Investigation of deformed nuclei with a new potential combination

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Abstract: We investigate the alpha-decay half-lives of non-spherical nuclei with the Yukawa potential as the proximity potential and an angle-dependent term that accounts for the deformation effects and apply the results to Ho, Tb, Lu, Tm, Ta, Hf, Yb, Re, Ir, Pt, Au, Po, etc. as examples. The comparison with the existing data is encouraging.

Key words: alpha decay, deformed nuclei, ground state, multiple approximations, Yukawa potential

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1 Introduction

The half-lives of isotopes can be determined from various theoretical and experimental techniques including beta, neutrino-less double beta and alpha decays [1]. The latter, however, is in many cases the only decay mode for the heavy and super-heavy nuclei. On the other hand, the study of alpha decay enables us to investigate many topics of nuclear structure such as nuclear spins and parities, shell effects, nuclear deformations, magic numbers and the island of stability [1, 2]. It can also provide us with notable clues in the search of new elements [2].

This field, just as any other scientific field, has been simultaneously investigated on both theoretical and experimental bases to find a reliable physical theory. To perform a useful experiment, however, primary theoretical estimations and calculations are inevitable [3, 4]. Here, we follow a theoretical approach to study the alpha decay and the consequent half-lives. In theoretical language, the alpha decay is regarded as the tunneling phenomenon of an alpha particle through the potential barrier between the daughter nucleus and the alpha particle [1–3]. A variety of theoretical techniques have been applied to the field including the cluster [5], relativistic mean field [6], the Skyrme-Hartree Fock mean field [7], density-dependent M3Y effective interaction [8–11], the coupled channel [12–14], the analytical supersymmetric [15] and the generalized liquid drop models [16–21]. Here, we work on the potential model which will be thoroughly reviewed in the next section. As many nuclei are in fact non-spherical, we will propose an angle-dependence in the potential relation and investigate the related effects [22, 23].

2 The alpha decay theory

Within the potential model, the barrier is normally considered as a sum of the Coulomb, proximity and centrifugal potentials for the touching configuration and the separated fragments [24]. The potential $V(r)$ is made up of two parts in the overlapping ($r<C_t$) and nonoverlapping ($r\geqslant C_t$) regions. In fact, we consider the potential as [24, 25]

$$V(\overline{r})=\begin{cases} a_0+a_1 r+a_2 r^2 & R_p\leqslant r<C_t, \\ V_c(r)+V_{prox}(z)+V_l(r) & r\geqslant C_t, \end{cases} \tag{1}$$

R_p is the radius of the parent nucleus, $V_l(r)=\dfrac{\hbar^2 l(l+1)}{2\mu r^2}$ stands for the centrifugal potential, $\mu=m\dfrac{A_d A_\alpha}{A_d+A_\alpha}$ and m, A_d and A_α respectively denote the masses of nucleon, daughter nucleus and the alpha. $V_{prox}(z)$ represents the proximity potential and z is the distance between the near surfaces of the fragments. The Coulomb potential V_c is given by $V_c(r)=\dfrac{Z_d Z_\alpha e^2}{r}$, with Z_d and Z_α being the atomic numbers of daughter and parent nuclei, respectively. In addition, $r=z+\overline{C}_d(\theta)+C_\alpha$ is the distance between the fragment centers with [26]

$$\overline{C}_t(\theta)=\overline{C}_d(\theta)+C_\alpha. \tag{2}$$

The Süssmann central radii $C_d(\theta)$ of the daughter nuclei are related to $R_d(\theta)$ via [27]

$$C_d(\theta)=R_d(\theta)-\frac{1}{2}kb^2, \tag{3}$$

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where k is the total curvature of the surface and $b\approx 1$ represents the width of the nuclear surface. The Süssmann central radius C_α of the emitted alpha is [27]

$$C_\alpha = R_\alpha - \frac{b^2}{R_\alpha}, \tag{4}$$

R_i, the sharp radii, are related to mass number A_i via [28]

$$R_\mathrm{i} = 1.28 A_\mathrm{i}^{1/3} - 0.76 + 0.8 A_\mathrm{i}^{-1/3}, \tag{5}$$

where i=p, d, α respectively denote the parent, the daughter and the alpha particle. If the nucleus is of deformed shape, θ , the angle between the axis of symmetry of the parent or daughter nucleus with the direction of emitted alpha, is given by [29]

$$R_\mathrm{i}(\theta) = R_\mathrm{i}\left[1+\sum_{n=0}^{\infty}\sqrt{\frac{2l+1}{4\pi}}\beta_l P_l(\cos(\theta))\right], \tag{6}$$

