

Re-examination of constraints on the Maxwell-Boltzmann distribution by Helioseismology^{*}

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Abstract: Nuclear reactions in stars occur between nuclei in the high-energy tail of the energy distribution and are sensitive to possible deviations from the standard equilibrium thermal-energy distribution, the well-known Maxwell-Boltzmann Distribution (MBD). Strong constraints on such deviations were made previously with the detailed helioseismic information of the solar structure. With a small deviation parameterized with a factor $\exp[-\delta(E/kT)^2]$, it was shown δ restricted between -0.005 and $+0.002$. These constraints have been carefully re-examined in the present work. We find that a normalization factor was missed in the previous modified MBD. In this work, the normalization factor c is calculated as a function of δ . It shows the factor c is almost unity within the range $0 < \delta \leq 0.002$, which supports the previous conclusion. However, it demonstrates that δ cannot take a negative value from the normalization point of view. As a result, a stronger constraint on δ is defined as $0 \leq \delta \leq 0.002$. The astrophysical implication on the solar neutrino fluxes is simply discussed based on a positive δ value of 0.003 . The reduction of the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes expected from the modified MBD can possibly shed alternative light on the solar neutrino problem. In addition, the resonant reaction rates for the ${}^{14}\text{N}(p, \gamma){}^{15}\text{O}$ reaction are calculated with a standard MBD and a modified MBD, respectively. It shows that the rates are quite sensitive even to a very small δ . This work demonstrates the importance and necessity of experimental verification or testing of the well-known MBD at high temperatures.

Key words: Solar interior, helioseismology, statistical mechanics, solar neutrinos

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1 Introduction

Under the ideal condition of non-interaction states, infinite volume and zero density, a single scale (the temperature or the average one-body energy) characterizes all the equilibrium distributions, which are described by the Maxwell-Boltzmann distribution (MBD) [1–3]. However, it is well-known that the actual distribution, which deviates from the standard MBD, is characterized by additional scales (total energy, Fermi energy, etc.) [1]. The ideal MBD has been widely accepted and utilized in the nuclear astrophysical community [4–6], thereinto, the MBD for the single particle energy distribution can be written as

$$f_{\text{MBD}}(E) = \frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{(kT)^{3/2}} e^{-E/kT}. \quad (1)$$

In the past, Degl’Innocenti et al. [1] made a strong constraint on possible deviations from the standard MBD based on the theoretical and observational knowledge of solar physics, the so called Helioseismology. In their work, small deviations of the modified MBD for the collision-energy distribution of the reacting nuclei i and j , was parameterized to first approximation by introducing a dimensionless parameter δ , and was expressed as (Eq. (4) in Ref. [1])

$$f_{ij}^{\delta}(E) = f_{\text{MBD}}(E) e^{-\delta(E/kT)^2}. \quad (2)$$

They found that δ should lie between -0.005 and $+0.002$ by analyzing the detailed helioseismic information of the solar structure with a stellar evolutionary code FRANCE [7]. It was shown that even a value of δ as small as 0.003 could give important effects on the solar neutrino fluxes.

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In this work, we will make an even stronger constraint on δ by carefully re-examining the previous work.

2 Re-examination

In this report, we will make a stronger constraint on δ by taking the normalization issue which was missed in the previous work into account. Actually, the standard MBD (Eq. (1)) is normalized to unity within the energy range from 0 to ∞ . But the modified MBD (Eq. (2)) was not normalized to unity in the previous work [1]. The above Eq. (2) can be normalized to unity by introducing a factor $c(\delta)$ (a function of δ), then Eq. (2) should be read as

$$f_{ij}^{\delta}(E) = c(\delta) \times f_{\text{MBD}}(E) e^{-\delta(E/kT)^2}. \quad (3)$$

The normalization factor $c(\delta)$ can be calculated by

$$c(\delta) = \frac{\sqrt{\pi}}{2} \frac{1}{\int_0^{\infty} \sqrt{x} \exp(-x - \delta x^2) dx}. \quad (4)$$

