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Direct CP violation in $\mathrm{B_{(s)}} \rightarrow \mathrm{J}/\psi \mathrm{P(V)}$ *

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Abstract: In the framework of factorization, we study direct CP violation in the decays of $\mathrm{B_{(s)}} \rightarrow \mathrm{J}/\psi \mathrm{P(V)}$ (P(V) refer to the pseudoscalar (vector) meson). The CP violation depends strongly on Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the effective parameter, N_c. The recent experimental data for the branching ratios of $\mathrm{B_{(s)}} \rightarrow \mathrm{J}/\psi \mathrm{P(V)}$ are accurate enough and we can give a strong constraint on the range of N_c. We find that the CP violating asymmetry is consistent with the available experiment values for the $\mathrm{b} \rightarrow \mathrm{d}$ transition, and a little smaller than the $\mathrm{b} \rightarrow \mathrm{s}$ transition. We also predict the CP violation of other decay channels for $\mathrm{B_{(s)}} \rightarrow \mathrm{J}/\psi \mathrm{P(V)}$. We expect our results can give valuable guidance for experiments.

Key words: branching ratio, N_c, CP violation

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1 Introduction

The study of nonleptonic two-body decays of B mesons is of importance for CP violation. But the decays of B mesons based on the $\mathrm{b} \rightarrow \mathrm{q\bar{c}c}$ transition are rarely analyzed. Fortunately, more b-quark and c-quark data will be collected by the Large Hadron Collider (LHC) experiment. In particular, the LHCb detector is designed to exploit the large numbers of b-hadrons produced at the LHC in order to make precise studies on CP asymmetries and on rare decays in b-hadron systems. The major renovation project at Beijing Electron-Positron Collider (BEPC) has been completed and the collision brightness will increase by two orders of magnitude as compared to the BEPC-I. All of these experiments will provide a new opportunity to search for more CP violation signals.

In general, the hadronic matrix elements are difficult to handle due to non-perturbative QCD. The naive factorization approximation will be used in which the hadronic matrix elements of four-quark operators are assumed to be saturated by vacuum intermediate states. During the decay process, a hadron containing a b-quark is relatively heavy, and is energetic. Thus the quarks generated during this time will be far away from the decay point and can not interact with others, and factorize out to form hadrons. This approximation can also be justified in QCD in the case of a large number of colors [1–3]. In recent years, the factorization scheme has been shown to be the leading order result in the framework of QCD factorization [4–8]. However, if the heavy meson of the final state is factorized out, such as in the research in this paper for the final state particles (with two heavy c-quarks), and flow through the weak interaction points, this method is proved to be outside of the range and could not give reliable theory results [4–8]. Based on this consideration, we analyze the CP violation when a B meson decays into heavy final state particles and another light meson in the framework of factorization.

The remainder of this paper is organized as follows. In Sec. 2, we present the form of the effective Hamiltonian and the values of the Wilson coefficients. In Sec. 3, we give the branching ratios for $\mathrm{B_{(s)}} \rightarrow \mathrm{J}/\psi \mathrm{P(V)}$ as a function of N_c. In Sec. 4, we calculate the CP violation in $\mathrm{B_{(s)}} \rightarrow \mathrm{J}/\psi \mathrm{P(V)}$. In Sec. 5, we present input parameters. We give the

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numerical results in Sec. 6. A summary and discussion are included in Sec. 7.

2 The effective Hamiltonian and the Wilson coefficients

With the operator product expansion [9], the effective Hamiltonian in bottom hadron decays is

$$H_{\Delta B=1} = \frac{G_{\rm F}}{\sqrt{2}}\left[\sum_{\rm q=d,s} V_{\rm cb}V^*_{\rm cq}(c_1O_1^{\rm c}+c_2O_2^{\rm c}) - V_{\rm tb}V^*_{\rm tq}\sum_{i=3}^{10}c_iO_i\right]+{\rm H.c.}, \quad (1)$$

where c_i $(i=1,\cdots,10)$ are the Wilson coefficients; $G_{\rm F}$ represents the Fermi coupling constant; $V_{\rm cb}$, $V_{\rm cq}$, $V_{\rm tb}$, and $V_{\rm tq}$ are the CKM matrix elements. The operators O_i have the following form:

