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Lattice calculation of the κ meson*

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Abstract: We study the κ meson in $2+1$ flavor QCD with sufficiently light u/d quarks. Using numerical simulations, we measure the point-to-point κ correlators in the "Asqtad" improved staggered fermion formulation. We then analyze these correlators using rooted staggered chiral perturbation theory (rSχPT), with particular attention paid to bubble contribution. After chiral extrapolation, we obtain the physical κ mass with 828±97 MeV, which is within the recent experimental value of 800–900 MeV. These numerical simulations are carried out with MILC $2+1$ flavor gauge configurations at a lattice spacing of $a \approx 0.12$ fm.

Key words: lattice QCD, kappa meson, chiral extrapolation

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1 Introduction

The so-called κ meson ($J^P=0^+$) is a scalar meson with strangeness. In 2010, the Particle Data Group (PDG) [1] listed the $K_0^*(800)$ or κ meson with a very broad width (550 MeV). A recent analysis [2] gave its mass at about 750^{30}_{50} MeV. Moreover, the resonance of a scalar meson was reported [2–4] to exist in the πK system with an κ meson mass of about 800 MeV.

Until now, four lattice simulations of the κ mass have been reported. Prelovsek et al. reported a crude estimation of the κ mass as 1.6 GeV by extrapolating the a_0 mass [5]. Mathur et al. [6] studied the $u\bar{s}$ scalar meson in the quenched approximation, and obtained the value of the κ mass to be 1.41 ± 0.12 GeV after removing the fitted $\pi\eta'$ ghost. With the dynamical $N_f=2$ sea quarks and a valence strange quark, the UKQCD collaboration [7] suggested an κ mass around 1000–1200 MeV. The SCALAR collaboration [8, 9] reported full QCD simulations on the κ meson using a dynamical fermion for the light u/d quark and valence approximation for the strange quark, which showed that the $I=1/2$ scalar meson had a mass around 1.8 GeV. In Ref. [10], a quenched QCD calculation was performed using Wilson fermions with plaquette gauge action, and estimated the κ mass to be about 1.7 GeV.

In the presence of 2+1 flavors of Asqtad improved staggered dynamical sea quarks, we obtained the κ mass as low as 826 ± 119 MeV in our previous study [11]. However, we neglected taste-symmetry breaking and used crude linear extrapolation. It is well known that, in the staggered fermion formulation of lattice QCD, due to taste-symmetry breaking there exist many multihadron states with $J^P=0^+$ which can proliferate between the source and sink of the κ correlator. Of special interest for us are the two-pseudoscalar states (i.e., bubble contributions) [12, 13]. With a sufficiently light u quark, the κ meson propagator is dominated at large time distances by these two-meson states. The bubble contributions are significantly affected by the unphysical approximations which are often used in lattice simulations [12, 13]. They are expected to disappear in the continuum limit.

In our previous work [14–17], we extended the analysis of Refs. [12, 13], examined the scalar meson correlators in lattice QCD with the inclusion of the disconnected diagrams, and carried out a quantitative comparison of the measured correlators and the predictions from rSχPT. Despite the considerable complexity of the scalar meson channels with dozens of spectral components, the rSχPT provides a strict framework which permits the analysis of the scalar

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meson correlator precisely in terms of only a small number of low-energy chiral couplings, which we may determine through fits to the data. In this work we extend the analyses of Refs. [14–17] to the κ meson; treat the u quark as a valence approximation quark, while the valence strange quark mass is fixed to its physical value [18] when considering that the κ meson contains a strange quark and a light u quark; perform a series of numerical simulations with MILC gauge configurations in the presence of $2+1$ flavors of Asqtad improved staggered dynamical sea quarks generated by the MILC collaboration [19]; and chirally extrapolate the mass of the κ meson to the physical π mass using the popular four parameter fit with the inclusion of chiral logarithms.

2 Pseudoscalar meson taste multiplets

In Refs. [14, 15], we give a brief review of the rooted staggered chiral perturbation theory with particular focus on the tree-level pseudoscalar mass spectrum, and achieve the rooted version of the theory through the replicated theory [20].

The tree-level masses of the pseudoscalar mesons are [15, 21]

$$M^2_{x,y,b} = \mu(m_x + m_y) + a^2\Delta_b, \tag{1}$$

where x, y are two quark flavor contents which make up, $b = 1,\cdots,16$ are the taste, $\mu = m^2_\pi/2m_\mathrm{q}$ is the low-energy chiral coupling constant of the point scalar current to the pseudoscalar field, and the term $a^2\Delta_b$ comes from taste symmetry breaking. m_x and m_y are the two valence quark masses in the pseudoscalar meson, and m_x is the light valence u/d quark mass by convention.

In this work we investigate degenerate u and d quarks, and treat the u quark as a valence approximation quark, while the valence strange quark mass is fixed to its physical value, m_s, thus it will be convenient to introduce the notations

$$\begin{aligned} M^2_{\mathrm{Ub}} &\equiv M_{\pi_\mathrm{b}} = 2\mu m_x + a^2\Delta_b, \\ M^2_{\mathrm{Sb}} &\equiv M_{\mathrm{ss,b}} = 2\mu m_\mathrm{s} + a^2\Delta_b, \\ M^2_{\mathrm{Kb}} &\equiv M_{\mathrm{K_b}} = \mu(m_x + m_s) + a^2\Delta_b, \end{aligned} \tag{2}$$

where M_U is the mass of the Goldstone pion with two light valence quark masses, M_K is the mass of the Goldstone kaon with one valence quark equal to the light valence quark and one at its physical mass m_s, and M_S is the mass of a fictitious meson $\mathrm{s\bar{s}}$ in a flavor non-singlet state [22] with two valence quarks at physical mass m_s.

