Parameterization of general Z-γ-Z$'$ mixing in an electroweak chiral theory *

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Abstract: A new general parameterization with eight mixing parameters among Z, γ and an extra neutral gauge boson Z$'$ is proposed and subjected to phenomenological analysis. We show that in addition to the conventional Weinberg angle θ_W, there are seven other phenomenological parameters, G', ξ, η, θ_l, θ_r, r and l, for the most general Z-γ-Z$'$ mixings, in which parameter G' arises due to the presence of an extra Stueckelberg-type mass coupling. Combined with the conventional Z-Z$'$ mass mixing angle θ', the remaining six parameters, ξ, η, $\theta_l-\theta'$, $\theta_r-\theta'$, r and l, are caused by general kinetic mixing. In all eight phenomenological parameters, θ_W, G', ξ, η, θ_l, θ_r, r and l, we can determine the Z-Z$'$ mass mixing angle θ' and the mass ratio $M_Z/M_{Z'}$. The Z-γ-Z$'$ mixing that we discuss are based on the model-independent description of the extended electroweak chiral Lagrangian (EWCL) previously proposed by us. In addition, we show that there are eight corresponding independent theoretical coefficients in our EWCL, which are fully fixed by our eight phenomenological mixing parameters. We further find that the experimental measurability of these eight parameters does not rely on the extended neutral current for Z$'$, but depends on the Z-Z$'$ mass ratio.

Key words: Z$'$ boson, chiral effective theory, parameterization

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1 Introduction

One of the simplest and more popular gauge extensions of the Standard Model (SM) is to add an extra $U(1)$ group associated with the Z$'$ gauge boson to the electroweak gauge group $SU(2)_L\otimes U(1)_Y$, which constitutes one of the "hot spots" in high energy physics today. The extra gauge boson Z$'$ is the carrier of a new gauge force corresponding to the smallest gauge group extensions that play a crucial role in cosmology, GUT, SUSY and various strong coupling new physics theories associated with new physics beyond the SM (for the latest review, see Ref. [1]). As long as a Z$'$ particle exists, it will shift observables from the present physics by mixing with the standard electroweak neutral gauge bosons, γ and Z. The corrections, however, depend on the details of the model set-up, and especially on the way the neutral gauge bosons mix. A model-independent way to figure out these mixings is through phenomenological requirements and constraints. Usually, theorists only consider minimal Z-Z$'$ mass mixing [2]. A massless photon constrains any possible extension of the mass mixings matrix to be of Stueckelberg-type [3]. However, theory and phenomenology do not forbid general three-body Z-γ-Z$'$ kinetic mixing. In the literature, only a few examples have been considered, such as the special kinetic mixings given in Refs. [4] and [5]. A general model-independent description of Z-γ-Z$'$ mixing is needed to enable data analysis and the experimental searches for Z$'$ to be more specific and effective, particularly in light of the progress made in the LHC experiments. With this motivation, we are prompted to study the most general gauge boson

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mixing. In fact, a general description of the Z′ interaction with SM particles has already been given in our previous work [3, 6], in which Z′ is regarded as a gauge boson of a broken $U(1)'$ symmetry and the conventional EWCL is extended to include this extra broken $U(1)'$ symmetry from the original $SU(2)_{\rm L}\otimes U(1)_{\rm Y}\to U(1)_{\rm em}$ to $SU(2)_{\rm L}\otimes U(1)_{\rm Y}\otimes U(1)'\to U(1)_{\rm em}$. In Ref. [3], the bosonic part up to order p^4 of the most general EWCL involving this Z′ boson and discovered particles has been proposed, and describes the most general Z-γ-Z′ mixing. In Ref. [6], various Z-γ-Z′ mixings that have appeared in the literature are shown to be included in our EWCL formalism and are further classified into five simple groupings. However, the expressions given in Refs. [3, 6] for these Z-γ-Z′ mixings are complex and are not suitable for phenomenological investigations.

The purpose of this paper is to improve this shortcoming by setting up a more general parameterization for all Z-γ-Z′ mixings to facilitate present and future phenomenological analysis in the EWCL given by Ref. [3]. We will discuss the physical meaning, origin and experimental measurability of these parameters within new parameterization. We show that there are eight independent degrees of freedom and all complexities of the mixing can be absorbed into eight phenomenological parameters, θ_W, G', ξ, η, θ_l, θ_r, r and l, for which all but the traditional Weinberg mixing angle θ_W and the Stueckelberg-type coupling G' combine with the conventional Z-Z′ mass mixing angle θ', and the remaining six parameters, ξ, η, $\theta_l-\theta'$, $\theta_r-\theta'$, r and l, are caused by general kinetic mixing. We will explicitly construct quantitative relations among these mixing parameters and those related to theoretical coefficients appearing in the underlying EWCL.

This paper is organized as follows. In Section 2, we give a short review of the relevant parts associated with the Z-γ-Z′ kinetic and mass mixings from the EWCL given in Ref. [3], and introduce the mixing matrix. In Section 3, we explain the physical meaning and origin of the eight parameters describing the mixing matrix by diagonalizing the mass-squared and kinetic matrices, and construct the relations among the various mixing matrix elements and coefficients in our EWCL. In Section 4, we first discuss the experimental measurability of the parameters arising in our new parameterization, and then express the EWCL coefficients related to Z-γ-Z′ mixing in these eight parameters, which transfer the measurability from the mixing parameters to the relevant EWCL coefficients. Section 5 then presents a summary.

2 Review of the kinetic and mass mixings from EWCL

We begin the discussion by first reviewing the EWCL of Z′ established in Ref. [3]. The general Lagrangian describing the gauge symmetry breaking $SU(2)_{\rm L}\otimes U(1)_{\rm Y}\otimes U(1)'\to U(1)_{\rm em}$ independent of the details of the symmetry breaking can be constructed in terms of 2×2 non-linear Goldstone field $\hat U$, with the following covariant derivative

$$D_\mu\hat U=\partial_\mu\hat U+{\rm i}gW_\mu\hat U-{\rm i}\hat U\left(g'\frac{\tau_3}{2}+\tilde g'\right)B_\mu-{\rm i}g''\hat U X_\mu,$$

where W_μ, B_μ and X_μ are gauge bosons corresponding to $SU(2)_{\rm L}$, $U(1)_{\rm Y}$ and $U(1)'$, respectively. Here, carets are used to distinguish the extended $U(1)'$ breaking quantities from the traditional electroweak breaking quantities in Ref. [7]. g, g', g'' and $\tilde g'$ are the $SU(2)_{\rm L}$ coupling, the conventional $U(1)_{\rm Y}$ coupling, the $U(1)'$ coupling and the special Stueckelberg-type gauge coupling, respectively.

