Study of singlet-triplet mixing via semileptonic decays *

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Abstract: The singlet-triplet mixing of $^1P_1$–$^3P_1$ is studied via calculating the branching ratios of semileptonic decay $B_s \to D_{s1} l\nu$ and $B \to D_1 l\nu$ by means of the instantaneous Bethe-Salpeter method. Special attention is paid to the relativistic corrections, since they are large for the P-wave states. Using the Mandelstam Formalism, we compute the transition form factors not only in the high-energy lepton end-point region but also in the full Q^2 region. In addition, the non-perturbative QCD effects are taken care of in the overlapping integral over the relativistic wave functions of the initial and final states.

Key words: singlet-triplet mixing, semileptonic decay, instantaneous Bethe-Salpeter method

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1 Introduction

The P-wave states $D^{\pm}_{s1}(2460, 2536)$, $D^{(\prime)0}_1$ and $D^{(\prime)\pm}_1$, which are composed of a heavy c-quark and a light antiquark $\bar{q}(\bar{q}=\bar{s},\bar{d},\bar{u})$, are generally acknowledged as singlet-triplet mixing states of $^1P_1$ and $^3P_1$ [1–3], so the mixing angle is an important quantity which should be first highlighted before we study the behaviors of the states.

There are four P-wave states: $^1P_1$ (1^+), $^3P_0(0^+)$, $^3P_1(1^+)$ and $^3P_2$ (2^+) totally. For equal-mass systems, like charmonium and bottomonium, the charge conjugation parity C can be used to distinguish two 1^+ states, which are 1^{+-} $(^1P_1)$ and 1^{++} $(^3P_1)$; in this case, $^1P_1$ and $^3P_1$ are physical states. But for quark and antiquark of different flavors, C is no longer a good quantum number, $^1P_1$ and $^3P_1$ do not need to be physical states. Actually the physical states are the mixtures of them. To simplify the theoretical description, we could take the infinitely heavy quark limit $m_Q\to\infty$ [4, 5]. In this case, the meson is similar to a hydrogen, thus the heavy quark spin s_Q decouples and all kinetic properties of the meson are determined by the light quark total angular momentum j_q. The spin of the light quark s_q couples with its orbital momentum $l(j_q=l\pm s_q)$, making the meson degenerate into two $j_q=3/2$ states $(J^P=1^+,2^+)$ and two $j_q=1/2$ states $(J^P=0^+,1^+)$, where $J=j_q+s_Q$ and P are the total angular momentum and parity of the meson respectively, so authors usually use labels of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ (j^P_q) to describe the two 1^+ states. Hence, the physical 1^+ states are mixtures of $^1P_1$ and $^3P_1$ naturally. The study of weak semileptonic decay channels $B^0_s\to D^+_{s1}(2460, 2536)l\nu$, $B^-\to D^{(\prime)0}_1 l\nu$ and $B^0\to D^{(\prime)+}_1 l\nu$ could provide an important test about the mixing, and other information about the mixing angle, which would deepen our understanding of the P-wave mesons.

Until recently, the semileptonic decays mentioned above have been calculated by several authors in different ways, such as the QCD sum rule (QSR) [6], the constituent quark model (CQM) [7, 8], the heavy quark effective theory (HQET) [9], ISGW2 [10], etc. Compared with the S-wave state, the P-wave state has a larger relativistic correction. It plays such an important role in the processes in which a heavy-light P-wave meson is involved that large error would be brought into the results if the relativistic correction were ignored. We can understand these properties through the shape of the wave functions. For the S-wave state, the wave function is a function of the relative momentum $\boldsymbol{q}$ between the quark and antiquark; usually the dominant contribution comes from the range of small momentum $\boldsymbol{q}$ close to the origin point, which means a small relativistic correction. But for the P-wave state, the dominant contribution comes mainly from the middle range of $\boldsymbol{q}$, therefore a large

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relativistic correction should be taken into account. Thus a more precise calculation needs a relativistic model. Hereby, we offer a relativistic scheme for handling the semileptonic decays involving P-waves.