$$\begin{aligned}
O_1^{\rm c} &= \bar{c}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\bar{q}_\beta\gamma^\mu(1-\gamma_5)c_\alpha,\\
O_2^{\rm c} &= \bar{c}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1-\gamma_5)c,\\
O_3 &= \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{\rm q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_4 &= \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{\rm q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,\\
O_5 &= \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{\rm q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_6 &= \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{\rm q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \qquad (2)\\
O_7 &= \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{\rm q'}e_{\rm q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_8 &= \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{\rm q'}e_{\rm q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_9 &= \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{\rm q'}e_{\rm q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_{10} &= \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{\rm q'}e_{\rm q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,
\end{aligned}$$

where α and β are the color indices, $O_1^{\rm c}$ and $O_2^{\rm c}$ are the tree operators, O_3-O_6 are QCD penguin operators which are isosinglets, O_7-O_{10} arise from electroweak penguin operators which have both isospin 0 and 1 components.

The Wilson coefficients, c_i, are known to the next-to-leading logarithmic order [10–14]. They are renormalization scheme dependent since the renormalization prescription involves an arbitrariness in the finite parts in the renormalization procedure. Therefore, we choose to use the renormalization scheme independent Wilson coefficients so that the CP violating asymmetries we obtain are renormalization scheme independent. The renormalization scale μ is chosen as the energy scale in the decays of b-hadrons, $O(m_{\rm b})$. When $\mu=5$ GeV, these renormalization scheme independent Wilson coefficients take the following values [13, 14]:

$$\begin{aligned}
&c_1=1.1502, & &c_2=-0.3125,\\
&c_3=0.0174, & &c_4=-0.0373,\\
&c_5=0.0104, & &c_6=-0.0459, \qquad (3)\\
&c_7=-1.050\times10^{-5}, & &c_8=3.839\times10^{-4},\\
&c_9=-0.0101, & &c_{10}=1.959\times10^{-3}.
\end{aligned}$$

To be consistent, the matrix elements of the operators O_i should also be renormalized to the one-loop order. This results in the effective Wilson coefficients, c'_i, which satisfy the constraint

$$c_i(m_{\rm b})\langle O_i(m_{\rm b})\rangle = c'_i\langle O_i\rangle^{\rm tree}, \quad (4)$$

where $\langle O_i\rangle^{\rm tree}$ is the matrix element at the tree level, which will be evaluated in the factorization approach. The relations between c'_i and c_i can be found in Refs. [10–12, 15].

Based on simple arguments at the quark level from the penguin diagrams, when ${\rm b}\to{\rm qd}\to\bar{\rm q}'{\rm q}'{\rm d}$ decay (${\rm q}'$ and $\bar{\rm q}'$ refer to the final state quark and antiquark), one can obtain $q^2=m_{\rm b}^2-2m_{\rm b}E_{\rm d}$ ($E_{\rm d}$ refer to d quark energy and $E_{\rm d}\sim\frac{1}{3}m_{\rm b}$). Hence we can get $\frac{q^2}{m_b^2}\sim\frac{1}{3}$ and the value of q^2 is chosen to be in the range $0.3<q^2/m_{\rm b}^2<0.5$ [16–19]. When $q^2/m_{\rm b}^2=0.3$,

$$\begin{aligned}
c'_1 &= 1.1502,\\
c'_2 &= -0.3125,\\
c'_3 &= 2.433\times10^{-2}+1.543\times10^{-3}{\rm i},\\
c'_4 &= -5.808\times10^{-2}-4.628\times10^{-3}{\rm i},\\
c'_5 &= 1.733\times10^{-2}+1.543\times10^{-3}{\rm i}, \qquad (5)\\
c'_6 &= -6.668\times10^{-2}-4.628\times10^{-3}{\rm i},\\
c'_7 &= -1.435\times10^{-4}-2.963\times10^{-5}{\rm i},\\
c'_8 &= 3.839\times10^{-4},\\
c'_9 &= -1.023\times10^{-2}-2.963\times10^{-5}{\rm i},\\
c'_{10} &= 1.959\times10^{-3},
\end{aligned}$$