or

$$R_\mathrm{i}(\theta) = R_\mathrm{i}\left[1+\beta_2\sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2(\theta)-\frac{1}{2}\right)\right.$$
$$+\beta_4\sqrt{\frac{9}{4\pi}}\frac{1}{64}(9+20\cos(2\theta)+35\cos(4\theta))$$
$$+\beta_6\sqrt{\frac{13}{4\pi}}\frac{1}{16}(231\cos^6(\theta)-315\cos^4(\theta)$$
$$\left.+105\cos^2(\theta)-5)\right], \tag{7}$$

where P_1 is the Legendre function and the deformation parameter $\beta_l = \beta_{l0}$ is determined via [30]

$$\beta_{lm} = \sqrt{4\pi}\frac{\int R_\mathrm{i}(\theta,\varphi)Y_1^m(\theta,\varphi)\mathrm{d}\Omega}{\int R_\mathrm{i}(\theta,\varphi)Y_0^0(\theta,\varphi)\mathrm{d}\Omega}, \tag{8}$$

where

$$Y_l^m(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}}P_1(\cos(\theta))\mathrm{e}^{\mathrm{i}m\varphi}. \tag{9}$$

The proximity potential $V_\mathrm{p}(z)$ is considered as [28]

$$V_\mathrm{prox}(z) = 4\pi\gamma b\frac{C_\mathrm{d}C_\alpha}{C_\mathrm{t}}\Phi\left(\frac{z}{b}\right), \tag{10}$$

with the nuclear surface tension coefficient being

$$\gamma = 0.9517\left[1-1.7826\frac{(N_\mathrm{p}-Z_\mathrm{p})^2}{A_\mathrm{p}^2}\right]\ (\mathrm{MeV/fm^2}). \tag{11}$$

For the proximity potential within the interval $0\leqslant\varepsilon\leqslant 1.9475$, we choose the choice of Ref. [25]:

$$\Phi(\varepsilon) = -1.7817+0.9270\varepsilon+0.0169\varepsilon^2-0.05148\varepsilon^3$$
$$0\leqslant\varepsilon\leqslant 1.9475. \tag{12a}$$

For $\varepsilon\geqslant 1.9475$, we have considered the Yukawa proximity potential

$$\Phi(\varepsilon) = \frac{b_0\mathrm{e}^{\frac{-\varepsilon}{b_1}}}{\varepsilon}\quad \varepsilon\geqslant 1.9475, \tag{12b}$$

where $\varepsilon = \frac{z}{b}$ and the constants b_0 and b_1 can be determined from the continuity conditions of $\Phi_{\mathrm{Eq.\ (12a)}}$ and $\Phi_{\mathrm{Eq.\ (12b)}}$, i.e.

$$\text{(i)}\,\Phi_{\mathrm{Eq.\ (12a)}}(C_\mathrm{t}+1.9475) = \Phi_{\mathrm{Eq.\ (12b)}}(C_\mathrm{t}+1.9475), \tag{13a}$$

$$\text{(ii)}\,\Phi'_{\mathrm{Eq.\ (12a)}}(C_\mathrm{t}+1.9475) = \Phi'_{\mathrm{Eq.\ (12b)}}(C_\mathrm{t}+1.9475). \tag{13b}$$

We have plotted the potential in Fig. 1. The decay energy Q is obtained via [31, 32]

$$Q = B(Z-2,A-2)+28.3-B(Z,A)\ (\mathrm{MeV}), \tag{14}$$

$$B(Z,A) = 7.298Z+8.071(A-Z)-M(A,Z)\ (\mathrm{MeV}), \tag{15}$$

where $B(A,Z)$ and $M(A,Z)$ respectively denote the binding energy and the excess mass.

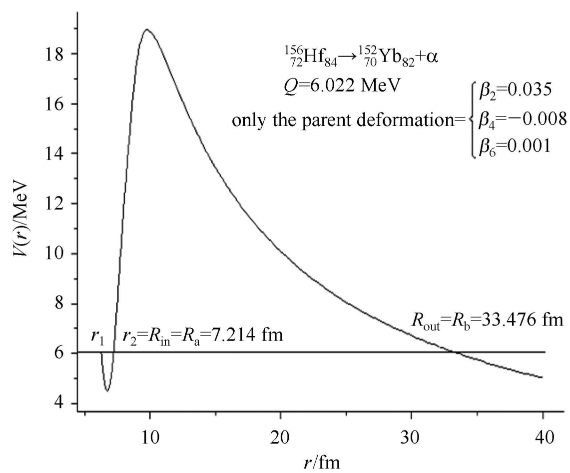

Fig. 1. The potential vs. the distance between the fragment centers (R_a and R_b are the first and second turning point respectively).