In this study, we calculate the decay widths and branching ratios in the Bethe-Salpeter (BS) method [11, 12] with the Mandelstam formalism [13]. First, we take the semileptonic decay of $\mathrm{B_s}$ meson as an example, use the Bethe-Salpeter Equation to obtain the relativistic wave functions of corresponding mesons, and then write down the transition matrix element with the relativistic BS wave functions as input in the Mandelstam formulation. Thus the transition matrix element provides the full phase space spectrum of the corresponding form factor, not only in the endpoint region, but also in the full Q^2 region. In this way, the recoil effect is considered, and will be shown in the text below. The non-perturbative QCD effects will be embodied in the overlapping integral of the wave functions of the initial and final states. By using this method, the relativistic corrections from both kinematics and dynamics are included. As for the result of B decays, we can easily draw out the result in the same way.

The remaining parts of this paper are organized as follows: in Section 2, the formulation of the exclusive semileptonic decay amplitude is presented; in Section 3, we show how to calculate the transition matrix element and give the form factors by the Bethe-Salpeter equation with the help of Mandelstam formalism; in Section 4, we give the relativistic Salpeter wave functions and their normalization conditions; finally, we present the numerical results and conclusion in Section 5.

2 Formulation of exclusive semileptonic decay amplitude

Taking the situation of $\overline{\mathrm{B}}{}^0_{\mathrm{s}}\to\mathrm{D}^+_{\mathrm{s1}}\mathrm{l}^-\overline{\nu}_{\mathrm{l}}$ as an instance to discuss, the Feynman diagram is shown in Fig. 1, where P, M are the momentum and mass of the initial state $\overline{\mathrm{B}}{}^0_{\mathrm{s}}$, P_{f}, M_{f} are those of the final meson $\mathrm{D}^+_{\mathrm{s1}}$, respectively. Other quantities such as quark masses and the corresponding momenta are all marked out.

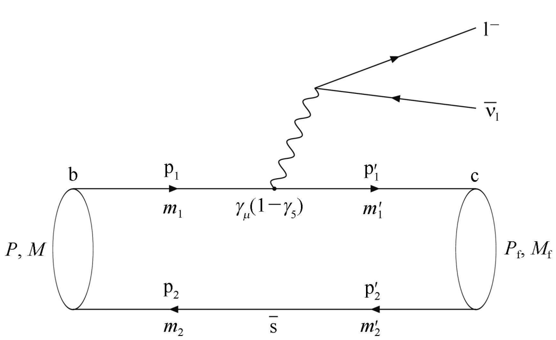

Fig. 1. The semileptonic decay of $\overline{\mathrm{B}}{}^0_{\mathrm{s}}\to\mathrm{D}^+_{\mathrm{s1}}\mathrm{l}^-\overline{\nu}_{\mathrm{l}}$.

The matrix element of the semileptonic decay can be generally written as [14]:

$$\mathcal{T}=\frac{G_{\mathrm{F}}}{\sqrt{2}}V_{\mathrm{cb}}\bar{u}_{\mathrm{l}}\gamma^{\mu}(1-\gamma^5)v_{\overline{\nu}_{\mathrm{l}}}\langle D_{\mathrm{s1}}(P_{\mathrm{f}})|J_{\mu}|B_{\mathrm{s}}(P)\rangle, \tag{1}$$

where V_{cb} is the CKM matrix element, J_{μ} is the current responsible for the decay, P and P_{f} are the momenta of initial state $\overline{\mathrm{B}}{}^0_{\mathrm{s}}$ and final state $\mathrm{D}^+_{\mathrm{s1}}$ respectively. Thus we have square of the matrix element:

$$|\mathcal{T}|^2=\frac{G_{\mathrm{F}}^2}{2}|V_{\mathrm{cb}}|^2 l^{\mu\nu}h_{\mu\nu}, \tag{2}$$

where the leptonic tensor is:

$$l^{\mu\nu}=\bar{u}_{\mathrm{l}}\gamma^{\mu}(1-\gamma^5)v_{\overline{\nu}_{\mathrm{l}}}\bar{v}_{\overline{\nu}_{\mathrm{l}}}(1+\gamma^5)\gamma^{\nu}u_{\mathrm{l}} \tag{3}$$

and the hadronic tensor is defined by:

$$h_{\mu\nu}\equiv\langle B_{\mathrm{s}}(P)|J^{\dagger}_{\nu}|D_{\mathrm{s1}}(P_{\mathrm{f}})\rangle\langle D_{\mathrm{s1}}(P_{\mathrm{f}})|J_{\mu}|B_{\mathrm{s}}(P)\rangle, \tag{4}$$