In the paper by [3], the bosonic part of the Lagrangian up to order p^4 is presented. Because of our interest here in the Z′ mixing effects, we focus only on the neutral gauge boson mixing parts, which can be divided into a mass part $\mathcal{L}_{\rm M}$

$$\begin{aligned}\mathcal{L}_{\rm M}=&-\frac14 f^2{\rm tr}[\hat V_\mu^2]+\frac14\beta_1 f^2\left({\rm tr}[T\hat V_\mu]\right)^2\\&+\frac14\beta_2 f^2{\rm tr}[\hat V_\mu]{\rm tr}[T\hat V^\mu]+\frac14\beta_3 f^2\left({\rm tr}[\hat V_\mu]\right)^2\\&\overset{U=1}{===}\frac{f^2}{8}(1-2\beta_1)(gW_\mu^3-g'B_\mu)^2\\&+\frac{f^2}{2}(1-2\beta_3)(g''X_\mu+\tilde g'B_\mu)^2\\&+\frac{f^2}{2}\beta_2(g''X_\mu+\tilde g'B_\mu)(gW^{3,\mu}-g'B^\mu)\\\equiv&\ \frac12\mathcal{V}_\mu^{\rm T}\mathcal{M}_0^2\mathcal{V}_\mu\end{aligned}\tag{1}$$

and the kinetic part $\mathcal{L}_{\rm K}$

$$\begin{aligned}\mathcal{L}_{\rm K}=&-\frac14 B_{\mu\nu}^2-\frac12{\rm tr}[W_{\mu\nu}^2]-\frac14 X_{\mu\nu}^2\\&+\frac12\alpha_1 gg'B_{\mu\nu}{\rm tr}[TW^{\mu\nu}]+\frac14\alpha_8 g^2\left({\rm tr}[TW_{\mu\nu}]\right)^2\\&+gg''\alpha_{24}X_{\mu\nu}{\rm tr}[TW^{\mu\nu}]+g'g''\alpha_{25}B_{\mu\nu}X^{\mu\nu}\\&\overset{U=1}{===}-\frac14 B_{\mu\nu}B_{\mu\nu}-\frac14 X_{\mu\nu}X^{\mu\nu}\\&-\frac14(1-\alpha_8 g^2)(\partial_\mu W_\nu^3-\partial_\nu W_\mu^3)^2\end{aligned}$$

$$+\left(\frac{1}{2}\alpha_1 g g' B_{\mu\nu}+g g''\alpha_{24}X^{\mu\nu}\right)(\partial_\mu W^3_\nu-\partial_\nu W^3_\mu)$$
$$+g'g''\alpha_{25}B_{\mu\nu}X^{\mu\nu}\equiv-\frac{1}{4}\mathcal{V}^{\rm T}_{\mu\nu}\mathcal{K}_0\mathcal{V}^{\mu\nu}. \qquad (2)$$

Here, $T\equiv\hat{U}^\dagger\tau_3\hat{U}$ and $\hat{V}_\mu\equiv(\hat{D}_\mu\hat{U})\hat{U}^\dagger$ are $SU(2)_{\rm L}$ covariant operators. In $\mathcal{L}_{\rm M}$, the first term is the conventional non-linear σ model term and the fourth term is a new non-linear σ model term due to the presence of the $U(1)'$ Goldstone boson. The second term is the conventional custodial symmetry breaking term, and the third term is the mixing of the second and fourth terms. For $\mathcal{L}_{\rm K}$, with the exception of the standard kinetic terms for the $U(1)_{\rm Y}$, $SU(2)_{\rm L}$ and $U(1)'$ gauge bosons, the terms with coefficients α_1, α_{24} and α_{25} are the kinetic mixing terms between $U(1)$ and the diagonal part of the $SU(2)_{\rm L}$ gauge fields, between $U(1)'$ and the diagonal part of the $SU(2)_{\rm L}$ gauge fields, and between the $U(1)$ and $U(1)'$ gauge fields, respectively. The term with coefficients α_8 is the correction term for the diagonal part of the $SU(2)_{\rm L}$ gauge field. These coefficients parameterize the most general kinetic mixing among the Z-γ-Z$'$ bosons. For convenience, all these terms have been abbreviated into matrix forms in the unitary gauge $\hat{U}=1$ in the gauge boson vector $\mathcal{V}^{\rm T}_\mu=(W^3_\mu,\ B_\mu,\ X_\mu)$, the field strength tensor $\mathcal{V}_{\mu\nu}\equiv\partial_\mu\mathcal{V}_\nu-\partial_\nu\mathcal{V}_\mu$, the mass-squared matrix $\mathcal{M}^2_0$ and the kinetic matrix $\mathcal{K}_0$. From $\mathcal{M}^2_0$ and $\mathcal{K}_0$, we see that three body Z-γ-Z$'$ mixing is controlled by 11 dimensionless coefficients: four gauge couplings, g, g', $\tilde{g}'$ and g'', three mass-mixing low-energy constants β_1, β_2 and β_3, and four kinetic-mixing low-energy constants, α_1, α_8, α_{24} and α_{25}. Among these, only nine play roles in the sense that we can redefine nine new coefficients by absorbing β_1 and β_3 as follows

$$\begin{aligned}
g' &= \frac{\bar{g}'}{\sqrt{1-2\beta_1}},\ g=\frac{\bar{g}}{\sqrt{1-2\beta_1}},\\
g'' &= \frac{\bar{g}''}{\sqrt{1-2\beta_3}},\ \tilde{g}'=\frac{\bar{\tilde{g}}'}{\sqrt{1-2\beta_3}},\\
\beta_2 &= \bar{\beta}_2\sqrt{1-2\beta_1}\sqrt{1-2\beta_3},
\end{aligned} \qquad (3)$$

$$\begin{aligned}
&\alpha_{\rm a}=gg'\bar{\alpha}_1,\ \alpha_{\rm b}=g^2\bar{\alpha}_8,\ \alpha_{\rm c}=gg''\bar{\alpha}_{24},\\
&\alpha_{\rm d}=g'g''\bar{\alpha}_{25}.
\end{aligned} \qquad (4)$$

Then, $\mathcal{M}^2_0$ and $\mathcal{K}_0$ of these nine redefined coefficients become

$$\mathcal{M}^2_0=f^2\begin{pmatrix}
\frac{\bar{g}^2}{4} & -\frac{\bar{g}\bar{g}'}{4}+\frac{\bar{g}\bar{\tilde{g}}'\bar{\beta}_2}{2} & \frac{\bar{g}\bar{g}''\bar{\beta}_2}{2}\\
\frac{2\bar{g}\bar{\tilde{g}}'\bar{\beta}_2-\bar{g}\bar{g}'}{4} & \frac{\bar{g}'^2}{4}+\bar{\tilde{g}}'^2-\bar{g}\bar{\tilde{g}}'\bar{\beta}_2 & \bar{g}''\bar{\tilde{g}}'-\frac{\bar{g}'\bar{g}''\bar{\beta}_2}{2}\\
\frac{\bar{g}\bar{g}''\bar{\beta}_2}{2} & -\frac{\bar{g}'\bar{g}''\bar{\beta}_2}{2}+\bar{g}''\bar{\tilde{g}}' & \bar{g}''^2
\end{pmatrix}, \qquad (5)$$

$$\mathcal{K}_0=-\frac{1}{4}\begin{pmatrix}
1-\alpha_{\rm b} & -\alpha_{\rm a} & -2\alpha_{\rm c}\\
-\alpha_{\rm a} & 1 & -2\alpha_{\rm d}\\
-2\alpha_{\rm c} & -2\alpha_{\rm d} & 1
\end{pmatrix}. \qquad (6)$$