The isosinglet states (η and η′) are modified both by the taste-singlet anomaly and by the two-trace (quark-line hairpin) taste-vector and taste-axial-vector operators [22, 23]. When the anomaly parameter m_0 is large, we obtain the usual result

$$M^2_{\eta,\mathrm{I}} = \frac{1}{3}M^2_{\mathrm{UI}} + \frac{2}{3}M^2_{\mathrm{SI}}, \quad M_{\eta',\mathrm{I}} = \mathcal{O}(m^2_0). \tag{3}$$

In the taste-axial-vector sector we have

$$\begin{aligned} M^2_{\eta\mathrm{A}} &= \frac{1}{2}\left[M^2_{\mathrm{UA}} + M^2_{\mathrm{SA}} + \frac{3}{4}\delta_\mathrm{A} - Z_\mathrm{A}\right], \\ M^2_{\eta'\mathrm{A}} &= \frac{1}{2}\left[M^2_{\mathrm{UA}} + M^2_{\mathrm{SA}} + \frac{3}{4}\delta_\mathrm{A} + Z_\mathrm{A}\right], \\ Z^2_\mathrm{A} &= (M^2_{\mathrm{SA}} - M^2_{\mathrm{UA}})^2 - \frac{\delta_\mathrm{A}}{2}(M^2_{\mathrm{SA}} - M^2_{\mathrm{UA}}) + \frac{9}{16}\delta^2_\mathrm{A}, \end{aligned} \tag{4}$$

and likewise for V → A, where $\delta_\mathrm{V} = a^2\delta'_\mathrm{V}$ is the hairpin coupling of a pair of taste-vector mesons, $\delta_\mathrm{A} = a^2\delta'_\mathrm{A}$ is the hairpin coupling of a pair of taste-axial mesons (δ'_V and δ'_A are tree-level (LO) taste-violating hairpin parameters [18]).

In the taste-pseudoscalar and taste-tensor sectors, in which there is no mixing of the isosinglet states, the masses of the η_b and η'_b by definition are

$$M^2_{\eta,\mathrm{b}} = M^2_{\mathrm{Ub}}; \quad M^2_{\eta',\mathrm{b}} = M^2_{\mathrm{Sb}}. \tag{5}$$

In Table 1, we list the masses of the resulting taste multiplets in lattice units for our chosen lattice ensemble with the taste-breaking parameters δ_A and δ_V determined in Refs. [18, 22]. For the Goldstone multiplet (taste P), we measured their corresponding correlators and fitted them with a single-exponential [18]. Then, using the taste splittings in Refs. [18, 22], we calculated the masses of other non-Goldstone taste multiplets. We do not need the η'_I masses in this work, hence, we do not list these values in Table 1.

We should remind the readers that the data in Table 1 are obtained with the valence strange quark mass fixed to its physical value. Here and below, we adopt the notation in Ref. [18], and the primes on the masses indicate that they are the dynamical quark masses used in the lattice simulations, and not the physical masses m_u, m_d and m_s.

Table 1. The mass spectrum of the pseudoscalar meson for the MILC coarse ($a=0.12$ fm) lattice ensemble with $\beta=6.76$, $am'_{\rm ud}=0.005$, $am'_{\rm s}=0.05$.

am_x	taste(B)	$a\pi_{\rm B}$	$a{\rm K}_{\rm B}$	$a\eta_{\rm B}$	$a\eta'_{\rm B}$
0.005	P	0.1598	0.3106	0.1598	0.4087
	A	0.2342	0.3546	0.1831	0.4332
	T	0.2688	0.3784	0.2688	0.4624
	V	0.2971	0.3990	0.2832	0.4755
	I	0.3198	0.4162	0.4434	—
0.010	P	0.2233	0.3291	0.2233	0.4087
	A	0.2814	0.3710	0.2400	0.4335
	T	0.3108	0.3938	0.3108	0.4624
	V	0.3356	0.4136	0.3233	0.4755
	I	0.3558	0.4302	0.4525	—
0.015	P	0.2718	0.3468	0.2718	0.4087
	A	0.3212	0.3867	0.2851	0.4339
	T	0.3473	0.4087	0.3473	0.4624
	V	0.3696	0.4278	0.3584	0.4756
	I	0.3881	0.4439	0.4612	—
0.020	P	0.3127	0.3637	0.3127	0.4087
	A	0.3565	0.4020	0.3235	0.4345
	T	0.3802	0.4231	0.3802	0.4624
	V	0.4007	0.4416	0.3902	0.4757
	I	0.4178	0.4572	0.4698	—
0.025	P	0.3489	0.3798	0.3489	0.4087
	A	0.3886	0.4166	0.3572	0.4356
	T	0.4105	0.4370	0.4105	0.4624
	V	0.4295	0.4550	0.4195	0.4759
	I	0.4455	0.4701	0.4782	—

3 The κ correlator from SχPT

In Refs. [14–17], using the language of the replica trick [21, 24] and through matching the point-to-point scalar correlators in chiral low energy effective theory and staggered fermion QCD, we re derive the "bubble" contribution to the a_0 channel of Ref. [13], and extend the result to the σ channel. Here we further extend these results to the κ channel.