while when $q^2/m_b^2 = 0.5$,

$$\begin{aligned}
c_1' &= 1.1502,\\
c_2' &= -0.3125,\\
c_3' &= 2.120\times10^{-2}+5.174\times10^{-3}\mathrm{i},\\
c_4' &= -4.869\times10^{-2}-1.552\times10^{-2}\mathrm{i},\\
c_5' &= 1.420\times10^{-2}+5.174\times10^{-3}\mathrm{i},\\
c_6' &= -5.729\times10^{-2}-1.552\times10^{-2}\mathrm{i},\\
c_7' &= -8.340\times10^{-5}-9.938\times10^{-5}\mathrm{i},\\
c_8' &= 3.839\times10^{-4},\\
c_9' &= -1.017\times10^{-2}-9.938\times10^{-5}\mathrm{i},\\
c_{10}' &= 1.959\times10^{-3},
\end{aligned} \tag{6}$$

where we have taken $\alpha_s(m_z) = 0.112$, $\alpha_{em}(\mu = 5\ \mathrm{GeV}) = 1/137$, and $m_c = 1.35$ GeV.

The CKM matrix, which should be determined from experiments, can be expressed in terms of the Wolfenstein parameters, A, λ, ρ and η [20–22]:

$$V=\begin{pmatrix} V_{ub} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}=\begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-\mathrm{i}\eta)\\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+\mathrm{i}\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2\\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+\mathrm{i}\eta)] & 1-\frac{1}{2}A^2\lambda^4\end{pmatrix}, \tag{7}$$

where $O(\lambda^6)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [22]:

$$\begin{aligned} &\lambda = 0.2253\pm0.0007, && A = 0.808^{+0.022}_{-0.015},\\ &\bar{\rho} = 0.132^{+0.022}_{-0.014}, && \bar{\eta} = 0.341\pm0.013, \end{aligned} \tag{8}$$

where

$$\bar{\rho} = \rho\left(1-\frac{\lambda^2}{2}+\cdots\right), \quad \bar{\eta} = \eta\left(1-\frac{\lambda^2}{2}+\cdots\right). \tag{9}$$

3 Branching ratios for $B_{(s)} \to J/\psi\, P(V)$ as a function of N_c

The matrix elements for $B\to P$ and $B\to V$ (where P and V denote the pseudoscalar and vector mesons, respectively) can be decomposed as follows [23, 24]:

$$\begin{aligned}
\langle P|J_\mu|B\rangle &= \left(p_B+p_P-\frac{m_B^2-m_P^2}{k^2}k\right)_\mu F_1(k^2)\\
&\quad+\frac{m_B^2-m_P^2}{k^2}k_\mu F_0(k^2),\\
\langle V|J_\mu|B\rangle &= \frac{2}{m_B+m_V}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p_V^\sigma V(k^2)\\
&\quad+\mathrm{i}\left\{\epsilon_\mu^*(m_B+m_V)A_1(k^2)\right.\\
&\quad-\frac{\epsilon^*\cdot k}{m_B+m_V}(p_B+p_V)_\mu A_2(k^2)\\
&\quad\left.-\frac{\epsilon^*\cdot k}{k^2}2m_V\cdot k_\mu A_3(k^2)\right\}\\
&\quad+\mathrm{i}\frac{\epsilon^*\cdot k}{k^2}2m_V\cdot k_\mu A_0(k^2),
\end{aligned} \tag{10}$$

where J_μ is the weak current ($J_\mu = \bar{q}\gamma^\mu(1-\gamma_5)b$ with q=u, d, s); $p_B(m_B)$, $p_P(m_P)$, $p_V(m_V)$ are the momenta (masses) of B, P, V, respectively; $k = p_B - p_P(p_V)$ for $B\to P(V)$ transition and ϵ_μ is the polarization vector of V. F_i (i=0, 1) and A_i (i=0, 1, 2, 3) in Eq. (10) are the weak form factors which satisfy $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$, and $A_3(k^2) = [(m_B+m_V)/2m_V]A_1(k^2)-[(m_B-m_V)/2m_V]A_2(k^2)$.