The constants a_0, a_1, a_2 in Eq. (1a) are determined from the continuity conditions of the potential and its derivative, i.e. [24]

$$\text{(i)}\,V(R_\mathrm{p}) = Q, \tag{16a}$$

$$\text{(ii)}\,V_{\mathrm{Eq.\ (1a)}}(C_\mathrm{t}) = V(C_\mathrm{t}), \tag{16b}$$

$$\text{(iii)}\,V'_{\mathrm{Eq.\ (1a)}}(C_\mathrm{t}) = V'(C_\mathrm{t}). \tag{16c}$$

On the other hand, according to the WKB approximation, the penetration probability is

$$P = \exp\left\{-\frac{2}{\hbar}\int_{R_\mathrm{a}}^{R_\mathrm{b}}\sqrt{2\mu(V(r)-Q)}\mathrm{d}r\right\}. \tag{17}$$

The turning boundaries are obtained from $V_{\text{Eq. (1a)}}(r)=Q$. The latter has two roots; $r_1=R_\text{p}$ and $r_2=R_\text{a}$ with $r_2\geqslant r_1$ and $V(R_\text{b})=Q$. Finally, the half-life can be calculated from [33]

$$T_{1/2}=\left(\frac{\ln 2}{\lambda}\right)=\left(\frac{\ln 2}{\nu P}\right)(\text{s}^{-1}), \tag{18}$$

where $\nu=(\omega/2\pi)=(2E/h)$ and λ are the frequency of collision with barrier per second and decay constant, respectively. E is the empirical zero point vibration energy given by [34]

$$E=Q\left\{0.056+0.039\exp\left[\frac{4-A_\alpha}{2.5}\right]\right\}\ (\text{MeV}). \tag{19}$$

Substitution of E and P in (17) determines the half-life. The results are reported in Table 1.

Table 1. Alpha decay half-lives for the ground state to ground state transition.

	I_i^π	I_f^π	parent deformation			daughter deformation		$T_{1/2}^\text{cal}$/s	$T_{1/2}^\text{exp}$/s	HF	
			β_2	β_4	β_6	β_2	β_4			our model	Ref. [33]
^{152}Ho→^{148}Tb	2^-	2^-	0	0	0	0	0	1.630×10^3	1.35×10^3	0.83	0.57
			0	0	0	−0.052	0.009	1.628×10^3			
			0.126	0.038	0.016	0	0	1.630×10^3			
			0.126	0.038	0.016	−0.052	0.009	1.628×10^3			
^{154}Ho→^{150}Tb	2^-	2^-	0	0	0	0	0	1.197×10^6	3.71×10^6	3.1	1.91
			0	0	0	0.143	0.048	1.187×10^6			
			0.170	0.044	−0.001	0	0	1.197×10^6			
			0.170	0.044	−0.001	0.143	0.048	1.187×10^6			
^{156}Lu→^{152}Tm	2^-	2^-	0	0	0	0	0	7.985×10^{-1}	4.94×10^{-1}	0.62	0.47
			0	0	0	−0.052	0.009	7.977×10^{-1}			
			−0.104	0.012	0.003	0	0	7.985×10^{-1}			
			−0.104	0.012	0.003	−0.052	0.009	7.977×10^{-1}			
^{159}Ta→^{155}Lu	2^-	2^-	0	0	0	0	0	3.217	2.44	0.76	0.80
			0	0	0	0.035	0	3.216			
			0.107	0.012	0.005	0	0	3.217			
			0.107	0.012	0.005	0.035	0	3.216			
^{160}Ta→^{156}Lu	2^-	2^-	0	0	0	0	0	3.681×10^1	$\geqslant$1.7	0.05	0.03
			0	0	0	−0.104	0.012	3.666×10^1			
			0.134	0.022	0.005	0	0	3.681×10^1			
			0.134	0.022	0.005	−0.104	0.012	3.666×10^1			
^{153}Tm→^{149}Ho	$\frac{11}{2}^-$	$\frac{11}{2}^-$	0	0	0	0	0	3.020	1.63	0.54	0.44
			0	0	0	−0.008	0	3.020			
			−0.018	0	0	0	0	3.020			
			−0.018	0	0	−0.008	0	3.020			
^{156}Hf→^{152}Yb	0^+	0^+	0	0	0	0	0	3.765	2.34×10^{-2}	0.62	2.35
			0	0	0	0	0	3.765×10^{-2}			
			0.035	−0.008	0.001	0	0	3.765×10^{-2}			
			0.035	−0.008	0.001	0	0	3.765×10^{-2}			
^{158}W→^{154}Hf	0^+	0^+	0	0	0	0	0	2.271×10^{-3}	1.37×10^{-3}	0.60	0.58
			0	0	0	0.008	0	2.227×10^{-3}			
			0.018	0	0.002	0	0	2.271×10^{-3}			
			0.018	0	0.002	0.008	0	2.271×10^{-3}			
^{163}Re→^{159}Ta	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	1.006	1.22	1.21	0.79
			0	0	0	0.107	0.012	1.002			
			0.125	0.006	0.001	0	0	1.006			
			0.125	0.006	0.001	0.107	0.012	1.002			
^{165}Re→^{161}Ta	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	3.737×10^1	$>5.22\times10^1$	1.40	0.68
			0	0	0	0.143	0.016	3.711×10^1			
			0.153	0	−0.002	0	0	3.737×10^1			
			0.153	0	−0.002	0.143	0.016	3.711×10^1			

