}m_V\Gamma(s_{ll})}\\ &+\frac{3}{2}c_3\frac{m_V^2}{m_V^2-s_{\pi\pi}-\mathrm{i}m_V\Gamma(s_{\pi\pi})}\\ &\times\frac{m_V^2}{m_V^2-s_{ll}-\mathrm{i}m_V\Gamma(s_{ll})},\end{aligned}\tag{7}$$

which goes to 1 as $s_{\pi\pi}$, s_{ll} goes to 0 in the on-shell case. The width of the vector meson Γ can be given by [15]

$$\Gamma(s)=g_{m_V}\left(\frac{s}{m_V^2}\right)\left(\frac{1-\dfrac{4m^2}{s}}{1-\dfrac{4m^2}{m_V^2}}\right)^{3/2}\Theta(s-4m_\pi^2),\tag{8}$$

which eliminates the singularities on the vector meson mass, where $g_{m_V}=149.1$ MeV. c_1, c_2 and c_3 are three different parameter combinations on which the Lagrangians depend and have to be fixed by comparison with data. The different VMD models can be derived with different choices of these parameter sets. In our work the parameters are chosen as

$$c_3=c_1-c_2=1\tag{9}$$

for the hidden gauge model.

The double off-shell decays of η/η' into two lepton pairs are related to the decay $\eta/\eta'\rightarrow\gamma\gamma$. The momenta of the decay $\mathrm{P(P)}\rightarrow \mathrm{e^+(p_1)e^-(p_2)}$ $\mu^+(p_3)\mu^-(p_4)$ are defined as $\mathrm{P}=\mathrm{p_1+p_2+p_3+p_4}$, and the relevant variables can be written as the same as the formulas in Eq. (1). The decay amplitude can be written as [15]

$$\begin{aligned}&\mathcal{A}(\mathrm{P}\rightarrow \mathrm{e^+e^-}\mu^+\mu^-)\\ =&\ \frac{M_1(s_{12}s_{34})}{s_{12}^2s_{34}^2}\epsilon_{\mu\nu\rho\sigma}(p_1+p_2)^\nu\\ &(p_3+p_4)^\sigma\bar{u}(p_2)\gamma^\mu v(p_1)\cdot\bar{u}(p_4)\gamma^\rho v(p_3).\end{aligned}\tag{10}$$

The squared matrix element $\mathcal{A}^2$ can be derived as

$$\begin{aligned}\overline{\mathcal{A}^2} = &\ \frac{e^4|M_1(s_{12}s_{34})|^2}{s_{12}s_{34}}\lambda(m_P^2,s_{12},s_{34})[2-\beta_{12}^2\sin^2\theta_{12}\\ &-\beta_{34}^2\sin^2\theta_{34}+\beta_{12}^2\beta_{34}^2\sin^2\theta_{12}\sin^2\theta_{34}\sin^2\phi].\end{aligned}\tag{11}$$

Here the polar angle θ_{ij} is the angle between p_i and p_P in the $l(p_i)l(p_j)$ rest frame, the azimuthal angle ϕ is the angle between the plane formed by $l(p_1)l(p_2)$ and $l(p_3)l(p_4)$.

The form factor can be given by the VMD model as [15]

$$M_1(s_{12}s_{34})=\mathcal{M}\times\mathrm{VMD}_1(s_{12},s_{34}),\tag{12}$$

where

$$\begin{aligned}\mathrm{VMD}_1(s_{12},s_{34})=&\ 1-c_3+c_3\times\frac{m_V^2}{m_V^2-s_{12}-\mathrm{i}m_V\Gamma(s_{12})}\\ &\times\frac{m_V^2}{m_V^2-s_{34}-\mathrm{i}m_V\Gamma(s_{34})},\end{aligned}\tag{13}$$

with $c_3=1$ for the hidden gauge model.

3 Event generators

Monte Carlo (MC) simulation is of great importance in the construction of the detector, the simulation of a detector response and comparison detector performance with expectations. The simulation of the BESIII detector is performed with a MC package based on GEANT4 [19], which gives the expectations of what a given process would look like in the BESIII detector. The generator framework is KKMC [20] combined with BesEvtGen [21], which is used to generate the charmonium decay events as close as possible to real data events taken at the BESIII detector. The former one is used to simulate the $\mathrm{e^+e^-}$ annihilation till $\mathrm{c\bar{c}}$ production and the latter is for generating charmonium decays.