3.1 The non-singlet κ correlator

To simulate the correct number of quark species, we use the fourth-root trick, which automatically performs the transition from four tastes to one taste per flavor for staggered fermions at all orders. We employ an interpolation operator with isospin $I=1/2$ and $J^P=0^+$ at the source and sink,

$$\mathcal{O}(x)\equiv\frac{1}{\sqrt{n_{\rm r}}}\sum_{a,g}\bar{s}_g^a(x)u_g^a(x), \tag{6}$$

where g is the indices of the taste replica, $n_{\rm r}$ is the number of taste replicas, a is the color indices, and we omit the Dirac-Spinor index. The time slice correlator $C(t)$ for the κ meson can be evaluated by

$$C(t)=\frac{1}{n_{\rm r}}\sum_{\boldsymbol{x},a,b}\sum_{g,g'}\left\langle\bar{s}_{g'}^b(\boldsymbol{x},t)u_{g'}^b(\boldsymbol{x}\ ,t)\ \bar{u}_g^a(0,0)s_g^a(0,0)\right\rangle,$$

where 0, $\boldsymbol{x}$ are the spatial points of the κ state at source, sink, respectively. After performing Wick contractions of fermion fields, and summing over the taste index [11], for the light u quark Dirac operator $M_{\rm u}$ and the s quark Dirac operator $M_{\rm s}$, we obtain

$$C(t)=\sum_{\boldsymbol{x}}(-)^x\left\langle{\rm Tr}\left[M_{\rm u}^{-1}(\boldsymbol{x},t;0,0)M_{\rm s}^{-1^\dagger}(\boldsymbol{x},t;0,0)\right]\right\rangle, \tag{7}$$

where Tr is the trace over the color index, and $x=(\boldsymbol{x},t)$ is the lattice position.

The mass of the κ meson can be reliably determined on the lattice simulation. However, many multihadron states with $J^P=0^+$ exist, which can propagate between the source and the sink. Of special interest for us is the intermediate state with two pseudoscalars, $\rm P_1P_2$, which we refer to as the bubble contribution (B) [13]. If the masses of $\rm P_1$ and $\rm P_2$ are small, the bubble term gives a considerable contribution to the κ correlator, and it should be included in the fit of the lattice correlator in Eq. (7), namely,

$$C(t)=A{\rm e}^{-m_\kappa t}+B(t), \tag{8}$$

where we omit the unimportant contributions from the excited states, the oscillating terms corresponding to a particle with opposite parity, and other high-order multihadron intermediate states.

3.2 Coupling of a scalar current to the pseudoscalar

Before we embark on the bubble contribution, here we first derive the coupling of a point scalar current $\bar{s}_r(x)u_r(x)$ to a pair of pseudoscalar fields at the lowest energy order of the staggered chiral perturbation theory (SχPT), where the subscript r in the expression $u_r(x)$ is the index of the taste replica for a given quark flavor u. The effective scalar current can be determined from the dependence of the lattice QCD Lagrangian and the staggered chiral Lagrangian on the spurion field $\mathcal{M}$, where $\mathcal{M}$ is the staggered quark mass matrix. For n Kogut-Susskind

(KS) flavors, $\mathcal{M}$ is a $4nn_{\rm r}\times 4nn_{\rm r}$ matrix.

$$\mathcal{M}=\begin{pmatrix} m_{\rm uu}I\otimes I_{\rm R} & m_{\rm ud}I\otimes I_{\rm R} & m_{\rm us}I\otimes I_{\rm R} & \cdots \\ m_{\rm du}I\otimes I_{\rm R} & m_{\rm dd}I\otimes I_{\rm R} & m_{\rm ds}I\otimes I_{\rm R} & \cdots \\ m_{\rm su}I\otimes I_{\rm R} & m_{\rm sd}I\otimes I_{\rm R} & m_{\rm ss}I\otimes I_{\rm R} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (9)$$

where I is a 4×4 unit matrix, and I_R is the $n_r\times n_r$ unit replica matrix. In short, $\mathcal{M}=\boldsymbol{m}\otimes I\otimes I_R$, and

$$\boldsymbol{m}=\begin{pmatrix} m_{\rm uu} & m_{\rm ud} & m_{\rm us} & \cdots \\ m_{\rm du} & m_{\rm dd} & m_{\rm ds} & \cdots \\ m_{\rm su} & m_{\rm sd} & m_{\rm ss} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (10)$$

is the $n\times n$ quark mass matrix. Since the staggered chiral lagrangian ($\mathcal{L}_{\rm S\chi PT}$) is an effective equivalent Lagrangian for $\mathcal{L}_{\rm QCD}$ in low energy limit, the effective current $\bar{s}_r(x)u_r(x)$ is obtained from

$$\bar{s}_r(x)u_r(x)=-\frac{\partial\mathcal{L}_{\rm S\chi PT}}{\partial\mathcal{M}_{s_ru_r}(x)}, \quad (11)$$

where

$$\mathcal{L}_{\rm S\chi PT}=\frac{1}{8}f_\pi^2{\rm Tr}[\partial^\mu\Sigma\partial_\mu\Sigma^\dagger]-\frac{1}{4}\mu f_\pi^2{\rm Tr}[\mathcal{M}^\dagger\Sigma+\Sigma^\dagger\mathcal{M}]$$