Table 1 (continued)

	I_i^π	I_f^π	parent deformation			daughter deformation		$T_{1/2}^{cal}$/s	$T_{1/2}^{exp}$/s	HF	
			β_2	β_4	β_6	β_2	β_4			our model	Ref. [33]
$^{166}Ir\rightarrow^{162}Re$	2^-	2^-	0	0	0	0	0	1.525×10^{-2}	1.13×10^{-2}	0.74	0.54
			0	0	0	0.116	0.013	1.518×10^{-2}			
			0.107	−0.004	0.004	0	0	1.525×10^{-2}			
			0.107	−0.004	0.004	0.116	0.013	1.518×10^{-2}			
$^{167}Ir\rightarrow^{163}Re$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	8.911×10^{-2}	7.19×10^{-1}	0.81	0.52
			0	0	0	0.125	0.006	8.866×10^{-2}			
			0.116	−0.011	0.002	0	0	8.911×10^{-2}			
			0.116	−0.011	0.002	0.125	0.006	8.866×10^{-2}			
$^{169}Ir\rightarrow^{165}Re$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	2.029	8.40×10^{-1}	0.41	0.22
			0	0	0	0.153	0	2.013			
			0.134	−0.009	−0.001	0	0	2.029			
			0.134	−0.009	−0.001	0.153	0	2.013			
$^{172}Pt\rightarrow^{168}Os$	0^+	0^+	0	0	0	0	0	2.886×10^{-1}	1.03×10^{-1}	0.36	0.24
			0	0	0	0.162	−0.006	2.861×10^{-1}			
			0.126	−0.010	−0.002	0	0	2.886×10^{-1}			
			0.126	−0.010	−0.002	0.162	−0.006	2.861×10^{-1}			
$^{170}Au\rightarrow^{166}Ir$	2^-	2^-	0	0	0	0	0	2.514×10^{-3}	2.00×10^{-3}	0.80	0.53
			0	0	0	0.107	−0.004	2.504×10^{-3}			
			−0.096	−0.012	−0.002	0	0	2.514×10^{-3}			
			−0.096	−0.012	−0.002	0.107	−0.004	2.504×10^{-3}			
$^{173}Au\rightarrow^{169}Ir$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	3.270×10^{-2}	2.07×10^{-2}	0.63	0.38
			0	0	0	0.134	−0.009	3.250×10^{-2}			
			−0.105	−0.011	0.001	0	0	3.270×10^{-2}			
			−0.105	−0.011	0.001	0.134	−0.009	3.250×10^{-2}			
$^{177}Au\rightarrow^{173}Ir$	$\left(\frac{1}{2},\frac{3}{2}\right)^+$	$\left(\frac{3}{2},\frac{5}{2}\right)^+$	0	0	0	0	0	7.439	3.66	0.49	0.68
			0	0	0	0.162	−0.023	7.360			
			−0.130	−0.009	0	0	0	7.439			
			−0.130	−0.009	0	0.162	−0.023	7.360			
$^{176}Hg\rightarrow^{172}Pt$	0^+	0^+	0	0	0	0	0	4.911×10^{-2}	2.10×10^{-2}	0.43	0.31
			0	0	0	0.126	−0.010	4.864×10^{-2}			
			−0.105	−0.027	0	0	0	4.911×10^{-2}			
			−0.105	−0.027	0	0.126	−0.010	4.864×10^{-2}			
$^{177}Tl\rightarrow^{173}Au$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	3.345×10^{-2}	2.47×10^{-2}	0.74	0.45
			0	0	0	−0.105	−0.011	3.331×10^{-2}			
			−0.053	−0.007	0	0	0	3.345×10^{-2}			
			−0.053	−0.007	0	−0.105	−0.011	3.331×10^{-2}			
$^{179}Tl\rightarrow^{175}Au$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0	0	0	0	0	6.069×10^{-1}	2.30×10^{-1}	0.38	0.22
			0	0	0	−0.122	−0.010	6.034×10^{-1}			
			−0.053	−0.007	0	0	0	6.069×10^{-1}			
			−0.053	−0.007	0	−0.122	−0.010	6.034×10^{-1}			
$^{181}Tl\rightarrow^{177}Au$	$\frac{1}{2}^+$	$\left(\frac{1}{2},\frac{3}{2}\right)^+$	0	0	0	0	0	4.840×10^{1}	$>3.20\times10^{1}$	0.66	0.80
			0	0	0	−0.130	−0.009	4.805×10^{1}			
			−0.053	−0.007	0	0	0	4.840×10^{1}			
			−0.053	−0.007	0	−0.130	−0.009	4.805×10^{1}			
$^{180}Pb\rightarrow^{176}Hg$	0^+	0^+	0	0	0	0	0	5.044×10^{-3}	4.00×10^{-3}	0.79	0.51
			0	0	0	−0.105	−0.027	5.018×10^{-3}			
			0.008	−0.008	0.003	0	0	5.044×10^{-3}			
			0.008	−0.008	0.003	−0.105	−0.027	5.018×10^{-3}			