Here, $c(\delta=0)=1.0$ becomes the standard MBD condition. For the positive δ values (a depleted high-energy tail), the factor $c(\delta)$ is numerically calculated and shown in Fig. 1. It shows that the factor c is quite close to unity (about 1% error) in the range $0 \leq \delta \leq 0.002$ constrained in the previous work [1]. Thus, the previous conclusions still hold for the positive δ . However, the integration in Eq. (4) has no solution (mathematically un-converged) for the negative δ values (an enhanced high-energy tail). This means that δ cannot take a negative value. Although this convergence issue was ‘‘mathematically’’ solved by the cut-off technique [1] for the negative δ values, this kind of cut-off is actually incorrect in physics. Therefore, a much stronger constraint has to be put on δ following the previous results (Eq. (16) in Ref. [1]), i.e., $0 \leq \delta \leq 0.002$. Back to Fig. 1 in Ref. [1], it is shown how three ratios ($Y_{\text{ph}}/Y_{\text{ph}}^{\text{SSM}}$, $\rho_{\text{b}}/\rho_{\text{b}}^{\text{SSM}}$ and $R_{\text{b}}/R_{\text{b}}^{\text{SSM}}$) can be fitted pretty well for the positive δ

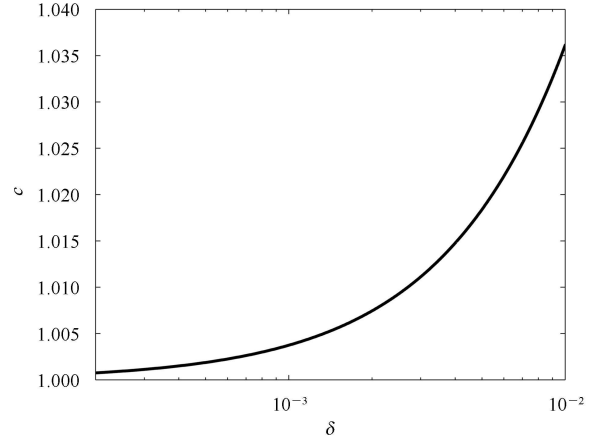


Fig. 1. Normalization factor c of the modified MBD as a function of δ .

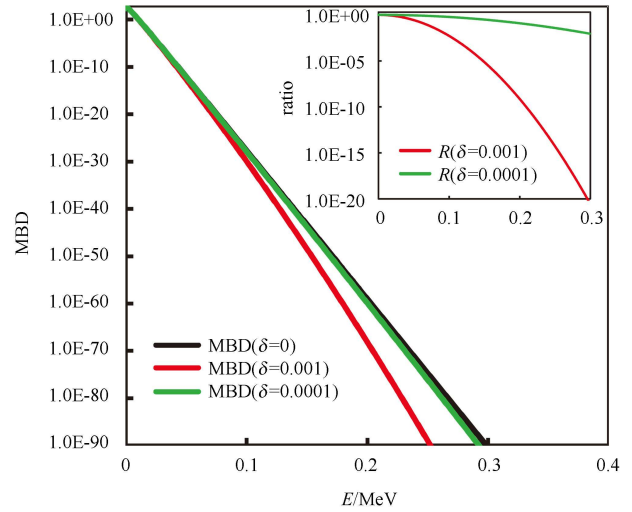


Fig. 2. (color online) The standard MBD (Eq. (1)) and the modified MBD (Eq. (3) with $\delta=0.001$, 0.0001) are shown for the core temperature ($T_9=0.016$) of our Sun. The ratios are inserted for comparison.

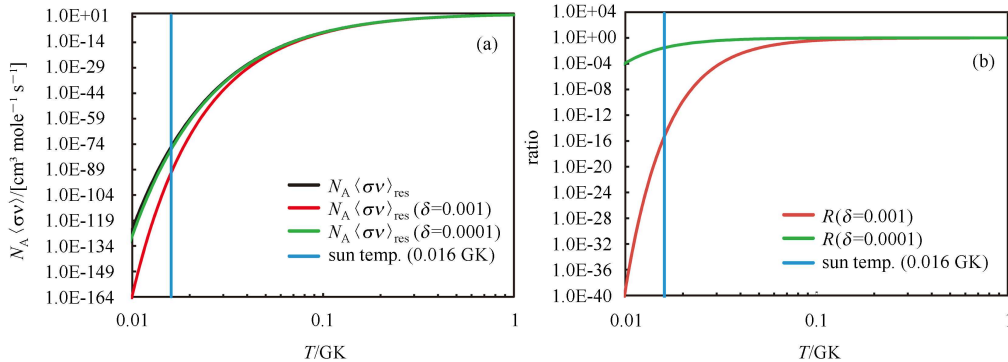


Fig. 3. (color online) Resonant reaction rates of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction for the resonance at $E_r=0.259$ MeV. (a) shows the rates calculated by the standard MBD (in black) and the modified MBD ($\delta=0.001$ in red, $\delta=0.0001$ in green), and (b) shows the corresponding ratios. See text for details.