where the summation is over the inner quantities, like the spin or polarization vector. Based on Lorentz covariance, $h_{\mu\nu}$ can be written as:

$$\begin{aligned}h_{\mu\nu} = & -\alpha g_{\mu\nu}+\beta_{++}(P+P_{\mathrm{f}})_{\mu}(P+P_{\mathrm{f}})_{\nu}\\ & +\beta_{+-}(P+P_{\mathrm{f}})_{\mu}(P-P_{\mathrm{f}})_{\nu}\\ & +\beta_{-+}(P-P_{\mathrm{f}})_{\mu}(P+P_{\mathrm{f}})_{\nu}\\ & +\beta_{--}(P-P_{\mathrm{f}})_{\mu}(P-P_{\mathrm{f}})_{\nu}\\ & +\mathrm{i}\gamma\epsilon_{\mu\nu\rho\sigma}(P+P_{\mathrm{f}})^{\rho}(P-P_{\mathrm{f}})^{\sigma},\end{aligned} \tag{5}$$

the factors α, β, γ are functions of Q^2 and will be defined in the next section.

3 The weak current matrix elements by Mandelstam formalism

To evaluate the exclusive weak decay of $\mathrm{B_s}$ meson, one needs to calculate the transition matrix element of current sandwiched between two single-hadron states, i.e., $\langle D_{\mathrm{s1}}|J_{\mu}|B_{\mathrm{s}}\rangle$. According to the Mandelstam formalism [13], at the leading order, the matrix element $\langle D_{\mathrm{s1}}(P_{\mathrm{f}})|J_{\mu}|B_{\mathrm{s}}(P)\rangle$ can be written as [15]:

$$\begin{aligned}\langle D_{\mathrm{s1}}(P_{\mathrm{f}})|J_{\mu}|B_{\mathrm{s}}(P)\rangle = \int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\mathrm{Tr}\Bigg[&\overline{\varphi^{++}_{P_{\mathrm{f}}}}\left(\boldsymbol{q}-\frac{m_{\mathrm{s}}}{m_{\mathrm{c}}+m_{\mathrm{s}}}\boldsymbol{P}_{\mathrm{f}}\right)\\ &\times\gamma_{\mu}(1-\gamma_5)\varphi^{++}_{\mathrm{P}}(\boldsymbol{q})\frac{\not{P}}{M}\Bigg],\end{aligned} \tag{6}$$

where we have chosen the center of mass system of initial meson B_{s}; $\boldsymbol{q}$ is the three dimensional inner relative momentum between quark and anti-quark, $\boldsymbol{P}_{\mathrm{f}}$ is the three dimensional momentum of final state, respectively; φ^{++}_{P} or $\overline{\varphi^{++}_{P_{\mathrm{f}}}}$ is the positive energy BS wave function of bound states, which will be obtained by solving the BS equation in the next section. One can see from Eq. (6) that the transition matrix element is formulated as an overlapping integral of the BS wave functions of the initial and final states. As claimed in the introduction, the non-perturbative effects are included in this overlapping integral, since the wave functions are obtained by solving the BS equation whose kernel is a QCD-inspired potential. Furthermore, the matrix element is a function of the final state momentum $\boldsymbol{P}_{\rm f}$, which plays an important role in the calculation since there is no constraint on its range, so our method is suitable not only for the zero recoil (endpoint) region, but also for all the recoil regions.

Generally, the matrix element $\langle D_{\rm s1}(P_{\rm f})|J^{\mu}|B_{\rm s}(P)\rangle$ can be expressed in various quantities and one can form it by the available kinematic variables such as $P_{\rm f}^{\mu}$, P^{μ}. The coefficients of these variables are Lorentz-invariant and are usually called form factors. The transition matrix element can be written as a function of the form factors:

$$\begin{aligned}&\langle D_{\rm s1}(P_{\rm f},\epsilon)|V_{\mu}|B_{\rm s}(P)\rangle\\ &\equiv {\rm i}g\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}(P+P_{\rm f})^{\rho}(P-P_{\rm f})^{\sigma},\end{aligned}\tag{7}$$

$$\begin{aligned}\langle D_{\rm s1}(P_{\rm f},\epsilon)|A_{\mu}|B_{\rm s}(P)\rangle \equiv\ & f\varepsilon^{*}_{\mu}+h_{+}(\varepsilon^{*}\cdot P)(P+P_{\rm f})_{\mu}\\ &+h_{-}(\varepsilon^{*}\cdot P)(P-P_{\rm f})_{\mu},\end{aligned}\tag{8}$$

where V_{μ} is the hadronic vector current, and A_{μ} is the axial vector current; g, f, h_{+} and h_{-} are the corresponding form factors, whose numerical values will be obtained by our model.