We consider the branching ratios of the weak decays in $B\to M_1M_2$ (M_1, M_2 refer to a pseudoscalar meson P or a vector meson V). The decay amplitudes have the form $A(B\to M_1M_2) = \alpha X^{(BM_1,\ M_2)}$ where α is related to the CKM matrix elements and Wilson coefficients, and $X^{(BM_1,\ M_2)}$ denotes the factorizable amplitude with the form $\langle M_2|(\bar{q}_2q_3)_{V-A}|0\rangle\langle M_1|(\bar{q}_1b)_{V-A}|\bar{B}\rangle$ where $(\bar{q}_2q_3)_{V-A}$ and $(\bar{q}_1b)_{V-A}$ denote $V-A$ weak currents. Factors of parameters $X^{(BM_1,M_2)}$ can be written as

$$\begin{aligned}
X^{(BP,V)} &= \langle V|(\bar{q}_2q_3)_{V-A}|0\rangle\langle P|(\bar{q}_1b)_{V-A}|\bar{B}\rangle\\
&= 2f_Vm_VF_1^{BP}(m_V^2)(\varepsilon^*\cdot p_B)
\end{aligned} \tag{11}$$

for the decay of $B\to VP$;

$$\begin{aligned}
X^{(BV_1,V_2)} &= \langle V_2|(\bar{q}_2q_3)_{V-A}|0\rangle\langle V_1|(\bar{q}_1b)_{V-A}|\bar{B}\rangle\\
&= -\mathrm{i}f_{V_2}m_2\Big[(\varepsilon_1^*\cdot\varepsilon_2^*)(m_B+m_1)A_1^{BV_1}(m_2^2)
\end{aligned}$$

$$-\ (\varepsilon_1^*\cdot p_{\rm B})(\varepsilon_2^*\cdot p_{\rm B})\frac{2A_2^{\rm BV_1}(m_2^2)}{m_{\rm B}+m_1}$$

$$\left.+{\rm i}\varepsilon_{\mu\nu\alpha\beta}\varepsilon_1^{*\nu}\varepsilon_2^{*\mu}p_{\rm B}^{\alpha}p_1^{\beta}\frac{2V^{\rm BV_1}(m_2^2)}{m_{\rm B}+m_1}\right] \quad (12)$$

for the decay of $\mathrm{B}\to\mathrm{VV}$.

Due to the above factorizable decay amplitudes, the decay amplitudes for $\mathrm{B_{(s)}}\to\mathrm{J}/\psi\mathrm{M}$, $\mathrm{B_{(s)}}\to\mathrm{J}/\psi\mathrm{N}$ are

$$A(\mathrm{B_{(s)}}\to\mathrm{J}/\psi\mathrm{M}) = \alpha_1 X^{(\mathrm{B_{(s)}M,J/\psi})}, \quad (13)$$

$$A(\mathrm{B_{(s)}}\to\mathrm{J}/\psi\mathrm{N}) = \alpha_2 X^{(\mathrm{B_{(s)}N,J/\psi})}, \quad (14)$$

where M refers to $\mathrm{b}\to\mathrm{d}$ transition, such as π, ρ, ω; N represents $\mathrm{b}\to\mathrm{s}$ transition, such as K, K^*, ϕ;

$$\alpha_1 = V_{\rm cb}V_{\rm cd}^*a_2-V_{\rm tb}V_{\rm td}^*(a_3+a_5+a_7+a_9), \quad (15)$$

$$\alpha_2 = V_{\rm cb}V_{\rm cs}^*a_2-V_{\rm tb}V_{\rm ts}^*(a_3+a_5+a_7+a_9), \quad (16)$$

with a_i $(i=1,2,\cdots,10)$ being defined as:

$$\begin{aligned} a_{2j} &= c'_{2j}+\frac{c'_{2j-1}}{N_{\rm c}}, \\ a_{2j-1} &= c'_{2j-1}+\frac{c'_{2j}}{N_{\rm c}} \text{ for } j=1,2,\cdots,5. \end{aligned} \quad (17)$$

All the decay amplitudes are multiplied by $\dfrac{G_{\rm F}}{\sqrt{2}}$.