In general, the BESIII simulation is accomplished by passing the four-vectors of all the particles produced by the event generator into a simulation package of the BESIII detector after taking into account the detector construction, the detector response, the interaction between the particles, and the materials. In this paper, to simplify the generator, we first generate the phase space events and then use the reject-accepted sampling method to model the rare decays of η/η' with the Doing It Yourself (DIY) [21] model in BesEvtGen.

The decay rate for the four-body phase space (Particle $\mathrm{P}\rightarrow 1+2+3+4$) can be written as [15, 22]

$$\begin{aligned}\mathrm{d}\Gamma = &\ \frac{|\mathcal{A}|^2|\tilde{\boldsymbol{k}}||\boldsymbol{p}_+^*||\boldsymbol{k}_+^{**}|}{2^{12}\pi^6m_P^2\sqrt{s_{k_+k_-}}\sqrt{s_{p_+p_-}}}\\ &\times\mathrm{d}s_{p_+p_-}\mathrm{d}s_{k_+k_-}\mathrm{d}\cos\theta_k\mathrm{d}\cos\theta_p\mathrm{d}\phi,\end{aligned}\tag{14}$$

where $\mathcal{A}$ is the decay amplitude for each specific process discussed in Section 2; $\tilde{\boldsymbol{k}}$ is the momentum of particle 3 in the P rest frame; $\boldsymbol{p}_+^*$ is the momentum of particle 1 in the $\mathrm{p_+p_-}$ rest frame and $\boldsymbol{k}_+^{**}$ is the momentum of particle 3 in the $\mathrm{k_+k_-}$ rest frame, as shown in Fig. 1. According to the kinematic conservation laws the decay rate can be written as

$$\mathrm{d}\Gamma = \frac{|\mathcal{A}|^2\beta_\mathrm{k}\beta_\mathrm{p}\lambda^{1/2}(m_\mathrm{P}^2, s_{\mathrm{p_+p_-}}, s_{\mathrm{k_+k_-}})}{2^{15}\pi^6 m_\mathrm{P}^3} \times \mathrm{d}s_{\mathrm{p_+p_-}}\mathrm{d}s_{\mathrm{k_+k_-}}\mathrm{d}\cos\theta_\mathrm{p}\mathrm{d}\cos\theta_\mathrm{k}\mathrm{d}\phi. \quad (15)$$

Using the relation $\sqrt{s_{\mathrm{p_+p_-}}}\mathrm{d}\sqrt{s_{\mathrm{p_+p_-}}} = \frac{1}{2}\mathrm{d}s_{\mathrm{p_+p_-}}$ and defining the invariant mass of $\mathrm{p_+p_-}$ as $m_{\mathrm{p_+p_-}} = \sqrt{s_{\mathrm{p_+p_-}}}$, the decay rate can be expressed as

$$\mathrm{d}\Gamma = \frac{|\mathcal{A}|^2\beta_\mathrm{k}\beta_\mathrm{p}\lambda^{1/2}(m_\mathrm{P}^2, m_{\mathrm{p_+p_-}}^2, m_{\mathrm{k_+k_-}}^2)}{2^{13}\pi^6 m_\mathrm{P}^3} m_{\mathrm{p_+p_-}} \times m_{\mathrm{k_+k_-}}\mathrm{d}m_{\mathrm{p_+p_-}}\mathrm{d}m_{\mathrm{k_+k_-}}\mathrm{d}\cos\theta_\mathrm{p}\mathrm{d}\cos\theta_\mathrm{k}\mathrm{d}\phi, \quad (16)$$

from which the theoretical curve of $\partial\Gamma/\partial m_{\mathrm{p_+p_-}}$ over the variable $m_{\mathrm{p_+p_-}}$ can be derived by integrating over the variables θ_p, θ_k and $m_{\mathrm{k_+k_-}}$. In the event generation, the mass dependent form factors are used for $\eta/\eta' \to \pi^+\pi^-l^+l^-$ and $e^+e^-\mu^+\mu^-$, which are described by Eq. (4) and Eq. (12), respectively. The masses of the mesons and leptons are set to be [23]

$$\begin{aligned} m_\eta &= 547.853\ \mathrm{MeV}, & m_{\eta'} &= 957.78\ \mathrm{MeV}, \\ m_{\pi_0} &= 134.9766\ \mathrm{MeV}, & m_\rho &= 775.49\ \mathrm{MeV}, \\ m_\mathrm{e} &= 0.511\ \mathrm{MeV}, & m_\mu &= 105.658\ \mathrm{MeV}. \end{aligned} \quad (17)$$