Table 1 (continued)

	I_i^π	I_f^π	parent deformation β_2	parent deformation β_4	parent deformation β_6	daughter deformation β_2	daughter deformation β_4	$T_{1/2}^{cal}$/s	$T_{1/2}^{exp}$/s	HF our model	HF Ref. [33]
^{188}Po→^{184}Pb	0^+	0^+	0	0	0	0	0	1.820×10^{-4}	2.70×10^{-4}	1.82	0.99
			0	0	0	0.009	−0.008	1.819×10^{-4}			
			0.293	0.007	−0.010	0	0	1.820×10^{-4}			
			0.293	0.007	−0.010	0.009	−0.008	1.820×10^{-4}			
^{192}Po→^{188}Pb	0^+	0^+	0	0	0	0	0	4.551×10^{-2}	3.23×10^{-2}	4.55	0.41
			0	0	0	0	−0.008	4.550×10^{-2}			
			−0.207	0.008	0.001	0	0	4.551×10^{-2}			
			−0.207	0.008	0.001	0	−0.008	4.550×10^{-2}			
^{193}Po→^{189}Pb	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	2.795×10^{-1}	4.20×10^{-1}	1.50	0.83
			0	0	0	0	−0.008	2.795×10^{-1}			
			−0.215	0.009	0.002	0	0	2.795×10^{-1}			
			−0.215	0.009	0.002	0	−0.008	2.795×10^{-1}			
^{197}Po→^{193}Pb	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	1.356×10^{2}	1.27×10^{2}	0.94	0.54
			0	0	0	0	−0.015	1.355×10^{2}			
			0.062	0.001	−0.002	0	0	1.356×10^{2}			
			0.062	0.001	−0.002	0	−0.015	1.355×10^{2}			
^{199}Po→^{195}Pb	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	3.910×10^{3}	2.74×10^{3}	0.70	0.42
			0	0	0	0.009	−0.015	3.909×10^{3}			
			0	−0.015	0	0	0	3.910×10^{3}			
			0	−0.015	0	0.009	−0.015	3.909×10^{3}			
^{201}Po→^{197}Pb	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	7.907×10^{4}	5.74×10^{4}	0.72	0.35
			0	0	0	0	−0.008	7.906×10^{4}			
			0	−0.015	0	0	0	7.907×10^{4}			
			0	−0.015	0	0	−0.008	7.906×10^{4}			
^{198}At→^{194}Bi	3^+	3^+	0	0	0	0	0	3.780	4.67	1.24	0.64
			0	0	0	−0.052	0.009	3.776			
			−0.207	−0.007	−0.001	0	0	3.780			
			−0.207	−0.007	−0.001	−0.052	0.009	3.776			
^{200}At→^{196}Bi	3^+	3^+	0	0	0	0	0	5.675×10^{1}	8.27×10^{1}	1.46	0.77
			0	0	0	0.052	0.009	5.653×10^{1}			
			0.089	−0.006	−0.001	0	0	5.675×10^{1}			
								5.626×10^{1}			
^{202}At→^{198}Bi	$(2,3)^+$	$(2,3)^+$	0	0	0	0	0	5.870×10^{2}	4.97×10^{2}	0.85	0.43
			0	0	0	0.009	−0.052	5.847×10^{2}			
			0.062	−0.007	0.001	0	0	5.870×10^{2}			
			0.062	−0.007	0.001	0.009	−0.052	5.847×10^{2}			
^{195}Rn→^{191}Po	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	1.539×10^{-2}	6.00×10^{-3}	0.39	0.21
			0	0	0	0.275	−0.031	1.498×10^{-2}			
			−0.240	0.013	0.003	0	0	1.539×10^{-2}			
			−0.240	0.013	0.003	0.275	−0.031	1.498×10^{-2}			
^{197}Rn→^{193}Po	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	1.330×10^{-1}	6.50×10^{-2}	0.49	0.26
			0	0	0	−0.215	0.009	1.307×10^{-1}			
			−0.232	0.012	0.002	0	0	1.330×10^{-1}			
			−0.232	0.012	0.002	−0.215	0.009	1.307×10^{-1}			
^{199}Rn→^{195}Po	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	1.398	6.60×10^{-1}	0.47	1.25
			0	0	0	0.071	0.002	1.395			
			−0.207	0.001	−0.001	0	0	1.398			
			−0.207	0.001	−0.001	0.071	0.002	1.395			