Then the decay rate is given by [25]

$$\Gamma(\mathrm{B}\to\mathrm{VP})=\frac{p_{\rm c}^3}{8\pi m_{\rm V}^2}|A(\mathrm{B}\to\mathrm{VP})/(\varepsilon\cdot P_{\rm B})|^2 \quad (18)$$

for the decay of $\mathrm{B}\to\mathrm{VP}$, where

$$p_{\rm c}=\frac{\sqrt{[m_{\rm B}^2-(m_{\rm P}+m_{\rm V})^2][m_{\rm B}^2-(m_{\rm P}-m_{\rm V})^2]}}{2m_{\rm B}}$$

is the c.m. momentum of the product particle and $A(\mathrm{B}\to\mathrm{VP})$ is the decay amplitude.

$$\Gamma(\mathrm{B}\to\mathrm{V_1V_2})=\frac{p_{\rm c}}{8\pi m_{\rm B}^2}|\alpha(m_{\rm B}+m_1)m_2f_{\rm V_2}A_1^{\rm BV_1}(m_2)|^2H \quad (19)$$

for the decay of $\mathrm{B}\to\mathrm{VV}$, where $f_{\rm V_2}$ is the decay constant of $\mathrm{V_2}$, $m_{\rm B}$ and $m_1(m_2)$ are the masses of the B meson and the vector meson $\mathrm{V_1(V_2)}$, respectively, and

$$H=(a-bx)^2+2(1+c^2y^2), \quad (20)$$

where

$$a = \frac{m_{\rm B}^2-m_1^2-m_2^2}{2m_1m_2},\ b=\frac{2m_{\rm B}^2p_{\rm c}^2}{m_1m_2(m_{\rm B}+m_1)^2},$$

$$c = \frac{2m_{\rm B}p_{\rm c}}{(m_{\rm B}+m_1)^2},$$

$$x = \frac{A_2^{\rm BV_1}(m_2^2)}{A_1^{\rm BV_1}(m_2^2)},\ y=\frac{V^{\rm BV_1}(m_2^2)}{A_1^{\rm BV_1}(m_2^2)},$$

$$p_{\rm c} = \frac{\sqrt{[m_{\rm B}^2-(m_1+m_2)^2][m_{\rm B}^2-(m_1-m_2)^2]}}{2m_{\rm B}}, \quad (21)$$

$A_1^{\rm BV_1}$, $A_2^{\rm BV_1}$ and $V^{\rm BV_1}$ in Eqs. (20) and (21) are the form factors associated with $\mathrm{B}\to\mathrm{V_1}$ transition.

4 *CP* violation in $\mathrm{B_{(s)}}\to\mathrm{J}/\psi\mathrm{P(V)}$

Letting A ($\bar{\rm A}$) be the amplitude for the decay $\mathrm{B_{(s)}}\to\mathrm{J}/\psi\mathrm{P(V)}$ one has:

$$A = \langle \mathrm{J}/\psi P(V)|H^{\rm T}|\bar{B}_{\rm (s)}\rangle+\langle \mathrm{J}/\psi P(V)|H^{\rm P}|\bar{B}_{\rm (s)}\rangle, \quad (22)$$

$$\bar{A} = \langle \mathrm{J}/\psi \bar{P}(\bar{V})|H^{\rm T}|B_{\rm (s)}\rangle+\langle \mathrm{J}/\psi \bar{P}(\bar{V})|H^{\rm P}|B_{\rm (s)}\rangle, \quad (23)$$

with $H^{\rm T}$ and $H^{\rm P}$ being the Hamiltonian for the tree and penguin operators, respectively.

We can define the relative magnitude and phases between the tree and penguin operator contributions as follows:

$$A = \langle \mathrm{J}/\psi P(V)|H^{\rm T}|\bar{B}_{\rm (s)}\rangle[1+r{\rm e}^{{\rm i}(\delta+\phi)}], \quad (24)$$

$$\bar{A} = \langle \mathrm{J}/\psi \bar{P}(\bar{V})|H^{\rm T}|B_{\rm (s)}\rangle[1+r{\rm e}^{{\rm i}(\delta-\phi)}], \quad (25)$$

where δ and ϕ are the strong and weak relative phases, respectively. The weak phase difference ϕ arises from the appropriate combination of the CKM matrix elements: $\phi=\arg[(V_{\rm cb}V_{\rm cq}^*)/(V_{\rm tb}V_{\rm tq}^*)]$ (q=d, s). As a result, $\sin\phi$ is equal to $\sin\gamma$ with γ being defined in the standard way [22]. The parameter r is the absolute value of the ratio of penguin and tree amplitudes:

$$r\equiv\left|\frac{\langle \mathrm{J}/\psi P(V)|H^{\rm P}|\bar{B}_{\rm (s)}\rangle}{\langle \mathrm{J}/\psi P(V)|H^{\rm T}|\bar{B}_{\rm (s)}\rangle}\right|. \quad (26)$$