Furthermore, there exists a scale symmetry for $\mathcal{M}^2_0$ and $\mathcal{K}_0$, i.e. these are invariant under the following transformation determined by an arbitrary parameter ζ,

$$\bar{g}\to\zeta\bar{g},\ \bar{g}'\to\zeta\bar{g}',\ \bar{g}''\to\zeta\bar{g}'',\ \bar{\tilde{g}}'\to\zeta\bar{\tilde{g}}',\ f\to\frac{1}{\zeta}f, \qquad (7)$$

with $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$, $\alpha_{\rm d}$ unchanged. Since the dimensional coefficient f does not enter into the final mixing matrix, the above scale symmetry implies that among the nine redefined theoretical coefficients, only eight of these are independent, and span the largest mixing space for an extra neutral gauge boson Z$'$. We take these eight theoretical coefficients as $\bar{g}/\bar{g}'$, $\bar{\tilde{g}}'/\bar{g}''$, $\bar{g}_{\rm Z}/\bar{g}''$, $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$, $\alpha_{\rm d}$ with

$$\bar{g}_{\rm Z}\equiv\sqrt{\bar{g}^2+\bar{g}'^2}. \qquad (8)$$

These will provide all the combinations of extra neutral vector boson corrections to low-energy EW physics via mixing. As discussed in Ref. [6], if we then input a different set of values for these coefficients, then the effective theory can recuperate the various Z$'$ models that have been presented in the literature. The mixing can be disentangled by diagonalizing the mass-squared matrix $\mathcal{M}^2_0$ and kinetic matrix $\mathcal{K}_0$ simultaneously, i.e. through introducing in a 3×3 real matrix U, which relates the interaction eigenstate $(W^3_\mu,\ B_\mu,\ X_\mu)$ to the mass eigenstate $(Z_\mu,\ A_\mu,\ Z'_\mu)$ in the following manner

$$\begin{pmatrix}W^3_\mu\\ B_\mu\\ X_\mu\end{pmatrix}=U\begin{pmatrix}Z_\mu\\ A_\mu\\ Z'_\mu\end{pmatrix}. \qquad (9)$$

The U matrix has to fulfill conditions

$$U^{\mathrm{T}}\mathcal{M}_0^2U=\mathrm{diag}(M_{\mathrm{Z}}^2,0,M_{\mathrm{Z}'}^2),\ U^{\mathrm{T}}\mathcal{K}_0U=-\frac{1}{4}I. \quad (10)$$

In Refs. [3, 6], we have already discussed the exact form of U, although in practice its physical meaning tends to get lost due to its complex form, and is not suitable for presenting phenomenological arguments. Here, we simplify its expression by re-parameterizing it as follows,

$$U\equiv\begin{pmatrix} s_W\xi+c_Wc_ll & s_Wa & s_W\eta+c_Ws_rr \\ c_W\xi-s_Wc_ll & c_Wa & c_W\eta-s_Ws_rr \\ (s_Wc_ll-c_W\xi)G'-s_ll & -c_WaG' & (s_Ws_rr-c_W\eta)G'+c_rr \end{pmatrix}=U_0U_1, \quad (11)$$

$$U_0\equiv\begin{pmatrix} c_W & s_W & 0 \\ -s_W & c_W & 0 \\ s_WG' & -c_WG' & 1 \end{pmatrix},\ U_1\equiv\begin{pmatrix} lc_l & 0 & rs_r \\ \xi & a & \eta \\ -ls_l & 0 & rc_r \end{pmatrix}, \quad (12)$$

in which there are three angle parameters, θ_W, θ_r and θ_l, establishing the trigonometric values $c_i\equiv\cos\theta_i$, $s_i\equiv\sin\theta_i$ for $i=W$, l and r, and six other mixing parameters, G', a, ξ, η and r,l; nine in total. Among these nine parameters, $a=a(\theta_W,\ \theta_r,\ \theta_l,\ G',\ \xi,\ \eta,$ r and $l)$, is a single relation determining one of the other eight parameters; a detailed dependence will be given later in (59). Thus only eight of the nine parameters in (11) are independent, and the degrees of freedom just match the number of independent theoretical coefficients for electroweak gauge boson mixing that we counted before. In fact, because of the massless photon, parameter a is a normalization constant and plays the role of rescaling the photon field, which does not cause observable effects in the two-point vertices involving electroweak gauge bosons. Note that in the SM tree diagram limit, U_0 is a pure Weinberg rotation with $G'=0$, and U_1 is a unit matrix with $\theta_l=\theta_r=\xi=\eta=0$ and $l=r=a=1$.

3 Phenomenological parameters in terms of diagonalization and EWCL coefficients

Next, we explain the physical meaning and origin of the eight parameters, θ_W, G', ξ, η, θ_r, θ_l, r and l, by diagonalizing the mass-squared matrix $\mathcal{M}_0^2$ and kinetic matrix $\mathcal{K}_0$. First, G' is defined in such a way that it relates to the Stueckelberg-type coupling $\bar{\tilde{g}}'$ as

$$G'\equiv\frac{\bar{\tilde{g}}'}{\bar{g}''}=\frac{\tilde{g}'}{g''}. \quad (13)$$

i.e. G' is derived from the Stueckelberg coupling as the ratio of the Stueckelberg coupling and conventional $U(1)'$ coupling. In our EWCL formalism, the deviation from SM has two sources: a Stueckelberg-type interaction for B_μ and the extra $U(1)'$ interaction from gauge boson X_μ, with G' the relative ratio of the interaction strengths between these two types of sources. Theoretically, G' can take arbitrary real numbers, in particular $G'=\infty$ and $G'=0$ correspond to $g''=0$, $\tilde{g}'$ finite and $\tilde{g}'=0$, g'' finite, respectively. However, phenomenological analysis shows that a very large G' is not physically realistic, as Ref. [8] gives $G'=\tilde{g}'/g''=1.9/149\approx0.013$. If we ignore G', then the rotation matrix U_0 then reverts to the standard Weinberg rotation with Weinberg angle θ_W defined as

$$c_W\equiv\frac{\bar{g}}{\bar{g}_{\mathrm{Z}}},\ s_W\equiv\frac{\bar{g}'}{\bar{g}_{\mathrm{Z}}},\ \text{or,}\ \tan\theta_W=\frac{\bar{g}'}{\bar{g}}=\frac{g'}{g}. \quad (14)$$