The coefficients defined in Eq. (5) can be expressed as functions of form factors and Q^2:

$$\alpha = f^2+4M^2g^2|\boldsymbol{P}_{\rm f}|^2,\tag{9}$$

$$\begin{aligned}\beta_{++} =\ & \frac{f^2}{4M_{\rm f}^2}-M^2g^2y+\frac{1}{2}\left[\frac{M^2}{M_{\rm f}^2}(1-y)-1\right]fh_{+}\\ &+\frac{M^2|\boldsymbol{P}_{\rm f}|^2}{M_{\rm f}^2}h_{+}^2,\end{aligned}\tag{10}$$

$$\begin{aligned}\beta_{+-} =\ & \beta_{-+}=g^2(M^2-M_{\rm f}^2)-\frac{f^2}{4M_{\rm f}^2}-\frac{1}{2}f(h_{+}+h_{-})\\ &-\frac{1}{2}f(h_{+}-h_{-})\frac{ME_{\rm f}}{M_{\rm f}^2}+h_{+}h_{-}\frac{M^2|\boldsymbol{P}_{\rm f}|^2}{M_{\rm f}^2},\end{aligned}\tag{11}$$

$$\begin{aligned}\beta_{--} =\ & -g^2(M^2+2ME_{\rm f}+M_{\rm f}^2)+\frac{f^2}{4M_{\rm f}^2}\\ &-fh_{-}\left(\frac{ME_{\rm f}}{M_{\rm f}^2}+1\right)+h_{-}^2\frac{M^2|\boldsymbol{P}_{\rm f}|^2}{M_{\rm f}^2},\end{aligned}\tag{12}$$

$$\gamma = 2gf,\tag{13}$$

in which $\boldsymbol{P}_{\rm f}$ is the three dimensional momentum of final state, $y=Q^2/M^2$, $Q=P-P_{\rm f}$.

4 The relativistic wave functions

The kinematic relativistic effect mainly comes from the wave functions, so we need relativistic wave functions to calculate the corresponding decay form factors. There are two aspects concerning this point: one is that the expression of the wave functions should be in a relativistic form, and the other one is that we need a relativistic dynamic equation to obtain the numerical values of the wave functions. To match the latter requirement, we have the famous Bethe-Salpeter equation [11], or its instantaneous one, the Salpeter equation [12]. For the relativistic expression of the wave functions, we will briefly review our recent results in the following subsections.

4.1 The wave function for $^1S_0$ state

The general relativistic expression for the relativistic Salpeter wave function of the bound state $J^P=0^-$ can be written as (in center-of-mass system) [16]:

$$\begin{aligned}\varphi_{^1S_0}(\boldsymbol{q}) =\ & M\times\left[\frac{\not{P}}{M}f_1(\boldsymbol{q})+f_2(\boldsymbol{q})+\frac{\not{q}_{_{P_\perp}}}{M}f_3(\boldsymbol{q})\right.\\ &\left.+\frac{\not{P}\not{q}_{_{P_\perp}}}{M^2}f_4(\boldsymbol{q})\right]\gamma_5,\end{aligned}\tag{14}$$

where $q_{_{P_\perp}}=(0,\boldsymbol{q})$, and M is the mass of the corresponding meson. Comparing with the usually used wave function, we have $f_1\neq f_2$, and there are extra terms f_3 and f_4 in proportion to relative momentum $\boldsymbol{q}$. The components of the wave function have the following relations [16]:

$$\begin{aligned}&f_3(\boldsymbol{q})=\frac{f_2(\boldsymbol{q})M(-\omega_1+\omega_2)}{m_2\omega_1+m_1\omega_2};\\ &f_4(\boldsymbol{q})=-\frac{f_1(\boldsymbol{q})M(\omega_1+\omega_2)}{m_2\omega_1+m_1\omega_2}.\end{aligned}\tag{15}$$