The *CP* violating asymmetry, a, can be written as

$$a\equiv\frac{|A|^2-|\bar{A}|^2}{|A|^2+|\bar{A}|^2}=\frac{-2r\sin\delta\sin\phi}{1+2r\cos\delta\cos\phi+r^2}. \quad (27)$$

5 Input parameters

The form factors depend on the inner structure of hadrons. We adopt the following form factor models: Model 1 (BSW model) [23, 24], Model 2 (QCD sum rule) [26], Model 3 (constituent quark picture) [27], Model 4 (light cone sum rule) [28], Model 5 (pQCD) [29–31], and Model 6 (light cone quark model in conjunction with soft collinear effective theory) [32]. In Table 1, we present the experimental values of branching ratios for $B \to J/\psi P(V)$ [22]. So we can determine the range of N_c by comparison with the experimental data.

Table 1. The experimental values of branching ratios for $B \to J/\psi P(V)$.

decay	experimental value
$B^+ \to J/\psi\pi^+$	$(4.9 \pm 0.4) \times 10^{-5}$
$B^+ \to J/\psi K^+$	$(1.014 \pm 0.034) \times 10^{-3}$
$B^+ \to J/\psi\rho^+$	$(5.0 \pm 0.8) \times 10^{-5}$
$B^+ \to J/\psi K^{*+}$	$(1.43 \pm 0.08) \times 10^{-3}$
$B^0 \to J/\psi K^0$	$(8.71 \pm 0.32) \times 10^{-4}$
$B^0 \to J/\psi\pi^0$	$(1.76 \pm 0.16) \times 10^{-5}$
$B^0 \to J/\psi K^{*0}$	$(1.33 \pm 0.06) \times 10^{-3}$
$B^0 \to J/\psi\rho^0$	$(2.7 \pm 0.4) \times 10^{-5}$
$B_s \to J/\psi\phi$	$(1.3 \pm 0.4) \times 10^{-3}$

6 Numerical results

6.1 Determination of the range of N_c value

As mentioned before, N_c includes the nonfactorizable effects and can be determined by the experimental data. We take $B^+ \to J/\psi\pi^+$ as example. Currently, the branching ratio for $B^+ \to J/\psi\pi^+$ is $(4.9 \pm 0.4) \times 10^{-5}$ [22]. In Fig. 1 we present the results of the branching ratio as a function of N_c in different models of form factors and CKM matrix elements.

From Fig. 1, one can see the branching ratio for $B^+ \to J/\psi\pi^+$ depends on the values of N_c, q^2/m_b^2, CKM matrix elements, and models of form factors. We present the branching ratios with different models of form factors for determining the N_c value accurately. One can see that the branching ratio is sensitive to the form factor models. We also present the experimental results in Fig. 1 in order to determine the N_c value. One can see that the experimental results divide Fig. 1 into two parts. Hence, we can determine the range of the N_c value accurately. Similarly, we can determine the ranges of the N_c values for the other decay channels through comparision with the experimental results.

The N_c ranges are summarized in Table 2. We present the ranges of N_c as function of q^2/m_b^2 due to different wilson coefficients. In Table 2, one can see that the N_c values vary very slightly for $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$. Since the experiments present accurate results of branching ratios for the decay channels $B_{(s)} \to J/\psi P(V)$, we can obtain the accurate range of N_c values in Table 2. One can see that the N_c range is mode dependent. With the Wilson coefficients, the