The Weinberg angle originates from the mixing of field $W^{3,\mu}$ and B^μ and the Weinberg rotation enables the part of the mass matrix associated with γ and Z to be diagonalized if the Z′ particle and the Stueckelberg coupling are neglected. Once the Stueckelberg coupling $\bar{\tilde{g}}'$ shows up, there will be off diagonal matrix elements involving γ-Z and γ-Z′ mixings. To disentangle these mixings, we add G' terms to the U_0 matrix, and after the U_0 rotation we find

$$U_0^{\mathrm{T}}\mathcal{M}_0^2U_0=f^2\begin{pmatrix} \frac{1}{4}\bar{g}_{\mathrm{Z}}^2 & 0 & \frac{1}{2}\bar{g}_{\mathrm{Z}}\bar{g}''\bar{\beta}_2 \\ 0 & 0 & 0 \\ \frac{1}{2}\bar{g}_{\mathrm{Z}}\bar{g}''\bar{\beta}_2 & 0 & \bar{g}''^2 \end{pmatrix}. \quad (15)$$

This is a typical Z-Z′ mixing matrix. We apply a further matrix $\tilde{U}_0$ with rotation angle θ' to diagonalize (15), i.e.

$$\tilde{U}_0=\begin{pmatrix} c' & 0 & s' \\ 0 & 1 & 0 \\ -s' & 0 & c' \end{pmatrix},$$

$$\tilde{U}_0^{\mathrm{T}}U_0^{\mathrm{T}}\mathcal{M}_0^2U_0\tilde{U}_0=\mathrm{diag}(M^2,0,M'^2) \quad (16)$$

with $c'=\cos\theta', s'=\sin\theta'$. We find that this fixes the rotation angle θ' as follows

$$\tan\theta' = \frac{\varDelta_{\rm g}-\sqrt{\varDelta_{\rm g}^2+16\bar{g}_{\rm Z}^2\bar{g}''^2\bar{\beta}_2^2}}{4\bar{\beta}_2\bar{g}''\bar{g}_{\rm Z}}, \quad (17)$$

$$\varDelta_{\rm g} = \bar{g}_{\rm Z}^2-4\bar{g}''^2.$$

Hence θ' originates from the Z-Z′ mass mixing, its role is to disentangle this mixing, and it appears in most of the new physics models involving the Z′ boson. With the zero eigenvalue in (16) corresponding to the massless photon, the two other nonzero eigenvalues in (16) are

$$\frac{M^2}{f^2} = \frac{1}{4}\bar{g}_{\rm Z}^2c'^2+\bar{g}''^2s'^2-s'c'\bar{g}_{\rm Z}\bar{g}''\bar{\beta}_2,$$
$$\frac{M'^2}{f^2} = \bar{g}''^2c'^2+\frac{1}{4}\bar{g}_{\rm Z}^2s'^2+s'c'\bar{g}_{\rm Z}\bar{g}''\bar{\beta}_2\,. \quad (18)$$

Here, M and M' are just the Z and Z′ masses if there are no Stueckelberg and kinetic mixings. For $g''=\tilde{g}'=0$, (15) is already diagonal with eigenvalues

$$\frac{1}{4}f^2\bar{g}_{\rm Z}^2, 0, 0,$$

and there is no need to apply further rotation; clearly, $\theta'=0$ is given by (17), resulting in a unit matrix $\tilde{U}_0$. This further simplifies the eigenvalues of (18) to

$$M^2/f^2 = \frac{1}{4}\bar{g}_{\rm Z}^2,$$

and $M'^2/f^2=0$. Here, $M'=0$ implies that the mass of Z′ is zero and Z′ decouples from Z and γ.

After diagonalizing the mass-squared matrix $\mathcal{M}_0^2$, the next logical step is to further diagonalize the kinetic matrix $\mathcal{K}_0$. Considering that after the rotation $U_0\tilde{U}_0$, which diagonalizes $\mathcal{M}_0^2$, the kinetic matrix $\mathcal{K}_0$ is already transformed to a symmetric form

$$\tilde{U}_0^{\rm T}U_0^{\rm T}\mathcal{K}_0U_0\tilde{U}_0 = \begin{pmatrix} k_1 & k_2 & k_3 \\ k_2 & k_4 & k_5 \\ k_3 & k_5 & k_6 \end{pmatrix} \quad (19)$$

with

$$\begin{aligned} k_1 =\ & 1-2s_Ws'c'G'+s_W^2c'^2G'^2+2c_Ws_Wc'^2\alpha_{\rm a} \\ & -c_W^2c'^2\alpha_{\rm b}+(4c_Ws'c'-4c_Ws_Wc'^2G')\alpha_{\rm c} \\ & +(-4s_Ws'c'+4s_W^2c'^2G')\alpha_{\rm d}, \end{aligned} \quad (20)$$

$$\begin{aligned} k_2 =\ & c_Ws'G'-c_Ws_Wc'G'^2+(s_W^2-c_W^2)c'\alpha_{\rm a} \\ & -c_Ws_Wc'\alpha_{\rm b}+[2s_Ws'+2(c_W^2-s_W^2)c'G']\alpha_{\rm c} \\ & +(2c_Ws'-4c_Ws_Wc'G')\alpha_{\rm d}, \end{aligned} \quad (21)$$

$$\begin{aligned} k_3 =\ & -s_W(s'^2-c'^2)G'+s_W^2s'c'G'^2+2c_Ws_Ws'c'\alpha_{\rm a} \\ & -c_W^2c's'\alpha_{\rm b}+[2c_W(s'^2-c'^2)-4c_Ws_Ws'c'G']\alpha_{\rm c} \\ & +[2s_W(c'^2-s'^2)+4s_W^2s'c'G']\alpha_{\rm d}\,, \end{aligned} \quad (22)$$

$$\begin{aligned} k_4 =\ & 1+c_W^2G'^2-2c_Ws_W\alpha_{\rm a}-s_W^2\alpha_{\rm b} \\ & +4c_Ws_WG'\alpha_{\rm c}+4c_W^2G'\alpha_{\rm d}\,, \end{aligned} \quad (23)$$

$$\begin{aligned} k_5 =\ & -c_Wc'G'-c_Ws_Ws'G'^2+(s_W^2-c_W^2)s'\alpha_{\rm a} \\ & -c_Ws_Ws'\alpha_{\rm b}-[2s_Wc'-2(c_W^2-s_W^2)s'G']\alpha_{\rm c} \\ & -(2c_Wc'+4c_Ws_Ws'G')\alpha_{\rm d}\,, \end{aligned} \quad (24)$$

$$\begin{aligned} k_6 =\ & 1+2s_Ws'c'G'+s'^2s_W^2G'^2+2c_Ws_Ws'^2\alpha_{\rm a} \\ & -c_W^2s'^2\alpha_{\rm b}-(4c_Ws'c'+4c_Ws_Ws'^2G')\alpha_{\rm c} \\ & +(4s_Ws'c'+4s'^2s_W^2G')\alpha_{\rm d}. \end{aligned} \quad (25)$$

Note that as long as we have a nonzero Stueckelberg coupling G', then the rotated kinetic matrix $\tilde{U}_0^{\rm T}U_0^{\rm T}\mathcal{K}_0U_0\tilde{U}_0$ is not diagonal, even if the kinetic mixing coefficients $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$ and $\alpha_{\rm d}$ all vanish. For the special case, $g''=\tilde{g}'=0$, the matrix elements reduce to $k_3=k_5=0$ and $k_6=1$.