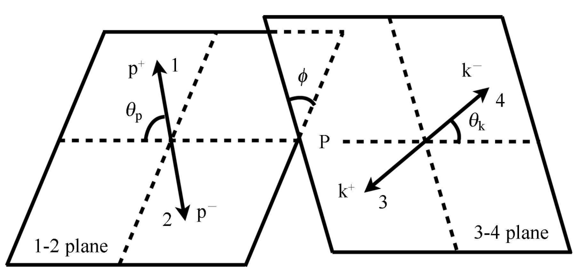

Fig. 1. Kinematics of the four-body decay of particle P.

To check the event generators, we presented the comparison of different variables between event generation and theoretical calculation at the truth level, which means we do not take the detector construction, the detector response, and the interaction between the particles and the materials into account. Take $\eta' \to \pi^+\pi^-e^+e^-$ for example, Fig. 2 shows the distributions of invariant mass of e^+e^- and $\pi^+\pi^-$ in which the histograms are the distributions from event generation and the curves show the theoretical calculation results with a normalization factor. The clear peak of invariant mass of $\pi^+\pi^-$ around 0.77 GeV in Fig. 2 is from ρ contribution. The theoretical curves coincide with the event generation results very well, which indicates that the event generators work very well after implementing in BESIII MC simulation package. The distributions obtained from the event generation with the phase space model are also displayed in the dashed histograms in Fig. 2. Compared with the phase space events, the discrepancy is obvious, which indicates that reliable event generators are very important for the determination of the detection efficiency of η/η' rare and forbidden decays. Fig. 3 shows the distributions of the polar angles $\cos\theta_\mathrm{l}$, $\cos\theta_\pi$ and the azimuthal angle ϕ from the event generation. The angle distributions are determined by the spin, the parity and the polarization of the initial states. For the conservation of parity, the helicity of the intermediate vector boson (virtual photon) can only be +1 or −1. Besides, the spin of leptons is 1/2 while the spin of pions is 0, the distributions of $\cos\theta_\mathrm{l}$ and $\cos\theta_\pi$ are different.

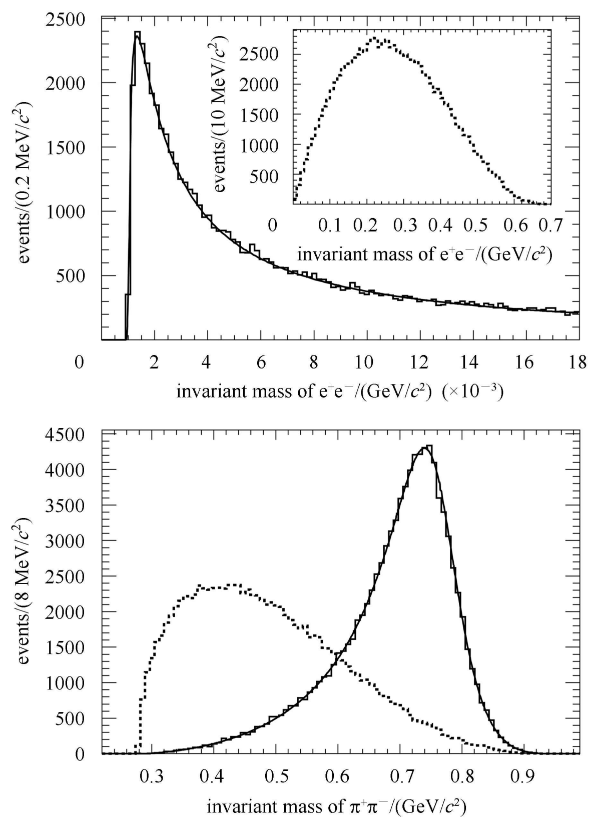

Fig. 2. Comparison of the distributions of invariant mass of e^+e^- and $\pi^+\pi^-$ of the process $\eta' \to \pi^+\pi^-e^+e^-$ between event generation and theoretical calculation. The histograms show the distributions derived with event generation and the curves show the theoretical calculation results. The phase space events are also shown by the dashed histograms.