Table 1 (continued)

	I_i^π	I_f^π	parent deformation			daughter deformation		$T_{1/2}^{cal}$/s	$T_{1/2}^{exp}$/s	HF	
			β_2	β_4	β_6	β_2	β_4			our model	Ref. [33]
$^{201}Rn\rightarrow^{197}Po$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	1.312×10^{1}	1.37×10^{1}	1.04	0.52
			0	0	0	0.062	0.001	1.310×10^{1}			
			−0.199	−0.016	−0.001	0	0	1.312×10^{1}			
			−0.199	−0.016	−0.001	0.062	0.001	1.310×10^{1}			
$^{203}Rn\rightarrow^{199}Po$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	1.114×10^{2}	6.67×10^{1}	0.60	0.30
			0	0	0	0	−0.015	1.114×10^{2}			
			−0.104	0.004	0.004	0	0	1.114×10^{2}			
			−0.104	0.004	0.004	0	−0.015	1.114×10^{2}			
$^{201}Fr\rightarrow^{197}At$	$\frac{9}{2}^-$	$\frac{9}{2}^-$	0	0	0	0	0	1.374×10^{-1}	6.20×10^{-2}	0.45	0.25
			0	0	0	−0.207	0.001	1.352×10^{-1}			
			−0.215	−0.006	0.001	0		1.374×10^{-1}			
			−0.215	−0.006	0.001	−0.207	0.001	1.352×10^{-1}			
$^{202}Fr\rightarrow^{198}At$	3^+	3^+	0	0	0	0	0	3.826×10^{-1}	3.00×10^{-1}	0.78	0.42
			0	0	0	−0.207	−0.007	3.762×10^{-1}			
			−0.207	−0.015	0.001	0	0	3.836×10^{-1}			
			−0.207	−0.015	0.001	−0.207	−0.007	3.762×10^{-1}			
$^{203}Fr\rightarrow^{199}At$	$\frac{9}{2}^-$	$\frac{9}{2}^-$	0	0	0	0	0	1.089	5.79×10^{-1}	0.53	1.78
			0	0	0	0.080	0.002	1.087			
			−0.190	−0.018	0.003	0	0	1.089			
			−0.190	−0.018	0.003	0.080	0.002	1.087			
$^{204}Fr\rightarrow^{200}At$	3^+	3^+	0	0	0	0	0	2.172	1.77	0.82	2.24
			0	0	0	0.089	−0.006	2.165			
			−0.190	−0.024	0	0	0	2.172			
			−0.190	−0.024	0	0.089	−0.006	2.165			
$^{206}Fr\rightarrow^{202}At$	$(2,3)^+$	$(2,3)^+$	0	0	0	0	0	1.848×10^{1}	1.90×10^{1}	1.03	1.52
			0	0	0	0.062	−0.007	1.845×10^{1}			
			−0.013	0.005	0.006	0		1.848×10^{1}			
			−0.013	0.005	0.006	0.062	−0.007	1.845×10^{1}			
$^{203}Ra\rightarrow^{199}Rn$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	6.038×10^{-2}	3.10×10^{-2}	0.51	0.30
			0	0	0	−0.207	0.001	5.940×10^{-2}			
			−0.207	−0.015	0.001	0	0	6.038×10^{-2}			
			−0.207	−0.015	0.001	−0.207	0.001	5.218×10^{-2}			
$^{205}Ra\rightarrow^{201}Rn$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0	0	0	0	0	4.144×10^{-1}	$\geqslant2.10\times10^{-1}$	0.51	0.27
			0	0	0	−0.199	−0.016	4.077×10^{-1}			
			−0.190	−0.024	0.003	0	0	4.144×10^{-1}			
			−0.190	−0.024	0.003	−0.199	−0.016	4.077×10^{-1}			
$^{207}Ra\rightarrow^{203}Rn$	$\left(\frac{5}{2},\frac{3}{2}\right)^-$	$\left(\frac{5}{2},\frac{3}{2}\right)^-$	0	0	0	0	0	2.244	1.44	0.64	0.51
			0	0	0	−0.104	0.004	2.235			
			−0.130	−0.002	0.007	0	0	2.244			
			−0.130	−0.002	0.007	−0.104	0.004	2.235			
$^{206}Ac\rightarrow^{202}Fr$	3^+	3^+	0	0	0	0	0	2.822×10^{-2}	2.20×10^{-2}	0.78	0.42
			0	0	0	−0.207	−0.015	2.774×10^{-2}			
			−0.207	−0.030	0.003	0	0	2.822×10^{-2}			
			−0.207	−0.030	0.003	−0.207	−0.015	2.774×10^{-2}			