values. For the largest $\delta=0.007$ data point, small deviations appear in all three ratios, which could be possibly explained by adding the small factor $c(\delta=0.007)$ (2%) which was ignored in the previous calculation. However, reproduction of Fig. 1 in Ref. [1] is beyond the scope of this work.

Degl'Innocenti et al. [1] calculated the effect of non-standard statistics on the solar neutrino fluxes, and showed that even for a small value of δ the Boron and Beryllium neutrino fluxes change substantially. For instance, the relative deviations from Standard Solar Models (SSMs) of the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes are, $\frac{\Delta\Phi_{\text{Be}}}{\Phi_{\text{Be}}}=-0.30$ and $\frac{\Delta\Phi_{\text{B}}}{\Phi_{\text{B}}}=-0.55$, respectively, for $\delta=+0.003$. These pronounced deviations imply that the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes estimated by the modified MBD are expected to be lower by 30% and 55% compared to those derived from the standard MBD utilized in the SSMs. These large deviations could possibly shed an alternative light on the famous solar neutrino problem [8].

3 Application

The standard MBD and modified MBD as a function of energy are calculated for the core temperature ($T_9=0.016$) of our Sun as shown in Fig. 2, and the corresponding ratios are also shown in a small figure inserted. It shows that the modified MBD is much smaller than the standard one at high-energy tail even for a small δ value of 0.0001. This depleted tail will dramatically change the reaction rate, which is of great nuclear astrophysical interest. For example, the well-known narrow resonance at $E_r=0.259$ MeV ($J^\pi=1/2^+$, $\omega\gamma=1.4\times 10^{-8}$ MeV) in the compound ${}^{15}\text{O}$ nucleus [9] dominates the reaction rate of the ${}^{14}\text{N}(p, \gamma){}^{15}\text{O}$ reaction, which rate governs the effi-

ciency of the CNO cycle, at $T_9\leq 1$. Its resonant reaction rates, according to the analytic narrow-resonance equation, can be calculated as [4]

$$N_A\langle\sigma v\rangle_{\text{res}}=1.54\times 10^{-11}\frac{\omega\gamma}{\mu T_9^{3/2}}\exp\left(-\frac{11.605E_r}{T_9}\right), \quad (5)$$

where resonant energy (E_r) and strength ($\omega\gamma$) are in units of MeV, and the reduced mass μ in amu. If the deviation from the standard MBD is taken into account, the above Eq. (5) reads as

$$N_A\langle\sigma v\rangle_{\text{res}}^*=1.54\times 10^{-11}\frac{\omega\gamma}{\mu T_9^{3/2}}\times\exp\left[-\frac{11.605E_r}{T_9}-\delta\left(\frac{11.605E_r}{T_9}\right)^2\right]. \quad (6)$$

The rates calculated by Eqs. (5, 6) are plotted in Fig. 3(a), where $\delta=0.001, 0.0001$ are used in the calculation. The corresponding ratios $R=\frac{N_A\langle\sigma v\rangle_{\text{res}}^*}{N_A\langle\sigma v\rangle_{\text{res}}}$ are shown in Fig. 3(b). It shows that the rates with the modified MBD are much smaller than those with the standard MBD. Even for a very small $\delta=0.0001$, the former rate is only about 3% of the standard value at a typical temperature of $T_9=0.016$ in the core of our Sun.

4 Conclusion

As a conclusion, a small deviation of the MBD can dramatically affect the nuclear reaction rates, which are very crucial input parameters for the astrophysical nucleosynthesis models. Therefore, a detailed experimental verification or test of the well-known MBD at high temperatures is extremely important and necessary for the nuclear astrophysical community.

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