is the staggered chiral Lagrangian [21]. We omit the high-order terms and the terms that are independent of $\mathcal{M}$. f_π is the tree-level pion decay constant [21], μ is the constant with the dimension of the mass [25], and $\Sigma=\exp\left(\frac{2{\rm i}\Phi}{f_\pi}\right)$ [21]. The field $\Phi=\sum_{b=1}^{16}\frac{1}{2}T^b\otimes\phi^b$ is described in terms of the mass eigenstate field ϕ^b [21], where ϕ^b is a 3×3 pseudoscalar matrix with flavor components $\phi^b_{{\rm f_r f'_{r'}}}$ with flavor f, f′, the index of the taste replica r, r′, and taste b which is given by generators $T^b=\{\xi_5,i\xi_5\xi_\mu,i\xi_\mu\xi_\nu,\xi_\mu,\xi_I\}$ [21]. Hence, the field Φ is $4nn_r\times 4nn_r$ pseudoscalar matrix in SχPT [21], and the subscripts u, s denote its valance flavor component. Therefore, Σ is also a $4nn_r\times 4nn_r$ matrix. The Tr is the full $4nn_r\times 4nn_r$ trace. Then, the effective current is [25]

$$\bar{s}_r(x)u_r(x)=\mu{\rm Tr_t}[\Phi(x)^2]_{s_ru_r}, \quad (12)$$

where the notation ${\rm Tr_t}$ stands for the trace over taste, and μ is the low-energy chiral coupling of the point scalar current to the pseudoscalar field $\Phi(x)$.

3.3 Bubble contribution to the κ correlator

In this subsection we compute the bubble contribution to the κ correlator in Eq. (7) from two intermediate states. From the above discussion, the point scalar current can be described in terms of the pseudoscalar field Φ by using SχPT [13, 25],

$$\begin{aligned}\bar{s}_r(x)u_r(x)=&\ \mu{\rm Tr_t}[\Phi(x)^2]_{s_ru_r},\\ \bar{u}_r(x)s_r(x)=&\ \mu{\rm Tr_t}[\Phi(x)^2]_{u_rs_r}.\end{aligned} \quad (13)$$

For concreteness, the bubble contribution to the κ correlator in a theory with n_r tastes per flavor and three flavors of KS dynamical sea quarks is [21]

$$\begin{aligned}B_\kappa^{\rm S\chi PT}(x)\ =&\ \frac{\mu^2}{n_r}\sum_{r,r'=1}^{n_r}\left\{\langle\bar{s}_r(x)u_r(x)\ \bar{u}_{r'}(0)s_{r'}(0)\rangle_{\rm Bubble}\right\}\\ =&\ \frac{\mu^2}{n_r}\sum_{r,r'=1}^{n_r}\left\{\left\langle{\rm Tr_t}[\Phi^2]_{s_ru_r}{\rm Tr_t}[\Phi^2]_{u_{r'}s_{r'}}\right\rangle\right\},\end{aligned}$$

where the subscript κ specifies the bubble contribution for the κ meson. If we consider this identity

$${\rm Tr_t}\left(T^{\rm a}T^{\rm b}\right)=4\delta_{\rm ab},$$

we arrive at

$$\begin{aligned}B_\kappa^{\rm S\chi PT}(x)\ =&\ \frac{\mu^2}{n_{\rm r}}\sum_{a=1}^{16}\sum_{b=1}^{16}\sum_{i=1}^{N_{\rm f}}\sum_{j=1}^{N_{\rm f}}\sum_{r,r'=1}^{n_r}\sum_{t,t'=1}^{n_r}\langle\phi^{\rm a}_{s_ri_t}(x)\\ &\times\phi^{\rm a}_{i_tu_r}(x)\ \phi^{\rm b}_{u_{r'}j_{t'}}(0)\phi^{\rm b}_{j_{t'}s_{r'}}(0)\rangle,\end{aligned}$$

where a, b, a', b' are the taste indices, i, j are the flavor indices, and r, t, r', t' are the indices of the taste replica. The Wick contractions result in the products of two propagators for the pseudoscalar fields. After summing over the index of the taste replica, and considering that u, d quarks are degenerate, the bubble contribution can be expressed in terms of the pseudoscalar propagators $\langle\phi^{\rm b}\ \phi^{\rm b}\rangle$.

$$\begin{aligned}B_\kappa^{\rm S\chi PT}(x)\ =\ n_r\mu^2\sum_{b=1}^{16}\Big\{&\langle\phi^{\rm b}_{\rm sd}(x)\phi^{\rm b}_{\rm us}(0)\rangle_{\rm conn}\langle\phi^{\rm b}_{\rm ss}(x)\phi^{\rm b}_{\rm ss}(0)\rangle_{\rm conn}+\langle\phi^{\rm b}_{\rm su}(x)\phi^{\rm b}_{\rm us}(0)\rangle_{\rm conn}\langle\phi^{\rm b}_{\rm uu}(x)\phi^{\rm b}_{\rm uu}(0)\rangle_{\rm conn}\\ &+\langle\phi^{\rm b}_{\rm su}(x)\phi^{\rm b}_{\rm us}(0)\rangle_{\rm conn}\langle\phi^{\rm b}_{\rm ss}(x)\phi^{\rm b}_{\rm ss}(0)\rangle_{\rm conn}\Big\}+\mu^2\sum_{\rm b=I,V,A}\Big\{\langle\phi^{\rm b}_{\rm su}(x)\phi^{\rm b}_{\rm us}(0)\rangle_{\rm conn}\langle\phi^{\rm b}_{\rm uu}(x)\phi^{\rm b}_{\rm uu}(0)\rangle_{\rm disc}\\ &+\langle\phi^{\rm b}_{\rm su}(x)\phi^{\rm b}_{\rm us}(0)\rangle_{\rm conn}\langle\phi^{\rm b}_{\rm ss}(x)\phi^{\rm b}_{\rm ss}(0)\rangle_{\rm disc}+2\langle\phi^{\rm b}_{\rm su}(x)\phi^{\rm b}_{\rm us}(0)\rangle_{\rm conn}\langle\phi^{\rm b}_{\rm uu}(x)\phi^{\rm b}_{\rm ss}(0)\rangle_{\rm disc}\Big\}.\end{aligned} \quad (14)$$