With these results, we introduce the matrix $\tilde{U}_1$ to further diagonalize the rotated kinetic matrix $\tilde{U}_0^{\rm T}U_0^{\rm T}\mathcal{K}_0U_0\tilde{U}_0$

$$\tilde{U}_1\equiv\tilde{U}_0^{-1}U_1 = \begin{pmatrix} l\cos(\theta_l-\theta') & 0 & r\sin(\theta_r-\theta') \\ \xi & a & \eta \\ -l\sin(\theta_l-\theta') & 0 & r\cos(\theta_r-\theta') \end{pmatrix}, \quad (26)$$

which changes the diagonal matrix diag $(M^2, 0, M'^2)$ to diag $(M_{\rm Z}^2, 0, M_{\rm Z'}^2)$ with

$$M_{\rm Z}^2 = M^2l^2\left\{\cos^2(\theta_l-\theta') + \frac{\cos(\theta_l-\theta')\sin(\theta_r-\theta')\sin(\theta_l-\theta')}{\cos(\theta_r-\theta')}\right\}, \quad (27)$$

$$M_{\rm Z'}^2 = M'^2r^2\left\{\cos^2(\theta_r-\theta') + \frac{\cos(\theta_r-\theta')\sin(\theta_r-\theta')\sin(\theta_l-\theta')}{\cos(\theta_l-\theta')}\right\}, \quad (28)$$

as long as we take

$$\frac{\tan(\theta_l-\theta')}{\tan(\theta_r-\theta')} = \frac{M^2}{M'^2}\,. \quad (29)$$

i.e.

$$\tilde{U}_1^{\rm T}\tilde{U}_0^{\rm T}U_0^{\rm T}\mathcal{K}_0U_0\tilde{U}_0\tilde{U}_1 = U_1^{\rm T}U_0^{\rm T}\mathcal{K}_0U_0U_1=U^{\rm T}\mathcal{K}_0U = -\frac{1}{4}{\rm diag}(1,1,1). \quad (30)$$

We see that the parameters in (26) play the role of generating the most general kinetic mixing. In particular, ξ and η originate from Z-γ and Z′-γ mixings, respectively, while l, r, $\theta_l-\theta'$ and $\theta_r-\theta'$ originate from the most general Z and Z′ redefinition and mixing, which need four independent parameters (two from redefinition and the other two from kinetic mixing).

The θ' appearing in (26) in the combinations of $\theta_l-\theta'$ and $\theta_r-\theta'$ is needed to subtract out Z-Z′ mass mixing from general Z-γ-Z′ mixing, leaving only the pure kinetic mixing. If there are no kinetic mixings, then

$$a=l=r=1,\ G'=\xi=\eta=0,\ \theta_l=\theta_r=\theta'. \quad (31)$$

By further requiring no Z-Z′ mass mixing by taking $\theta'=0$ in the above result, we recover the SM tree diagram limit mentioned previously.

Using (30), we then find

$$\frac{1}{a^2}=k_4, \quad (32)$$

which only rescales the photon field to a normalized kinetic form. Equation (29) gives one relation between the angle combinations $\theta_l-\theta'$ and $\theta_r-\theta'$, (30) further fixes $\tan(\theta_l-\theta')$ through the following quadratic equation

$$\left\{\frac{k_2k_5}{k_4}-k_3\right\}M^2M'^2\frac{\tan^2(\theta_l-\theta')}{M^4}+\left\{k_3-\frac{k_2k_5}{k_4}\right\} +\left\{\left(k_1-\frac{k_2^2}{k_4}\right)M'^2+\left(\frac{k_5^2}{k_4}-k_6\right)M^2\right\}\frac{\tan(\theta_l-\theta')}{M^2} =0. \quad (33)$$

There are two solutions from the above equation: one of these is chosen so that it vanishes in the limit $k_1=k_4=k_6=1$, $k_2=k_3=k_5=0$ for fixed M^2 and M'^2, and the other nonzero solution corresponds to the Z mass vanishing and γ receiving a nonzero mass. Combining the solution of (33) with Equation (29), we obtain $\theta_l-\theta'$ and $\theta_r-\theta'$. r and l can be determined by

$$\frac{1}{l^2} = \cos^2(\theta_l-\theta')\left\{\left(k_6-\frac{k_5^2}{k_4}\right)\tan^2(\theta_l-\theta') +2\left(\frac{k_2k_5}{k_4}-k_3\right)\tan(\theta_l-\theta')+k_1-\frac{k_2^2}{k_4}\right\}, \quad (34)$$

$$\frac{1}{r^2} = \cos^2(\theta_r-\theta')\left\{\left(k_1-\frac{k_2^2}{k_4}\right)\tan^2(\theta_r-\theta') +2\left(k_3-\frac{k_2k_5}{k_4}\right)\tan(\theta_r-\theta')+k_6-\frac{k_5^2}{k_4}\right\}. \quad (35)$$

With l, r, $\theta_l-\theta'$ and $\theta_r-\theta'$, ξ known, and η reexpressible

$$\frac{\xi}{l} = \frac{k_5\sin(\theta_l-\theta')-k_2\cos(\theta_l-\theta')}{k_4}, \quad (36)$$

$$\frac{\eta}{r} = -\frac{k_2\sin(\theta_r-\theta')+k_5\cos(\theta_r-\theta')}{k_4}. \quad (37)$$

As an example, we give the explicit result for the special case $g''=\tilde{g}=0$ (the present situation is the 0/0 case, here in the limiting procedure we let $\tilde{g}$ approach zero first, and then take g'' to zero, because as we mentioned before G' is small from purely phenomenological estimations), where the above considerations program gives the result:

$$\theta_l = \theta_r=\theta'=G'=\eta=0, \quad (38)$$

$$\frac{1}{a^2} = k_4,\ \frac{1}{l^2}=k_1-\frac{k_2^2}{k_4},\ r=1,\ \xi=-\frac{k_2l}{k_4}, \quad (39)$$

$$M_{\rm Z}^2 = M^2l^2,\ M^2=\frac{1}{4}\bar{g}_{\rm Z}^2f^2,\ M_{\rm Z'}^2=M'^2=0. \quad (40)$$

Up to this stage, once we know the coefficients in mass-squared matrix $\mathcal{M}_0^2$ and kinetic matrix $\mathcal{K}_0$, i.e. f and eight theoretical coefficients of EWCL $\bar{g}/\bar{g}'$, $\bar{\tilde{g}}'/\bar{g}''$, $\bar{g}_{\rm Z}/\bar{g}''$, $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$, $\alpha_{\rm d}$, we can obtain the final phenomenological mixing parameters, θ_W, θ_r, θ_l, G', ξ, η, l and r, and the intermediate mixing angle θ' and photon normalization factor a. In particular, the intermediate mass-squared ratio M^2/M'^2 is determined from (29) and the physical mass ratio $M_{\rm Z}/M_{\rm Z'}$ can be expressed as

$$\frac{M_{\rm Z}}{M_{\rm Z'}}=\frac{l}{r}\frac{\sin^{1/2}(2\theta_l-2\theta')}{\sin^{1/2}(2\theta_r-2\theta')}. \quad (41)$$

This result offers hope in predicting the Z′ mass in mixing parameters. Unfortunately, the mixing parameters themselves are not easy to test. In the next section, we will discuss the experimental measurability of the mixing parameters. Here we would rather treat the above relation as an additional constraint used in determining parameters for a given Z-Z′ mass ratio.