From this wave function we can obtain the wave functions corresponding to the positive projection [16]:

$$\begin{aligned}\varphi^{++}_{^1S_0}(\boldsymbol{q}) =\ & \frac{M}{2}\left(f_1(\boldsymbol{q})+f_2(\boldsymbol{q})\frac{m_1+m_2}{\omega_1+\omega_2}\right)\left[\frac{\omega_1+\omega_2}{m_1+m_2}+\frac{\not{P}}{M}\right.\\ &\left.-\frac{\not{q}_{_{P_\perp}}(m_1-m_2)}{m_2\omega_1+m_1\omega_2}+\frac{\not{q}_{_{P_\perp}}\not{P}(\omega_1+\omega_2)}{M(m_2\omega_1+m_1\omega_2)}\right]\gamma_5,\end{aligned}\tag{16}$$

where the numerical values of wave function $f_1(\boldsymbol{q})$ and $f_2(\boldsymbol{q})$ are obtained by solving the Salpeter equation. The normalization condition is [16]

$$\int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}4f_1(\boldsymbol{q})f_2(\boldsymbol{q})M^2\left\{\frac{\omega_1+\omega_2}{m_1+m_2}+\frac{m_1+m_2}{\omega_1+\omega_2}+\frac{2|\boldsymbol{q}|^2(\omega_1m_1+\omega_2m_2)}{(\omega_1m_2+\omega_2m_1)^2}\right\}=2M. \tag{17}$$

4.2 The wave function for $^1P_1$ state

Similar to that of $^1S_0$ state, we can easily get the positive projection of the $^1P_1$ wave function [17]:

$$\begin{aligned}\varphi^{++}_{^1P_1}(\boldsymbol{q}) = &\frac{1}{2}\left[g_1(\boldsymbol{q})+g_2(\boldsymbol{q})\frac{\omega_1+\omega_2}{m_1+m_2}\right](q_{P_\perp}\cdot\varepsilon_{P_\perp})\\ &\times\left[1+\frac{\not{P}}{M}\frac{m_1+m_2}{\omega_1+\omega_2}-\frac{\not{q}_{P_\perp}(\omega_1-\omega_2)}{m_2\omega_1+m_1\omega_2}\right.\\ &\left.+\frac{\not{q}_{P_\perp}\not{P}(m_1+m_2)}{M(m_2\omega_1+m_1\omega_2)}\right].\end{aligned} \tag{18}$$

The normalization condition for the $^1P_1$ wave function is [17]:

$$\int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\frac{16g_1(\boldsymbol{q})g_2(\boldsymbol{q})\omega_1\omega_2|\boldsymbol{q}|^2}{3(m_1\omega_2+m_2\omega_1)}=2M. \tag{19}$$

4.3 The wave function for $^3P_1$ state

Similarly, we also have [17]

$$\begin{aligned}\varphi^{++}_{^3P_1}(\boldsymbol{q}) = &\frac{1}{2}\left[\varphi_1(\boldsymbol{q})+\varphi_2(\boldsymbol{q})\frac{\omega_1+\omega_2}{m_1+m_2}\right]\\ &\times\left[\frac{\not{\varepsilon}_{P_\perp}\not{q}_{P_\perp}\not{P}-\not{P}q_{P_\perp}\cdot\varepsilon_{P_\perp}}{M}\right.\\ &+\left(\not{\varepsilon}_{P_\perp}\not{q}_{P_\perp}-q_{P_\perp}\cdot\varepsilon_{P_\perp}\right)\frac{m_1+m_2}{\omega_1+\omega_2}\\ &+\frac{(\not{\varepsilon}_{P_\perp}\not{P}q^2_{P_\perp}-\not{q}_{P_\perp}\not{P}q_{P_\perp}\cdot\varepsilon_{P_\perp})(\omega_1-\omega_2)}{M(m_2\omega_1+m_1\omega_2)}\\ &\left.-\frac{(\not{\varepsilon}_{P_\perp}q^2_{P_\perp}-\not{q}_{P_\perp}q_{P_\perp}\cdot\varepsilon_{P_\perp})(m_1+m_2)}{m_2\omega_1+m_1\omega_2}\right]\gamma_5.\end{aligned} \tag{20}$$