The subscripts conn and disc stand for the connected contribution and the disconnected contribution, respectively. The propagators $\langle\phi^{\rm b}\phi^{\rm b}\rangle$ for the pseudoscalar field $\phi^{\rm b}$ for various b tastes are intensively studied in Ref. [21]. The propagators for all tastes (I, V, A, T, P) have connected contributions, while only tastes I, V and A have disconnected contributions [21].

$$\langle\phi^{\rm b}_{\rm uu}(x)\phi^{\rm b}_{\rm uu}(0)\rangle^{\rm V}_{\rm disc}=\frac{-\delta_{\rm V}(k^2+M^2_{\rm S_V})}{(k^2+M^2_{\rm U_V})(k^2+M^2_{\eta_{\rm V}})(k^2+M^2_{\eta'{\rm V}})},$$

$$\langle\phi^{\rm b}_{\rm ss}(x)\phi^{\rm b}_{\rm ss}(0)\rangle^{\rm V}_{\rm disc}=\frac{-\delta_{\rm V}(k^2+M^2_{\rm U_V})}{(k^2+M^2_{\rm S_V})(k^2+M^2_{\eta_{\rm V}})(k^2+M^2_{\eta'{\rm V}})},$$

$$\langle\phi^{\rm b}_{\rm uu}(x)\phi^{\rm b}_{\rm ss}(0)\rangle^{\rm V}_{\rm disc}=-\frac{\delta_{\rm V}}{(k^2+M^2_{\eta_{\rm V}})(k^2+M^2_{\eta'{\rm V}})},$$

$$\langle\phi^{\rm b}_{\rm uu}(x)\ \phi^{\rm b}_{\rm uu}(0)\rangle^{\rm I}_{\rm disc}=-\frac{4}{3}\frac{k^2+M^2_{\rm S_I}}{(k^2+M^2_{\rm U_I})(k^2+M^2_{\eta{\rm I}})},$$

$$\langle\phi^{\rm b}_{\rm ss}(x)\ \phi^{\rm b}_{\rm ss}(0)\rangle^{\rm I}_{\rm disc}=-\frac{4}{3}\frac{k^2+M^2_{\rm U_I}}{(k^2+M^2_{\rm S_I})(k^2+M^2_{\eta{\rm I}})},$$

$$\langle\phi^{\rm b}_{\rm uu}(x)\ \phi^{\rm b}_{\rm ss}(0)\rangle^{\rm I}_{\rm disc}=-\frac{4}{3}\frac{1}{k^2+M^2_{\eta{\rm I}}}.$$

And likewise for the axial taste (A), we only require V → A. By plugging in the above mesonic propagators (namely, carrying out the Wick contractions) and switching to momentum space, Eq. (14) can be rewritten as

$$\begin{aligned}
B^{\rm S\chi PT}_{\kappa}(p)\ =\ \mu^2\sum_k\Bigg\{&n_r\sum_{b=1}^{16}\left[2\frac{1}{(k+p)^2+M^2_{\rm K_b}}\frac{1}{k^2+M^2_{\rm U_b}}+\frac{1}{(k+p)^2+M^2_{\rm K_b}}\frac{1}{k^2+M^2_{\rm S_b}}\right]\\
&-2\frac{1}{(k+p)^2+M^2_{\rm K_I}}\frac{1}{k^2+M^2_{\rm U_I}}+\frac{2}{3}\frac{1}{(k+p)^2+M^2_{\rm K_I}}\frac{1}{k^2+M^2_{\eta{\rm I}}}-4\frac{1}{(k+p)^2+M^2_{\rm K_I}}\frac{1}{k^2+M^2_{\rm S_I}}\\
&-\frac{\delta_{\rm V}}{(k+p)^2+M^2_{\rm K_V}}\frac{k^2+M^2_{\rm S_V}}{(k^2+M^2_{\rm M_V})(k^2+M^2_{\eta_{\rm V}})(k^2+M^2_{\eta'{\rm V}})}\\
&-\frac{\delta_{\rm A}}{(k+p)^2+M^2_{\rm K_A}}\frac{k^2+M^2_{\rm S_A}}{(k^2+M^2_{\rm U_A})(k^2+M^2_{\eta_{\rm A}})(k^2+M^2_{\eta'{\rm A}})}\\
&-\frac{\delta_{\rm V}}{(k+p)^2+M^2_{\rm K_V}}\frac{k^2+M^2_{\rm U_V}}{(k^2+M^2_{\rm S_V})(k^2+M^2_{\eta_{\rm V}})(k^2+M^2_{\eta'{\rm V}})}\\
&-\frac{\delta_{\rm A}}{(k+p)^2+M^2_{\rm K_A}}\frac{k^2+M^2_{\rm U_A}}{(k^2+M^2_{\rm S_A})(k^2+M^2_{\eta_{\rm A}})(k^2+M^2_{\eta'{\rm A}})}\\
&-2\frac{\delta_{\rm V}}{(k+p)^2+M^2_{\rm K_V}}\frac{1}{(k^2+M^2_{\eta_{\rm V}})(k^2+M^2_{\eta'{\rm V}})}\\
&-2\frac{\delta_{\rm A}}{(k+p)^2+M^2_{\rm K_A}}\frac{1}{(k^2+M^2_{\eta_{\rm A}})(k^2+M^2_{\eta'{\rm A}})}\Bigg\}.
\end{aligned}\tag{15}$$

And likewise for the axial taste (A), we only require V → A. In Table 1, we list all the pseudoscalar masses needed for the current study.