Table 1 (continued)

	I_i^π	I_f^π	parent deformation			daughter deformation		$T_{1/2}^{\text{cal}}$/s	$T_{1/2}^{\text{exp}}$/s	HF	
			β_2	β_4	β_6	β_2	β_4			our model	Ref. [33]
^{208}Ac→^{204}Fr	3^+	3^+	0	0	0	0	0	1.415×10^{-1}	9.60×10^{-2}	0.68	0.37
			0	0	0	−0.190	−0.024	1.393×10^{-1}			
			−0.190	−0.024	0.006	0	0	1.415×10^{-1}			
			−0.190	−0.024	0.006	−0.190	−0.024	1.393×10^{-1}			
^{217}Pa→^{213}Ac	$\frac{9}{2}^-$	$\frac{9}{2}^-$	0	0	0	0	0	2.095×10^{-3}	3.48×10^{-3}	1.66	0.89
			0	0	0	−0.044	−0.015	2.093×10^{-3}			
			0	0.008	0	0	0	2.095×10^{-3}			
			0	0.008	0	−0.044	−0.015	2.093×10^{-3}			
^{174}Ir→^{170}Re	3^+	5^+	0	0	0	0	0	8.194×10^{2}	1.58×10^{3}	1.93	1.55
			0	0	0	0.199	−0.019	8.077×10^{2}			
			0.171	−0.022	−0.001	0	0	8.194×10^{2}			
			0.171	−0.022	−0.001	0.199	−0.019	8.077×10^{2}			
^{183}Pb→^{179}Hg	$\frac{3}{2}^-$	$\frac{7}{2}^-$	0	0	0	0	0	5.068×10^{-1}	8.11×10^{-1}	1.60	1.29
			0	0	0	−0.122	−0.018	5.035×10^{-1}			
			0.009	0.015	0.004	0	0	5.068×10^{-1}			
			0.009	0.015	0.004	−0.122	−0.018	5.035×10^{-1}			
^{185}Pb→^{181}Hg	$\frac{3}{2}^-$	$\frac{1}{2}^-$	0	0	0	0	0	3.606	3.15×10^{3}	873.46	692.31
			0	0	0	−0.122	−0.018	3.582			
			0.009	0.015	0.002	0	0	3.606			
			0.009	0.015	0.002	−0.122	−0.018	3.582			
^{187}Pb→^{183}Hg	$\frac{3}{2}^-$	$\frac{1}{2}^-$	0	0	0	0	0	5.559×10^{1}	2.17×10^{2}	3.90	3.03
			0	0	0	−0.130	−0.017	5.516×10^{1}			
			0	−0.015	0.001	0	0	5.559×10^{1}			
			0	−0.015	0.001	−0.130	−0.017	5.516×10^{1}			
^{186}Bi→^{182}Tl	3^+	2^-	0	0	0	0	0	0.993×10^{-3}	1.48×10^{-2}	14.91	10.80
			0	0	0	−0.053	−0.007	0.992×10^{-3}			
			−0.052	0.016	−0.001	0		0.993×10^{-3}			
			−0.052	0.016	−0.001	−0.053	−0.007	0.993×10^{-3}			
^{191}Bi→^{187}Tl	$\frac{9}{2}^-$	$\frac{1}{2}^+$	0	0	0	0	0	8.708×10^{1}	2.05×10^{1}	0.24	0.76
			0	0	0	−0.053	−0.007	8.700×10^{1}			
			−0.052	0.009	0	0	0	8.708×10^{1}			
			−0.052	0.009	0	−0.053	−0.007	8.700×10^{1}			
^{193}Bi→^{189}Tl	$\frac{9}{2}^-$	$\frac{1}{2}^+$	0	0	0	0	0	7.093×10^{3}	1.91×10^{3}	0.27	0.83
			0	0	0	−0.053	−0.007	7.086×10^{3}			
			−0.052	0.009	0	0	0	7.093×10^{3}			
			−0.052	0.009	0	−0.053	−0.007	7.086×10^{3}			
^{194}Bi→^{190}Tl	3^+	2^-	0	0	0	0	0	1.121×10^{4}	2.05×10^{4}	1.83	1.07
			0	0	0	−0.061	−0.007	1.119×10^{4}			
			−0.052	0.009	0	0	0	1.121×10^{4}			
			−0.052	0.009	0	−0.061	−0.007	1.119×10^{4}			
^{195}Bi→^{191}Tl	$\frac{9}{2}^-$	$\frac{1}{2}^+$	0	0	0	0	0	9.577×10^{5}	5.72×10^{5}	0.60	1.70
			0	0	0	−0.053	−0.007	9.569×10^{5}			
			−0.052	0.009	0	0	0	9.577×10^{5}			
			−0.052	0.009	0	−0.053	−0.007	9.569×10^{5}			
^{216}Ac→^{212}Fr	1^-	5^+	0	0	0	0	0	1.065×10^{-4}	4.40×10^{-4}	4.13	13.25
			0	0	0	−0.008	0.008	1.065×10^{-4}			
			−0.018	0.008	−0.003	0	0	1.065×10^{-4}			
			−0.018	0.008	−0.003	−0.008	0.008	1.065×10^{-4}			