The normalization condition for the $^3P_1$ wave function is [17]:

$$\int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\frac{32\varphi_1(\boldsymbol{q})\varphi_2(\boldsymbol{q})\omega_1\omega_2(\omega_1\omega_2-m_1m_2+|\boldsymbol{q}|^2)}{3(m_1+m_2)(\omega_1+\omega_2)}=2M. \tag{21}$$

4.4 The wave function for the physical 1^+ state

The two physical 1^+ states of P-wave, which are also written as $\frac{1}{2}^+$ and $\frac{3}{2}^+$ in HQET, can be described as a pair of mixing states of $^1P_1$ and $^3P_1$ as follows [18]:

$$\begin{aligned}\left|\frac{1}{2}\right\rangle&=|^1P_1\rangle\cos\theta+|^3P_1\rangle\sin\theta;\\ \left|\frac{3}{2}\right\rangle&=-|^1P_1\rangle\sin\theta+|^3P_1\rangle\cos\theta,\end{aligned} \tag{22}$$

where $\left|\frac{1}{2}\right\rangle$ and $\left|\frac{3}{2}\right\rangle$ are two physical states denoted by the angular momentum j_{q} of the light quark, and θ is the mixing angle between $^1P_1$ and $^3P_1$ states.

$\mathrm{D}^+_{\mathrm{s1}}(2460)$ and $\mathrm{D}^+_{\mathrm{s1}}(2536)$ are corresponding to $\left|\frac{1}{2}\right\rangle$ and $\left|\frac{3}{2}\right\rangle$ respectively. If the heavy-quark limit $m_{\mathrm{Q}}\to\infty$ is taken, $\theta=\theta_{\mathrm{i}}\cong35.26°$ and the relations become [1, 2]:

$$\begin{aligned}|D^+_{\mathrm{s1}}(2460)\rangle&=\sqrt{\frac{2}{3}}|^1P_1\rangle_{(\mathrm{c\bar{s}})}+\sqrt{\frac{1}{3}}|^3P_1\rangle_{(\mathrm{c\bar{s}})};\\ |D^+_{\mathrm{s1}}(2536)\rangle&=-\sqrt{\frac{1}{3}}|^1P_1\rangle_{(\mathrm{c\bar{s}})}+\sqrt{\frac{2}{3}}|^3P_1\rangle_{(\mathrm{c\bar{s}})}.\end{aligned} \tag{23}$$

Meanwhile, $D'^{0,\pm}_1$ and $D^{0,\pm}_1$ are corresponding to $\left|\frac{1}{2}\right\rangle$ and $\left|\frac{3}{2}\right\rangle$ respectively, thus:

$$\begin{aligned}\left|D^{0/\pm}_1\right\rangle&=\sqrt{\frac{2}{3}}|^1P_1\rangle_{(\mathrm{c\bar{u},c\bar{d}})}+\sqrt{\frac{1}{3}}|^3P_1\rangle_{(\mathrm{c\bar{u},c\bar{d}})};\\ \left|D'^{0/\pm}_1\right\rangle&=-\sqrt{\frac{1}{3}}|^1P_1\rangle_{(\mathrm{c\bar{u},c\bar{d}})}+\sqrt{\frac{2}{3}}|^3P_1\rangle_{(\mathrm{c\bar{u},c\bar{d}})}.\end{aligned} \tag{24}$$

5 The numerical results

In our model, there are some model-dependent parameters to be fixed and they are chosen as: $m_{\mathrm{b}}=4.96$ GeV, $m_{\mathrm{c}}=1.62$ GeV, $m_{\mathrm{s}}=0.500$ GeV, $m_{\mathrm{u}}=0.305$ GeV, $V_{\mathrm{cb}}=0.0406$. The life-times of corresponding mesons are: $\tau_{\mathrm{B_s}}=1.472\times10^{-12}$ s, $\tau_{\mathrm{B^-}}=1.638\times10^{-12}$ s, $\tau_{\mathrm{B^0}}=1.525\times10^{-12}$ s [19].

In Tables 1 and 2, we list our results in the infinity heavy-quark limit along with other theoretical results and the concerned experimental data, which are from D0 Collaboration [22].