In the continuum limit, taste-symmetry is restored, namely, $\delta_{\rm V}=a^2\delta'_{\rm V}\to0$, $\delta_{\rm A}=a^2\delta'_{\rm A}\to0$, then Eq. (15) reduces to

$$\begin{aligned}
B_{\kappa}(p)\ =\ \mu^2\sum_k\Bigg\{&2n_r\sum_{b=1}^{16}\frac{1}{(k+p)^2+M^2_{\rm K_b}}\frac{1}{k^2+M^2_{\rm U_b}}+\sum_{b=1}^{16}\frac{n_r}{(k+p)^2+M^2_{\rm K_b}}\frac{1}{k^2+M^2_{\rm S_b}}-\frac{2}{(k+p)^2+M^2_{\rm K_I}}\frac{1}{k^2+M^2_{\rm U_I}}\\
&+\frac{2}{3}\frac{1}{(k+p)^2+M^2_{\rm K_I}}\frac{1}{k^2+M^2_{\eta{\rm I}}}-\frac{4}{(k+p)^2+M^2_{\rm K_I}}\frac{1}{k^2+M^2_{\rm S_I}}\Bigg\}.
\end{aligned}$$

Here, the total contribution from the pairs of the states with mass M_{U} and M_{S} is proportional to

$$(16n_r-4), \tag{16}$$

which vanishes when $n_r=1/4$. The negative threshold has nicely canceled out the unphysical threshold KS. The surviving thresholds are the physical ηK.

4 Simulations and results

We use the MILC lattices with $2+1$ dynamical flavors of the Asqtad-improved staggered dynamical fermions. A detailed description of the simulation parameters can be found in Refs. [19, 22, 26]. We analyzed the κ correlators on the 0.12 fm MILC ensemble of 520 $24^3\times64$ gauge configurations with bare quark masses $am'_{\mathrm{ud}}=0.005$ and $am'_{\mathrm{s}}=0.05$, and bare gauge coupling $10/g^2=6.76$ of the lattice spacing $a^{-1}=1.679^{+49}_{-13}$ GeV, which has a physical volume of approximately 2.5 fm. The mass of the dynamical strange quark is close to its physical value, $am_{\mathrm{s}}=0.0344$ [19, 26]. The masses of the u and d quarks are degenerate. In Table 1, we list all the pseudoscalar masses used in our fits, with the exception of the masses $M_{\eta_{\mathrm{A}}}$, $M_{\eta'_{\mathrm{A}}}$, $M_{\eta_{\mathrm{V}}}$, and $M_{\eta'_{\mathrm{V}}}$. These masses can vary with the fit parameters δ_{A} and δ_{V}.

For the light u quark Dirac operator M_{u} and the s quark Dirac operator M_{s}, we measure the point-to-point quark-line connected correlator, which is described by Eq. (7). We use the conjugate gradient method (CG) to obtain the required matrix element of the inverse fermion matrix M_{u} and M_{s}.

In order to improve the statistics, we place the source on all the time slices, $t_{\mathrm{s}}=0,\ \cdots,\ T-1$, and therefore we perform $T=64$ inversions for each configuration and average these correlators. Note that the time extent of our lattices is more than twice the spatial extent. The rather large effort required to generate propagators allows us to evaluate the correlators with high precision, which is important to extract the desired κ masses reliably.

Since the κ meson contains a strange s quark and a light u quark, we should treat the u quark as a valence approximation quark, while the valence strange quark mass is fixed to its physical value [18]. The physical value of the strange quark mass of the lattice ensemble used in the present work has been precisely determined by MILC simulations [19], namely, $am_{\mathrm{s}}=0.0344$, where a is the lattice spacing.

The propagators of the κ meson are calculated with the same configurations using five u valence quarks, namely, we choose $am_x=0.005$, 0.01, 0.015, 0.02 and 0.025, where m_x is the light valence u quark mass. In order to obtain the physical mass of the κ meson, we then perform extrapolation to the chiral (physical π mass, obtained from PDG) limit guided by chiral perturbation theory. The correlators of the π, K meson and fictitious meson $\mathrm{s\bar{s}}$ are also measured with the same configurations used for calculating the pseudoscalar masses in Table 1.

Figure 1 shows the κ propagators with five different light valence u quark masses and their predicted bubble contributions. For our chosen MILC configurations used in the present study, we obtain the positive predicted bubble contributions, since the small negative contribution in the bubble term is most likely outweighed by the positive contribution, as is discussed for the $\mathrm{a_0}$ correlators in Ref. [13]. We add a constant 7.8E-6 to all data points and the corresponding bubble terms in the y axis, simply for good visualization. Fig. 1 clearly shows that the predicted bubble contributions dominate the κ propagators after $t\geqslant12$.

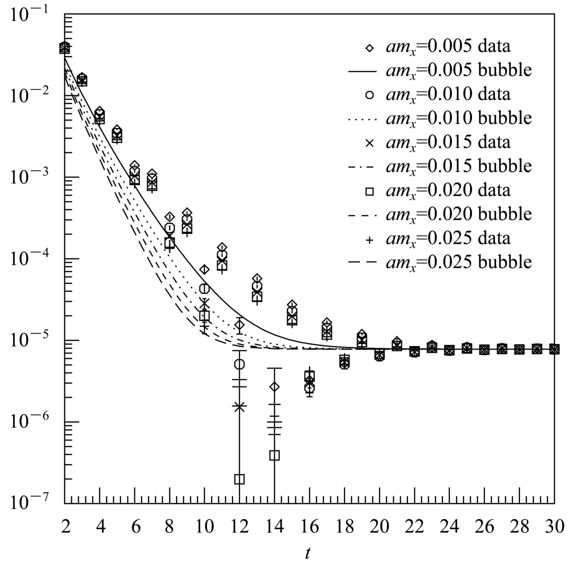

Fig. 1. The κ propagators for five valence u quarks. Overlaid on the data are their corresponding predicted bubble contributions.