Phenomenologically, a more important question is, once we know the eight phenomenological mixing parameters, θ_W, θ_r, θ_l, G', ξ, η, l and r, from fitting the experimental data, how can we obtain the corresponding eight theoretical coefficients, $\bar{g}/\bar{g}'$, $\bar{\tilde{g}}'/\bar{g}''$,

$\bar{g}_{\rm Z}/\bar{g}''$, $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$ and $\alpha_{\rm d}$? Considering that the mixing parameter $G'=\bar{\tilde{g}}'/g''$ has already appeared in $\mathcal{M}_0^2$, i.e. it is both a theoretical coefficient and a phenomenological parameter, the remaining problem is to fix the other seven coefficients, $\bar{g}/\bar{g}'$, $\bar{g}_{\rm Z}/\bar{g}''$, $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$ and $\alpha_{\rm d}$, in eight phenomenological parameters, θ_W, θ_r, θ_l, G', ξ, η, l and r. Since the computation details are very complex, here we only outline the calculations. We choose seven equations, (14), (29), (33), (34), (35), (36) and (37), for which the auxiliary quantity θ' is further determined by (17), M^2/M'^2 by (18), and k_1, k_2, k_3, k_4, k_5 and k_6 by (23) to (25). By solving these equations, we can in principle express these theoretical coefficients in phenomenological parameters.

With the expressions of the EWCL coefficients of the phenomenological parameters, and with the help of (17) and (32), the conventional Z-Z$'$ mass mixing angle θ', the ratio $M_{\rm Z}/M_{\rm Z'}$ and a can all be expressed in the eight phenomenological mixing parameters.

The above procedure yields completely general results. To the terms of order p^4, we give explicit expressions for six phenomenological parameters, θ_r, θ_l, ξ, η and l,r, in terms of theoretical coefficients $\bar{g}'/\bar{g}$, $\bar{\tilde{g}}'/\bar{g}''$, $\bar{g}_{\rm Z}/\bar{g}''$, $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$, $\alpha_{\rm d}$:

$$\begin{aligned}\theta_r \approx\ & \theta'+\frac{4s_W\bar{g}''^2}{\Delta_{\rm g}}G'+\frac{s_W(5\bar{g}_{\rm Z}^2+12\bar{g}''^2)}{\Delta_{\rm g}}G'\theta'^2\\ &-\frac{4c_W(-2\bar{g}_{\rm Z}^2s_W^2+\Delta_{\rm g})\bar{g}''^2}{\Delta_{\rm g}^2}G'\alpha_{\rm a}+\frac{8s_W\bar{g}''^2}{\Delta_{\rm g}}\alpha_{\rm d}\\ &-\frac{4s_Wc_W^2\bar{g}_{\rm Z}^2\bar{g}''^2}{\Delta_{\rm g}^2}G'\alpha_{\rm b}-\frac{8c_W\bar{g}''^2}{\Delta_{\rm g}}\alpha_{\rm c},\end{aligned}\tag{42}$$

$$\begin{aligned}\theta_l \approx\ & \theta'+\frac{s_W\bar{g}_{\rm Z}^2}{\Delta_{\rm g}}G'+\frac{s_W(3\bar{g}_{\rm Z}^2+20\bar{g}''^2)}{\Delta_{\rm g}}G'\theta'^2\\ &-\frac{c_W\bar{g}_{\rm Z}^2(-2\bar{g}_{\rm Z}^2s_W^2+\Delta_{\rm g})}{\Delta_{\rm g}^2}G'\alpha_{\rm a}\\ &-\frac{s_Wc_W^2\bar{g}_{\rm Z}^4}{\Delta_{\rm g}^2}G'\alpha_{\rm b}-\frac{2c_W\bar{g}_{\rm Z}^2}{\Delta_{\rm g}}\alpha_{\rm c}+\frac{2s_W\bar{g}_{\rm Z}^2}{\Delta_{\rm g}}\alpha_{\rm d},\end{aligned}\tag{43}$$

$$\begin{aligned}r \approx\ & 1-s_WG'\theta'+\frac{2c_Ws_W(\bar{g}_{\rm Z}^2+4\bar{g}''^2)}{\Delta_{\rm g}}G'\alpha_{\rm c}\\ &+\frac{2(c_W^2\bar{g}_{\rm Z}^2+4(c_W^2-2)\bar{g}''^2)}{\Delta_{\rm g}}G'\alpha_{\rm d},\end{aligned}\tag{44}$$

$$\begin{aligned}l \approx\ & 1+s_WG'\theta'-s_Wc_W\alpha_{\rm a}+\frac{c_W^2}{2}\alpha_{\rm b}\\ &-\frac{2c_Ws_W(\bar{g}_{\rm Z}^2+4\bar{g}''^2)}{\Delta_{\rm g}}G'\alpha_{\rm c}\\ &+\frac{2s_W^2(\bar{g}_{\rm Z}^2+4\bar{g}''^2)}{\Delta_{\rm g}}G'\alpha_{\rm d},\end{aligned}\tag{45}$$

$$\begin{aligned}\xi \approx\ & -c_WG'\theta'+(2c_W^2-1)\alpha_{\rm a}+c_Ws_W\alpha_{\rm b}\\ &+\frac{8(2c_W^2-1)\bar{g}''^2}{\Delta_{\rm g}}G_3\alpha_{\rm c}-\frac{16c_Ws_W\bar{g}''^2}{\Delta_{\rm g}}G'\alpha_{\rm d},\end{aligned}\tag{46}$$

$$\begin{aligned}\eta \approx\ & c_WG'-\frac{c_W}{2}G'\theta'^2+\frac{2s_W(c_W^2\bar{g}_{\rm Z}^2-2\bar{g}''^2)}{\Delta_{\rm g}}G'\alpha_{\rm a}\\ &+\frac{\bar{g}_{\rm Z}^2c_Ws_W^2}{\Delta_{\rm g}}G'\alpha_{\rm b}+2s_W\alpha_{\rm c}+2c_W\alpha_{\rm d}.\end{aligned}\tag{47}$$

Here, $\theta'\approx-2\bar{g}_{\rm Z}\bar{g}''\bar{\beta}_2/\Delta_{\rm g}$, $\theta_W=\arctan\bar{g}'/\bar{g}$ and $G'=\bar{\tilde{g}}'/\bar{g}''$. Moreover, we obtain

$$\begin{aligned}a \approx\ & 1+c_Ws_W\alpha_{\rm a}+\frac{s_W^2}{2}\alpha_{\rm b}-2c_Ws_WG'\alpha_{\rm c}\\ &-2c_W^2G'\alpha_{\rm d}\,,\end{aligned}\tag{48}$$

$$\theta' \approx \frac{\bar{g}_{\rm Z}^2\theta_r-4\bar{g}''^2\theta_l}{\Delta_{\rm g}}\,.\tag{49}$$