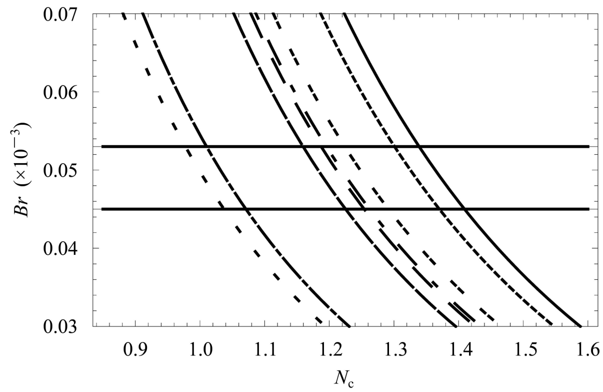

Fig. 1. Branching ratio for $B^+ \to J/\psi\pi^+$ as a function of N_c in Model 1, 2, 3 and 4, respectively, when $k^2/m_b^2 = 0.3$ and for limiting values of the CKM matrix elements. The dashed (solid) curve corresponds to the minimum (maximum) CKM matrix elements for Model 1; The line-line (dot-line) line corresponds to the minimum (maximum) CKM matrix elements for Model 2; The dot-dot (line-dot-line) line corresponds to the minimum (maximum) CKM matrix elements for Model 3; The line-blank-line (dot-dot-blank) line corresponds to the minimum (maximum) CKM matrix elements for Model 4. The solid straight line corresponds to the experimental data.

Table 2. The N_c value range of $B \to J/\psi P(V)$.

decay	q^2/m_b^2	the range of N_c value
$B^+ \to J/\psi\pi^+$	0.3	1.19 ± 0.22
	0.5	1.21 ± 0.22
$B^+ \to J/\psi K^+$	0.3	1.54 ± 0.38
	0.5	1.55 ± 0.38
$B^+ \to J/\psi\rho^+$	0.3	1.97 ± 0.18
	0.5	2.00 ± 0.15
$B^+ \to J/\psi K^{*+}$	0.3	1.81 ± 0.16
	0.5	1.83 ± 0.16
$B^0 \to J/\psi K^0$	0.3	1.61 ± 0.26
	0.5	1.63 ± 0.26
$B^0 \to J/\psi\pi^0$	0.3	1.33 ± 0.14
	0.5	1.35 ± 0.14
$B^0 \to J/\psi K^{*0}$	0.3	2.48 ± 0.10
	0.5	2.47 ± 0.12
$B^0 \to J/\psi\rho^0$	0.3	2.80 ± 0.11
	0.5	2.80 ± 0.11
$B_s \to J/\psi\phi$	0.3	2.46 ± 0.27
	0.5	2.46 ± 0.27

CKM matrix elements, and the determined N_c values in Table 2, we can calculate the CP violation accurately.

6.2 The CP violation for $B_{(s)} \rightarrow J/\psi P(V)$

The CP violation depends on q^2, N_c and the CKM matrix elements. From what is mentioned above, we have determined the ranges of N_c values, so we can present the CP violation accurately. We also take the decay channel of $B^+ \rightarrow J/\psi\pi^+$ as example. The results are shown in Fig. 2 for $q^2/m_b^2 = 0.3$. One can find the CP violating parameter is sensitive to N_c, q^2/m_b^2, and the CKM matrix elements. In Table 2, we have determined the accurate ranges of the N_c value. Hence, comparing with the N_c value, one can obtain the exact CP violating parameter, a (see Fig. 2 for $B^+ \rightarrow J/\psi\pi^+$).

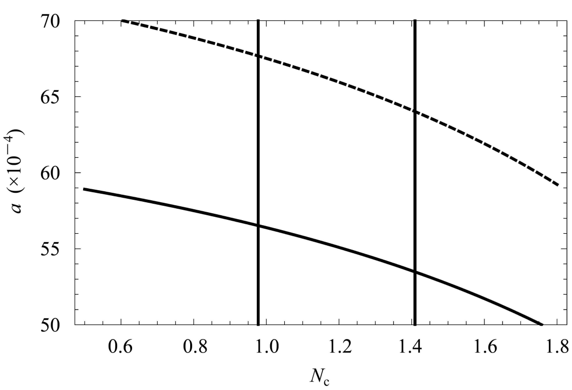

Fig. 2. The CP violating parameter, a, as function of N_c value for $B^+ \rightarrow J/\psi\pi^+$ when $q^2/m_b^2 = 0.3$. The dashed (solid) curve corresponds to the minimum (maximum) CKM matrix elements. The vertical solid straight line corresponds to the experimental data.