For staggered quarks, the meson propagators have the generic single-particle form.

$$\mathcal{C}(t)=\sum_i A_i\mathrm{e}^{-m_it}+\sum_i A'_i(-1)^t\mathrm{e}^{-m'_it}+(t\rightarrow N_{\mathrm{t}}-t), \tag{17}$$

where the oscillating terms correspond to a particle with opposite parity. For the κ meson correlator, we consider only one mass with each parity in the fits of Eq. (17), namely, in our concrete calculation, our operator is the state with spin-taste assignment $I\otimes I$

and its oscillating term with spin-taste assignment $\gamma_0\gamma_5\otimes\gamma_0\gamma_5$ [15]. From the aforementioned discussion, we must consider the bubble contribution. Therefore, all five κ correlators were then fit to the following physical model

$$C_{\kappa}(t)=C_{\kappa}^{\text{meson}}(t)+B_{\kappa}(t), \tag{18}$$

here

$$C_{\kappa}^{\text{meson}}(t)=b_{\kappa}e^{-m_{\kappa}t}+b_{K_A}(-1)^t e^{-M_{K_A}t}+(t\rightarrow N_t-t),$$

where b_{K_A} and b_{κ} are two overlap factors, and the bubble term B_{κ} in the fitting function Eq. (18) is given in momentum space by Eq. (15). Its time-Fourier transform yields $B_{\kappa}(t)$.

This fitting model contains the explicit κ pole, together with the corresponding negative-parity state K_A and the bubble contribution. There are four fit parameters (i.e., M_{κ}, M_{K_A}, b_{K_A}, and b_{κ}) for each κ correlator with a given valence u quark mass m_x. The bubble term $B_{\kappa}(t)$ was parameterized by three low-energy couplings, μ, δ_A, and δ_V. In our concrete fit, they were fixed to the values of the previous MILC determinations [18]. The taste multiplet masses in the bubble terms were fixed as listed in Table 1. The sum over intermediate momenta was cut off when the total energy of the two-body state exceeded $2.0/a$ or any momentum component exceeded $\pi/(4a)$. We determined that such a cutoff gave an acceptable accuracy for $t\geqslant 8$. The lightest intermediate state in the bubble term is πK. Therefore, this fitting model can remove a number of unwanted πK states with different tastes and slightly different energies.

For $am_x=0.005$, the effective mass plots of the κ meson are shown in Fig. 2. We find that the effective κ mass suffers from large errors, especially in larger minimum time distance regions. To avoid possible large errors coming from the data at large minimum time distance $D_{\min}$, we fit the effective mass of the κ meson only in the time range $9\leqslant D_{\min}\leqslant 11$, where the effective masses are almost constant with small errors.

In our fit, five κ propagators were fit using a minimum time distance of $10a$. At this distance, the contamination from the excited states is comparable to the statistical errors. We can neglect the systematic effect due to excited states, therefore we can extract the mass of the κ meson efficiently.

The fitted masses of the κ correlators are summarized in Table 2. The second block shows the masses of the κ meson in lattice units, and column four shows the time range for the chosen fit. As a consistency check, we also list the fitted masses of their corresponding negative parity state, K_A, in column three. We can note that the fitted values of the pseudoscalar meson K_A masses are consistent with our calculated values in Table 1 within small errors. Column five shows the number of degrees of freedom (dof) for the fit.

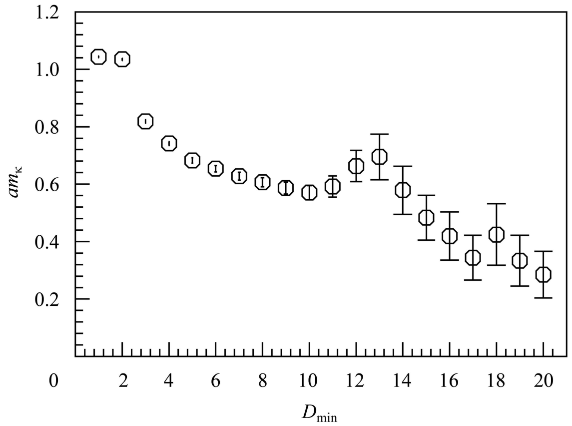

Fig. 2. κ masses as a function of minimum time distance. The effective mass plots will be a plateau in the time range $9\leqslant D_{\min}\leqslant 11$.

Table 2. Summary of the results for the fitted κ masses. The second block shows the κ masses in lattice units. The third block shows the fitted K_A masses.

am_x	am_{κ}	aM_{K_A}	range	χ^2/dof
0.005	0.545(26)	0.3568(25)	10 − 25	12.7/12
0.010	0.590(23)	0.3719(17)	10 − 25	11.8/12
0.015	0.625(21)	0.3868(14)	10 − 25	11.2/12
0.020	0.653(19)	0.4031(12)	10 − 25	11.0/12
0.025	0.673(18)	0.4182(11)	10 − 25	11.4/12

In order to obtain the physical mass of the κ meson, we carry out chiral extrapolation of the κ mass m_{κ} to the physical π mass using the popular three parameter fit with the inclusion of the next-to-next-to-leading order (NNLO) chiral logarithms. The general structure of the pion mass dependence of m_{κ} can be written as

$$m_{\kappa}=c_0+c_2m_{\pi}^2+c_3m_{\pi}^3+c_4m_{\pi}^4\ln(m_{\pi}^2), \tag{19}$$

where c_0, c_2, c_3 and c_4 are the fitting parameters, and the fourth term is the NNLO chiral logarithms.