Note that since (31) tells us that if there are no kinetic mixings, $\theta_l=\theta_r=\theta'$, then the differences $\theta_l-\theta'$ and $\theta_r-\theta'$ reflect the effects caused by kinetic mixing. Substituting (42) and (43) into (29), we find the result for M^2/M'^2, which just matches the results that we obtained from (18). Although our result here already includes all possible mixing cases, pure Z-Z$'$ mass mixing is worthy of a special discussion. We find that the limit $G'=\alpha_{\rm c}=\alpha_{\rm d}=0$ can not be taken at the very beginning, since this will lead to $\theta_r=\theta_l=\theta'$ from (42) to (43) and then limit problems 0/0 in (29) for M^2/M'^2. To obtain the correct result, we first need to maintain G' and $\alpha_{\rm c}$, $\alpha_{\rm d}$ with nonzero values through completion of the computation of the ratio M^2/M'^2, and then take its vanishing limit. This is an interesting new phenomenon, i.e. nonzero G' and $\alpha_{\rm c}$, $\alpha_{\rm d}$ extensions make it possible for M^2/M'^2 to be expressed in the mixing parameters. In contrast with the pure Z-Z$'$ mass mixing case from (18), we find that just the mixing angle θ' cannot fully fix the value of M^2/M'^2, as we are left with $\bar{\beta}_2$ degrees of freedom remaining.

4 Measurability of the parameters and relevant EWCL coefficients

Compared with the coefficients in EWCL, our eight parameters, θ_W, G', ξ, η, θ_l, θ_r, r and l, are closer to the experimental data and more easily determined experimentally. Once these are known, the relevant EWCL coefficients can be further determined by establishing the relations between these parameters and the EWCL coefficients. In this section, we begin by discussing how these parameter values can be fixed in principle from the experiment, and then construct the relations among the EWCL coefficients and parameters.

Experimentally, with the exception of $SU(2)_{\rm L}$ coupling g, which can be determined from charged currents, the main means to determine the mixing parameters are by testing the structure of the electro-magnetic and neutral currents. The corresponding Lagrangian is $gW^3_\mu J^{3,\mu}+g'B_\mu J^\mu_Y+g''X_\mu J^\mu_X$, where $J^{3,\mu}$ is the third component of the conventional weak isospin current, J^μ_Y is the hypercharge current, and J^μ_X is the current coupled to the extra X_μ boson. The physical bosons Z, γ, Z′, the Lagrangian of the electro-magnetic and the neutral currents becomes $eJ^\mu_{\rm em}A_\mu+g_{\rm Z}J^\mu_{\rm Z}Z_\mu+g''J^\mu_{\rm Z'}Z'_\mu$. With the help of (9), we can read off

$$\begin{aligned} eJ^\mu_{\rm em} &= gU_{1,2}J^{3,\mu}+g'U_{2,2}J^\mu_Y+g''U_{3,2}J^\mu_X \\ &= gs_Wa[J^{3,\mu}+J^\mu_Y]+g''U_{3,2}J^\mu_X, \end{aligned} \tag{50}$$

$$\begin{aligned} g_{\rm Z}J^\mu_{\rm Z} &= gU_{1,1}J^{3,\mu}+g'U_{2,1}J^\mu_Y+g''U_{3,1}J^\mu_X \\ &= g[(s_W\xi+c_Wc_l l)J^{3,\mu} \\ &\quad +(s_W\xi-s_Wc_l l\tan\theta_W)J^\mu_Y]+g''U_{3,1}J^\mu_X, \end{aligned} \tag{51}$$

$$g''J^\mu_{\rm Z'} = gU_{1,3}J^{3,\mu}+g'U_{2,3}J^\mu_Y+g''U_{3,3}J^\mu_X\,, \tag{52}$$

with $U_{i,j}$ a general matrix element of mixing matrix U, and we have used the result $gU_{1,2}=g'U_{2,2}$ combined with (11) and (14). In principle, once the experiments finally fix the coefficients $U_{i,j}$, then from (11), we can determine all eight parameters, θ_W, G', ξ, η, θ_l, θ_r, r and l. Considering the fact that Z′ has not been discovered as yet in current experiments, we divide the present experimental measurability of the parameters into two stages.

1) Suppose we can measure $eJ^\mu_{\rm em}$ and $g_{\rm Z}J^\mu_{\rm Z}$ experimentally but do not know what $J^\mu_{\rm Z'}$ and J^μ_X are. This is the present SM situation as it stands and is independent of details of the Z′ model. Then (50) implies that we can determine gs_Wa and the electro-magnetic coupling e now must be identified as $e=gs_Wa$. Compared with the conventional relation in SM, we find that an extra correction factor a appears in the relation. Considering that e and g can be measured from electro-magnetic and charge currents, respectively, we can then derive s_Wa. Further, from (51), we find $g(s_W\xi+c_Wc_l l)$ and $g(s_W\xi-s_Wc_l l\tan\theta_W)$. Then, in this first stage, combined with known g, we can obtain four combinations of the eight parameters: g, s_Wa, $s_W\xi+c_Wc_l l$ and $s_W\xi-s_Wc_l l\tan\theta_W$.

2) Suppose in addition to $eJ^\mu_{\rm em}$ and $g_{\rm Z}J^\mu_{\rm Z}$, we also know J^μ_X. This can be realized if we have prior $U(1)'$ charges for the SM fermions, which is Z′ model-dependent. Then from (50) and (11), $g''U_{3,2}=g''(s_W\eta+c_Ws_rr)$ is obtainable; and from (51) and (11), $g''U_{3,1}=g''(c_W\eta-s_Ws_rr)$ is calculable. We find at this second stage that we can obtain two further combinations of the eight parameters.

Therefore, before needing to measure $g''J^\mu_{\rm Z'}$, the above two stages already enable us to evaluate seven of the eight parameters. Using (41), the remaining unknown parameter can be determined once we assume a Z-Z′ mass ratio. Thus, even without the knowledge of $g''J^\mu_{\rm Z'}$, and as long as the Z-Z′ mass ratio is fixed, we can now measure all eight phenomenological parameters.