We also worked out the generators of the processes $\eta' \to \pi^+\pi^-\mu^+\mu^-$, $\eta' \to e^+e^-\mu^+\mu^-$, $\eta \to e^+e^-\mu^+\mu^-$.

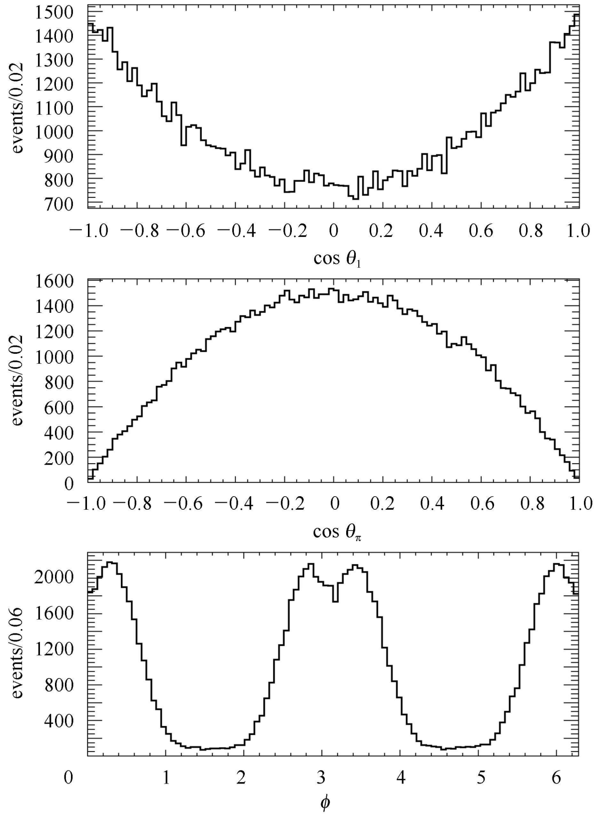

Fig. 3. The polar angles $\cos\theta_1$, $\cos\theta_\pi$ and the azimuthal angle ϕ distributions of the process $\eta' \to \pi^+\pi^- e^+e^-$.

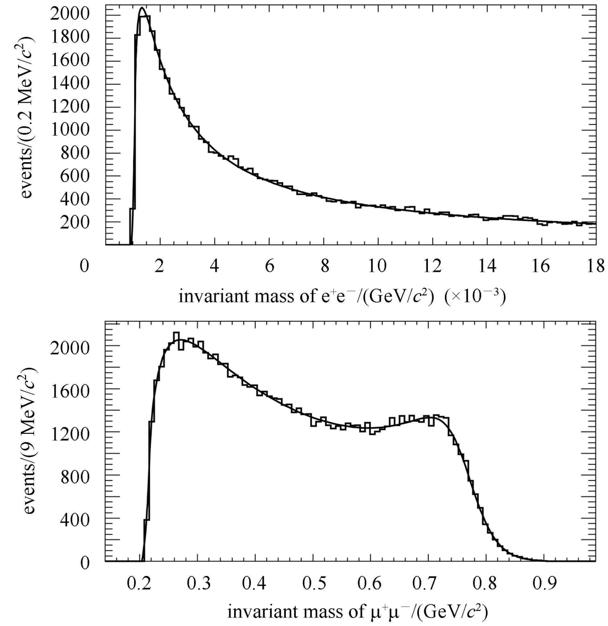

Fig. 5. Comparison of the distributions of invariant mass of e^+e^- and $\mu^+\mu^-$ of the process $\eta' \to e^+e^-\mu^+\mu^-$ between event generation and theoretical calculation. The histograms show the distributions derived with event generation and the curves show the theoretical results.