We obtain the physical π mass from PDG [1] and use it as a chiral limit. In Fig. 3, we show how physical value m_{κ} is extracted, which gives $\chi^2/\text{dof}=0.16/1$. The blue dashed line in Fig. 3 is the linear extrapolation of the mass of the κ meson to the physical pion mass m_π. The chirally extrapolated κ mass is $m_\kappa = (828 \pm 97)$ MeV, which is in good accordance with the result in our previous study on a MILC “medium” coarse lattice ensemble [11]. The fancy diamond in Fig. 3 indicates this value.

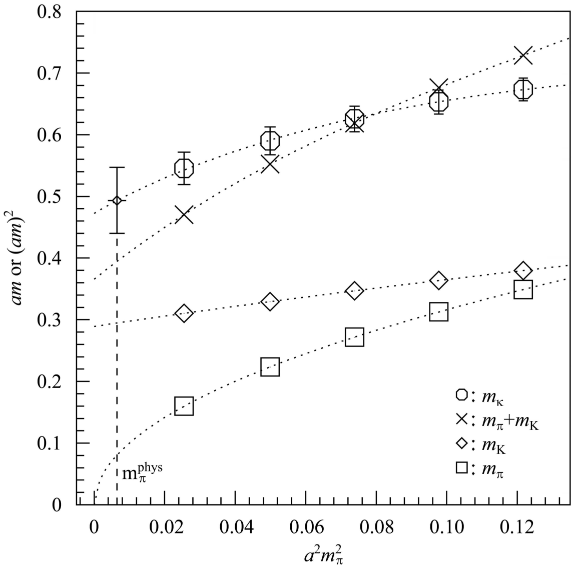

Fig. 3. Characteristics of m_κ, m_K, m_π and $m_{\pi+K}$ in lattice units as a function of pion mass. The chiral limit is obtained at the physical pion mass m_π.

Using the fitting model in Ref. [22], we extract kaon masses. In Fig. 3, we display m_κ, m_K, m_π, and $m_{\pi+K}$ in lattice units as a function of the pion mass m_π. We observe that, as the valence quark mass increases, the πK threshold grows faster than the κ mass and, as a consequence, the πK threshold is higher than the κ mass for large pion mass (about $am_x \geqslant 0.151$), and the πK threshold is lower than the κ meson for small pion mass (about $am_x \leqslant 0.151$). Hence, the κ meson can decay on our lattice for small quark mass, but for large quark mass the decay $\kappa \to \pi K$ is no longer allowed kinematically, which is in good agreement with the results in Ref. [27].

5 Summary and outlook

In the present study, we have extended the analyses of the scalar mesons in Refs. [14, 15], and derived the two-pseudoscalar-meson “bubble” contribution to the κ correlator in the lowest order SχPT. We used this physical model to fit the lattice simulation data of the point-to-point scalar κ correlators for the MILC coarse ($a \approx 0.12$ fm) lattice ensembles in the presence of the 2+1 flavors of Asqtad improved staggered dynamical sea quarks generated by the MILC Collaboration [22, 23]. We treated the light u quark as a valence approximation quark, while the strange valence s quark mass is fixed to its physical value, and we chirally extrapolated the mass of the κ meson to the physical pion mass. We achieved the physical mass of the κ meson with 828±97 MeV, which is very close to the recent experimental value of 800–900 MeV. This may probably be identified with the κ meson observed in the experiments.

Most of all, we find that the κ meson is heavier than the πK threshold for a small enough u quark mass. Therefore, it can decay on our lattice for small quark mass. This preliminary lattice simulation will stimulate people to study the decay mode $\kappa \to \pi K$. We are beginning the lattice study of this decay channel with an isospin representation of $I = 1/2$.

Since the appearance of the bubble contribution is a consequence of the fermion determinant, an analysis of the κ correlator in this work also provides a direct useful test of the fourth-root recipe. The bubble term in SχPT provides a useful explanation of the lattice artifacts induced by the fourth-root approximation [14, 15]. The artifacts include the thresholds at unphysical energies and the thresholds with negative weights. These contributions are clearly present in the κ channel in our QCD simulation with the Asqtad action at $a \approx 0.12$ fm. We find that the “bubble” term must be included in a successful spectral analysis of the κ correlator. The rSχPT predicts further that these lattice artifacts vanish in the continuum limit, leaving only physical two-body thresholds. It would be good to be able to investigate whether this expectation is ruled out in lattice simulations at smaller lattice spacing.

In this work, we reported our preliminary results on a one lattice ensemble. A more physical one should be in the continuum limit. We are beginning a series of numerical simulations with the MILC fine and super-fine lattice ensembles. Furthermore, the kappa meson is a resonance, i.e. a state with a considerable width under strong interactions. In order to map out “avoided level crossings” between the resonance and its decay products in a finite box volume, as proposed by Lüscher [28], we are beginning to measure the πK scattering $K + \pi \to K + \pi$ channel and the $\kappa \to K + \pi, K + \pi \to \kappa$ cross-correlators, in addition to the $\kappa \to \kappa$ correlator. Hopefully, we can also obtain the resonance parameters of the κ resonance.

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