In consequence, we can express the EWCL coefficients in these parameters. Up to order p^4, the theoretical coefficients $\bar g_{\rm Z}/\bar g''$, $\bar\beta_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$ and $\alpha_{\rm d}$ in phenomenological parameters θ_W, θ_r, θ_l, G', ξ, η, l and r can be written as

$$\frac{\bar g_{\rm Z}}{\bar g''}\approx\frac{2(\theta_l-\theta')}{\theta_r-\theta'}, \tag{53}$$

$$\bar\beta_2\approx-\frac{\bar g^2_{\rm Z}\theta_r-4\bar g''^2\theta_l}{2\bar g_{\rm Z}\bar g''}, \tag{54}$$

$$\begin{aligned} \alpha_{\rm a} = &-\frac{1}{4s_Wc_W\bar g''^2\varDelta_{\rm g}}\Big\{8s^2_W\bar g''^2\varDelta_{\rm g}(l-1) \\ &+s_W(\bar g^2_{\rm Z}s^2_W+2(c^2_W-2)\bar g''^2)\varDelta_{\rm g}G'\theta' \\ &+(\bar g^2_{\rm Z}s^2_W+(4-2c^2_W)\bar g''^2)\varDelta_{\rm g}(r-1) \\ &-4c_Ws_W\bar g''^2\varDelta_{\rm g}\xi \\ &+c_W(-\bar g^4_{\rm Z}s^2_W-2\bar g''^2c^2_W\varDelta_{\rm g})G'\eta\Big\}, \end{aligned} \tag{55}$$

$$\begin{aligned} \alpha_{\rm b} = &-\frac{1}{4c^2_W\bar g''^2\varDelta_{\rm g}}\Big\{((1-2c^2_W)\bar g^2_{\rm Z}+4\bar g''^2s^2_W)\varDelta_{\rm g}(r-1) \\ &+s_W(\varDelta_{\rm g}-2c^2_W\bar g^2_{\rm Z}+4c^2_W\bar g''^2)\varDelta_{\rm g}G'\theta' \\ &+8\bar g''^2(1-2c^2_W)\varDelta_{\rm g}(l-1)-8s_Wc_W\bar g''^2\varDelta_{\rm g}\xi \\ &+c_W(-\bar g^4_{\rm Z}s^2_W-16\bar g''^4s^2_W+\bar g^2_{\rm Z}c^2_W\varDelta_{\rm g})G'\eta\Big\}, \end{aligned} \tag{56}$$

$$\begin{aligned} \alpha_{\rm c} = &\frac{1}{8s_Wc_W\bar g''^2\varDelta_{\rm g}}\Big\{s_Wc^2_W\varDelta^2_{\rm g}\theta'-c^2_Ws_W\varDelta^2_{\rm g}\theta_r \\ &+s^2_Wc^2_W(5\bar g^2_{\rm Z}+14s_W\bar g''^2)\varDelta_{\rm g}G'\theta'^2 \\ &-8s^2_W\bar g''^2(-\bar g^2_{\rm Z}s^2_W+4\bar g''^2)G'(l-1) \\ &+(s^4_W\bar g^4_{\rm Z}-16\bar g''^4+2\bar g''^2c^4_W\bar g^2_{\rm Z}+8\bar g''^4c^2_W)G'(r-1) \end{aligned}$$

$$+4s_Wc_W\bar{g}''^2(\bar{g}_{\rm Z}^2(c_W^2-2)+4\bar{g}''^2)G'\xi$$

$$+4s_W^2c_W\bar{g}''^2\Delta_{\rm g}\eta\Big\}, \quad (57)$$

$$\alpha_{\rm d} = -\frac{1}{8\bar{g}''^2\Delta_{\rm g}}\Big\{(5\bar{g}_{\rm Z}^2s_W^2+(12-14c_W^2)\bar{g}''^2)\Delta_{\rm g}G'\theta'^2$$

$$+s_W\Delta_{\rm g}^2\theta'+4\bar{g}''^2\Delta_{\rm g}G'-8s_W^2\bar{g}_{\rm Z}^2\bar{g}''^2G'(l-1)$$

$$-s_W\Delta_{\rm g}^2\theta_r+\bar{g}_{\rm Z}^2(-\bar{g}_{\rm Z}^2s_W^2+(-4+2c_W^2)\bar{g}''^2)G'(r-1)$$

$$+4s_Wc_W\bar{g}_{\rm Z}^2\bar{g}''^2G'\xi-4c_W\bar{g}''^2\Delta_{\rm g}\eta\Big\}, \quad (58)$$

where θ' is given by (49) and $\bar{g}_{\rm Z}/\bar{g}''$ is given by (53). The remaining two theoretical coefficients $\bar{g}'/\bar{g}$ and $\bar{\bar{g}}'/\bar{g}''$, which are already determined in (14) and (13), respectively, are not displayed with the above formulae. Substituting the results back into (32) and combining with (23), we further obtain

$$a = 1-\frac{1}{8c_W^2\bar{g}''^2\Delta_{\rm g}}\Big\{s_W(s_W^2\bar{g}_{\rm Z}^2-4\bar{g}''^2)\Delta_{\rm g}G'\theta'$$

$$+8s_W^2\bar{g}''^2\Delta_{\rm g}(l-1)+(\bar{g}_{\rm Z}^2s_W^2+4\bar{g}''^2)\Delta_{\rm g}(r-1)$$

$$-8s_Wc_W\bar{g}''^2\Delta_{\rm g}\xi-c_W(\bar{g}_{\rm Z}^4s_W^2-4\bar{g}_{\rm Z}^2\bar{g}''^2c_W^2$$

$$+16\bar{g}''^4)G'\eta\Big\}. \quad (59)$$

The results (49) to (59) indicate that once we know the eight phenomenological parameters, θ_W, G', ξ, η, θ_l, θ_r, r and l, the conventional Z-Z$'$ mixing angle θ', then the general Z-γ-Z$'$ mixing coefficients $\bar{g}/\bar{g}'$, $\bar{g}_{\rm Z}/\bar{g}''$, G', $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$ and $\alpha_{\rm d}$, and a are fixed, where the a parameter, although it appears in the phenomenological role as discussed earlier, is derivable from the other eight parameters through (59).

5 Summary

To summarize our results, based on the extended electroweak chiral Lagrangian previously proposed by us, we have found that there are eight independent degrees of freedoms to describe the most general Z-γ-Z$'$ mixings that correspond to the eight independent theoretical coefficients, $\bar{g}/\bar{g}'$, $\bar{\bar{g}}'/\bar{g}''$, $\bar{g}_{\rm Z}/\bar{g}''$, $\bar{\beta}_2$, $\alpha_{\rm a}$, $\alpha_{\rm b}$, $\alpha_{\rm c}$ and $\alpha_{\rm d}$, in our electroweak chiral Lagrangian. For convenience in the phenomenological analysis, we have proposed a new general parameterization involving these eight parameters that describes the Z-γ-Z$'$ mixings, including the conventional Weinberg angle θ_W and a Stueckelberg-type coupling G'. Combined with the conventional Z-Z$'$ mass mixing parameter θ', we find that parameters ξ, η, $\theta_l-\theta'$, $\theta_r-\theta'$, r and l reflect the general kinetic mixings among the Z-γ-Z$'$. With this parameterization, θ_W, G', ξ, η, θ_l, θ_r, r and l, we can fully determine the Z-Z$'$ mass mixing angle θ' and the mass ratio $M_{\rm Z}/M_{\rm Z'}$. Experimentally, with the knowledge of charge currents, neutral currents and the current for the extra gauge boson X_μ, combined with mass ratio $M_{\rm Z}/M_{\rm Z'}$, we can in principle measure all eight parameters.

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