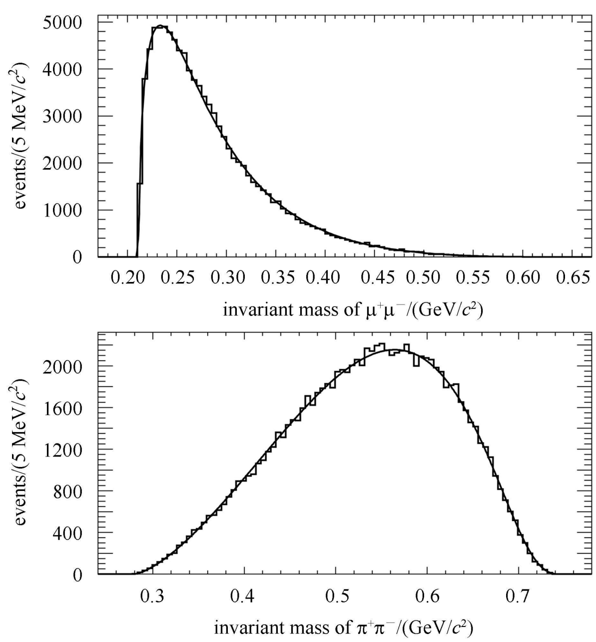

Fig. 4. Comparison of the distributions of invariant mass of $\mu^+\mu^-$ and $\pi^+\pi^-$ of the process $\eta' \to \pi^+\pi^-\mu^+\mu^-$ between event generation and theoretical calculation. The histograms show the distributions derived with event generation and the curves show the theoretical results.

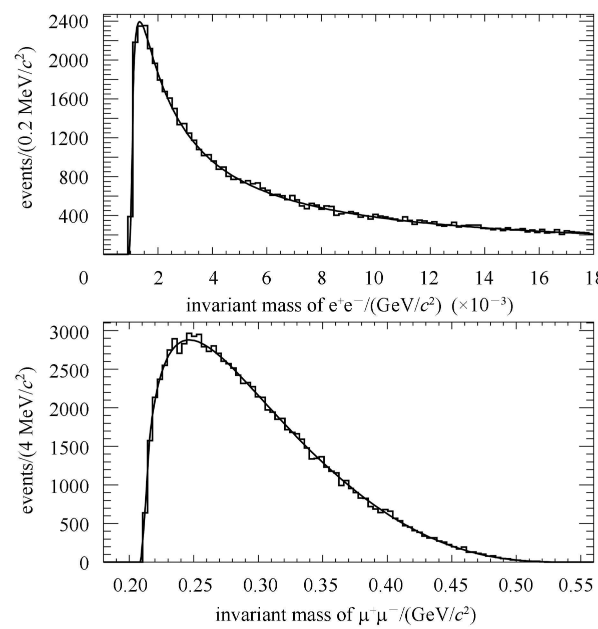

Fig. 6. Comparison of the distribution of invariant mass of e^+e^- and $\mu^+\mu^-$ of the process $\eta \to e^+e^-\mu^+\mu^-$ between event generation and theoretical calculation. The histograms show the distributions derived with event generation and the curves show the theoretical results.

The comparisons between event generation and theoretical calculation are displayed in Fig. 4, Fig. 5 and Fig. 6. All of them coincide with each other very well.

4 Summary

In summary, we have developed several event generators for generating the rare decays of η/η' to $\pi^+\pi^-e^+e^-$, $\pi^+\pi^-\mu^+\mu^-$, and $e^+e^-\mu^+\mu^-$. For the above decay modes, comparisons of various distributions from event generation to the theoretical calculation curves were performed. The results are in good agreement with each other, which indicates the simulation program works very well. At present the simulation of $\eta'\to\pi^+\pi^-e^+e^-$ and $\eta'\to\pi^+\pi^-\mu^+\mu^-$ is used in BESIII physics analysis and the MC simulation is in reasonable agreement with data [24]. In the BESIII physics analysis, the performance of these event generators will be improved in accordance with the distributions of different variables of η/η' from data and the improvement on the theoretical calculations.

For the other rare and forbidden decays of η/η' (e.g., such as four electrons, four muons, multi-lepton events), we will try to figure out their decay amplitudes and put them into the BESIII simulation package. Also we would like to update the theoretical parameters with the development of the latest theoretical predication and experimental results. All of them will provide a reliable simulation tool for studying η/η' physics at BESIII.

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