Although the semileptonic experimental measurement listed in Table 2 is not electronic but muonic, we are still able to get confidence from the good matching of our result and the existing experimental data. Ebert et al. have calculated the branching ratios of these two

Table 1. Different results for the branching ratios of $\overline{B}_s^0 \to D_{s1}^+(2460) l^- \overline{\nu}_l$ in percent (%).

ours ($\theta_i \cong 35.26°$)	Ebert [20]	QSR [6]	QSR in HQET [9]	CQM [7]
0.181	0.18	$\cong$0.49	0.08–0.10	0.752–0.869

Table 2. Different results for the branching ratios of $\overline{B}_s^0 \to D_{s1}^+(2536) l^- \overline{\nu}_l$ in percent (%).

ours ($\theta_i \cong 35.26°$)	Ebert [20]	ISGW2 [10]	Mayorga [21]	D0($B_s^0 \to D_{s1}^-(2536)\mu^+\nu X$) [22]
0.972	1.06	0.53	0.195	1.03±0.20±0.17±0.14

Table 3. Our results for the branching ratios of $B \to D_1^{(\prime)} l^- \overline{\nu}_l$ in percent (%).

$B^- \to D_1^{\prime 0} l^- \overline{\nu}_l$	$B^- \to D_1^0 l^- \overline{\nu}_l$	$\overline{B}^0 \to D_1^{\prime +} l^- \overline{\nu}_l$	$\overline{B}^0 \to D_1^+ l^- \overline{\nu}_l$
0.227	0.666	0.215	0.623

Table 4. Other results for the branching ratios of similar processes in percent (%).

	ours	CQM [8]	Belle [24]	BABAR [25, 26]
$Br(B^+ \to \overline{D}_1^{\prime 0} l^+ \nu_l)\ Br(\overline{D}_1^{\prime 0} \to D^{*-}\pi^+)$	0.151	0.132	<0.07	0.27±0.04±0.05
$Br(B^+ \to \overline{D}_1^0 l^+ \nu_l)\ Br(\overline{D}_1^0 \to D^{*-}\pi^+)$	0.222	0.257	0.42±0.07±0.07	0.297±0.17±0.17
$Br(B^0 \to D_1^{\prime -} l^+ \nu_l)\ Br(D_1^{\prime -} \to \overline{D}^{*0}\pi^-)$	0.143	0.123	<0.5	0.31±0.07±0.05
$Br(B^0 \to D_1^- l^+ \nu_l)\ Br(D_1^- \to \overline{D}^{*0}\pi^-)$	0.415	0.239	0.54±0.19±0.09	0.278±0.024±0.025

semileptonic decays in their own way, which is also a relativistic quark model. One can see that our results are in good agreement with theirs.

As for the case of $B \to D_1^{(\prime)0,\pm} l^- \overline{\nu}_l$, which also contains the singlet-triplet mixing of $^1P_1$ and $^3P_1$ that we are concerned with as the final states, the masses are $m_{D_1^{\prime 0}(2430)} = 2427\ \mathrm{MeV}/c^2$, $m_{D_1^0(2420)} = 2421.4\ \mathrm{MeV}/c^2$ [23]. Considering that people usually prefer to discuss $c\bar{q}(\bar{q}=\bar{u},\bar{d})$ together since the masses of $\bar{u}$-quark and $\bar{d}$-quark are so close to each other that the predicted masses of $D_1^{(\prime)\pm}$ are almost the same as those of $D_1^{(\prime)0}$ in the $c\bar{q}$ picture, we decide to take the values below to carry on the calculation:

$$m_{D_1^{\prime 0,+}(2430)} = 2427\ \mathrm{MeV}/c^2;$$

$$m_{D_1^{0,+}(2420)} = 2421.4\ \mathrm{MeV}/c^2.$$

By the same token, the branching ratios of semileptonic B decays can be easily calculated. We demonstrate our results of $B \to D_1^{(\prime)} l^- \overline{\nu}_l$ at the angle of $\theta_i \cong 35.26°$ in Table 3.

Since the strong decays $D_1^\prime \to D^{*+}\pi^-$ and $D_1^\prime \to D^{*0}\pi^0$ are the only OZI (Okubo-Zweig-Iizuka)-allowed decay channel of $D_1^\prime$ meson, isospin symmetry predicts an approximate branching fraction $\dfrac{2}{3}$ for $D_1^\prime \to D^{*+}\pi^-$ over $D_1^\prime \to D^*\pi$; if we consider their charge conjugate decays to have the same branching ratio (with CP violation ignored), the estimation from our calculation is also listed along with another column of theoretical predictions and the experimental measurements for similar processes which are taken as comparable results are listed in Table 4.