In Table 3 and Table 4, we summarize the results of CP violation and the experimental values. Currently, the experimental results of CP violation are scarce for $B \rightarrow J/\psi P(V)$. Hence, we only present the experimental value of four decay channels from the Particle Data Group (PDG) [22]. One can find that when $q^2/m_b^2 = 0.3(0.5)$, the CP violating parameter value is in the range of the experimental value for $b \rightarrow d$ transition, such as the decays of $B^+ \rightarrow J/\psi\pi^+$ and $B^+ \rightarrow J/\psi\rho^+$. However, our results are a little smaller than the available experimental values for $b \rightarrow s$ transition from the decays of $B^+ \rightarrow J/\psi K^+$ and $B^+ \rightarrow J/\psi K^{*+}$.

As can be seen from Table 3, the CP violating parameter value is a little larger in the case of $q^2/m_b^2 = 0.5$ than in the case of $q^2/m_b^2 = 0.3$. We also predict the CP violation of other decay channels from the final state of J/ψ meson in $B_{(s)}$ decays.

Table 3. The CP violation for $B \rightarrow J/\psi P(V)$.

decay	q^2/m_b^2	the range of N_c value
$B^+ \rightarrow J/\psi\pi^+$	0.3	0.0061 ± 0.0007
	0.5	0.0196 ± 0.0023
$B^+ \rightarrow J/\psi K^+$	0.3	-0.0003 ± 0.0001
	0.5	-0.0010 ± 0.0002
$B^+ \rightarrow J/\psi\rho^+$	0.3	0.0052 ± 0.0008
	0.5	0.0155 ± 0.0014
$B^+ \rightarrow J/\psi K^{*+}$	0.3	-0.0003 ± 0.0001
	0.5	-0.0010 ± 0.0002
$B^0 \rightarrow J/\psi K^0$	0.3	0.0057 ± 0.0008
	0.5	0.0182 ± 0.0027
$B^0 \rightarrow J/\psi\pi^0$	0.3	-0.0003 ± 0.0001
	0.5	-0.0010 ± 0.0001
$B^0 \rightarrow J/\psi K^{*0}$	0.3	-0.0002 ± 0.0001
	0.5	-0.0006 ± 0.0001
$B^0 \rightarrow J/\psi\rho^0$	0.3	0.0026 ± 0.0012
	0.5	0.0078 ± 0.0039
$B_s \rightarrow J/\psi\phi$	0.3	-0.0002 ± 0.0001
	0.5	-0.0007 ± 0.0003

Table 4. The experimental values of CP violation for $B \rightarrow J/\psi P(V)$.

decay	the experimental value
$B^+ \rightarrow J/\psi\pi^+$	0.01 ± 0.07
$B^+ \rightarrow J/\psi K^+$	0.009 ± 0.008
$B^+ \rightarrow J/\psi\rho^+$	-0.11 ± 0.14
$B^+ \rightarrow J/\psi K^{*+}$	-0.048 ± 0.033

7 Summary and discussion

In this paper, we have studied the CP violation in $B_{(s)} \rightarrow J/\psi P(V)$. It has been found that the CP violating asymmetries are in agreement with the experimental values for the decay channels of $B^+ \rightarrow J/\psi\pi^+$, $B^+ \rightarrow J/\psi\rho^+$ from $b \rightarrow d$ transition, and a little smaller for the channels of $B^+ \rightarrow J/\psi K^+$, $B^+ \rightarrow J/\psi K^{*+}$ from $b \rightarrow s$ transition. At present, there are no other experimental values of CP violation in $B_{(s)} \rightarrow J/\psi P(V)$. Therefore, we also predict the CP violation for other decay channels of $B \rightarrow J/\psi P(V)$. We expect our results can provide valuable guidance for searching for the CP violation in experiments.

QCD factorization gives large error ranges, although most of the data are consistent with the experimental results [4–7, 33]. Currently, more precise data of B decay channels are available in experiments. We compare naive factorization with QCD factorization in many channels of B decays [34], and find in some channels naive factorization results in data that is closer to the available experimental data. Especially, QCD factorization could not give reliable theoretical results including final state particles with two heavy c-quarks in B decay. In our work, we have used naive factorization approximation, in which there is an effective parameter N_c. It is introduced in this paper, including the non-factorizable contribution. Fortunately, we have more precise experimental data for the branching ratios of $B_{(s)} \to J/\psi P(V)$ in PDG [22]. So we can control the effective parameter N_c precisely.

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