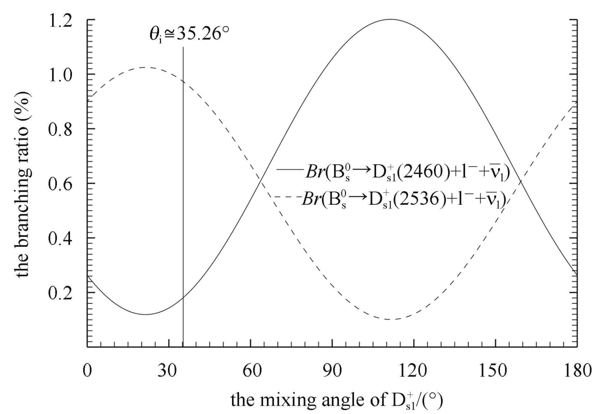

Fig. 2. The branching ratios of $\overline{B}_s^0 \to D_{s1}^+ l^- \overline{\nu}_l$ in different mixing angles.

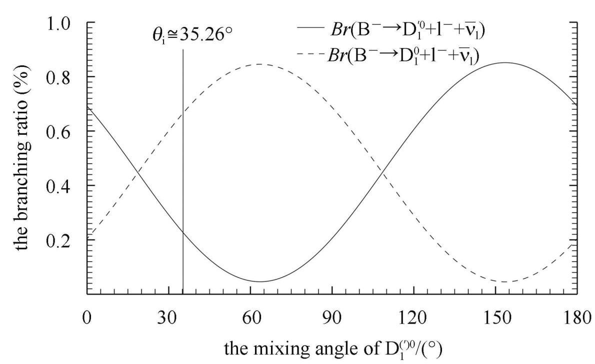

Fig. 3. The branching ratios of $B^- \to D_1^{(\prime)0} l^- \overline{\nu}_l$ in different mixing angles.

From the comparisons, one can find that although our results are close to the existing experimental data in the heavy quark limit, there are still discrepancies. That deviation might be brought up by the hypothesis of the heavy quark limit, which is not quite appropriate to the $c\bar{q}$ system. Because our listed results are based on heavy

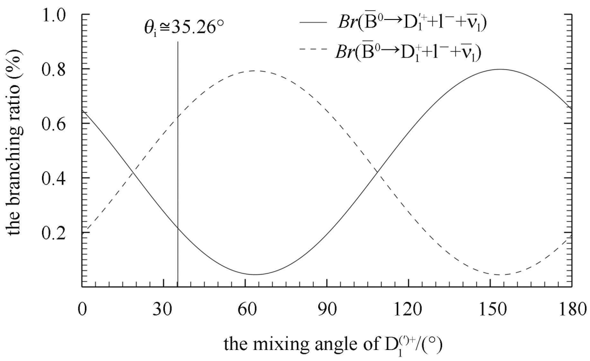

Fig. 4. The branching ratios of $\overline{B}^0 \to D_1^{(\prime)+} l^- \overline{\nu}_l$ in different mixing angles.

quark limit $m_Q \to \infty$, whereas the c-quark is not heavy enough compared with the antiquark in the bound state of the final meson, the heavy quark limit seems not quite natural here. One possibility for further study is to take into account the effect that breaks the hypothesis by modifying the mixing angle. In view of this, we draw Figs. 2, 3 and 4 to demonstrate how the branching ratios vary with different mixing angles, from which one can still get rough values according to our method as references, or to assess the reasonability degree of our results when experiments with improved accuracy determine the mixing angle more precisely in the future.

In conclusion, we calculate the exclusive semileptonic decays to the orbitally excited P-wave mesons $\overline{B}_s^0 \to D_{s1}^+ l^- \overline{\nu}_l$, $B^- \to D_1^{(\prime)0} l^- \overline{\nu}_l$ and $\overline{B}^0 \to D_1^{(\prime)+} l^- \overline{\nu}_l$ by means of the relativistic Bethe-Salpeter method, and derive the curves of the branching ratios versus mixing angles. Special attention is paid to the relativistic corrections.

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