Table 2. Alpha decay half-lives of some even-even nuclei for the ground state to ground state transition.

	the parent deformation			the daughter deformation		$T_{1/2}^{\text{cal}}/s$	$T_{1/2}^{\text{exp}}/s$ Ref. [20]	HF
	β_2	β_4	β_6	β_2	β_4			
^{214}Rn	0	0	0	0	0	2.704×10^{-7}	2.70×10^{-7}	0.999
	0	0	0	0	0.008	2.704×10^{-7}		
	0.008	0.008	0	0	0	2.704×10^{-7}		
	0.008	0.008	0	0	0.008	2.704×10^{-7}		
^{212}Po	0	0	0	0	0	2.324×10^{-7}	2.99×10^{-7}	1.29
	0	0	0	0	0	2.324×10^{-7}		
	0	0.008	0	0	0	2.324×10^{-7}		
	0	0.008	0	0	0	2.324×10^{-7}		
^{214}Po	0	0	0	0	0	3.305×10^{-4}	1.64×10^{-4}	0.50
	0	0	0	0	0	3.305×10^{-4}		
	−0.008	0.008	0	0	0	3.305×10^{-4}		
	−0.008	0.008	0	0	0	3.305×10^{-4}		
^{216}Po	0	0	0	0	0	5.448×10^{-1}	1.45×10^{-1}	0.30
	0	0	0	0	0.008	5.448×10^{-1}		
	0.020	0.018	0.004	0	0	5.448×10^{-1}		
	0.020	0.018	0.004	0	0.008	5.448×10^{-1}		
^{216}Ra	0	0	0	0	0	2.202×10^{-7}	1.82×10^{-7}	0.82
	0	0	0	0	0.008	2.202×10^{-7}		
	0.008	0.008	0.001	0	0	2.202×10^{-7}		
	0.008	0.008	0.001	0	0.008	2.202×10^{-7}		
^{218}Ra	0	0	0	0	0	8.397×10^{-5}	2.56×10^{-5}	0.30
	0	0	0	0.008	0.008	8.397×10^{-5}		
	0.020	0.010	0.003	0	0	8.397×10^{-5}		
	0.020	0.010	0.003	0.008	0.008	8.397×10^{-5}		
^{220}Ra	0	0	0	0	0	8.975×10^{-2}	1.81×10^{-2}	0.20
	0	0	0	0.008	0.008	8.974×10^{-2}		
	0.103	0.072	0.007	0	0	8.975×10^{-2}		
	0.103	0.072	0.007	0.008	0.008	8.974×10^{-2}		
^{218}Th	0	0	0	0	0	1.789×10^{-7}	1.17×10^{-7}	0.65
	0	0	0	0.008	0.008	1.789×10^{-7}		
	0.008	0.008	0.001	0	0	1.789×10^{-7}		
	0.008	0.008	0.001	0.008	0.008	1.789×10^{-7}		
^{222}Th	0	0	0	0	0	9.411×10^{-3}	2.29×10^{-3}	0.24
	0	0	0	0.020	0.010	9.408×10^{-3}		
	0.111	0.081	0.006	0	0	9.411×10^{-3}		
	0.111	0.081	0.006	0.020	0.010	9.408×10^{-3}		
^{222}U	0	0	0	0	0	5.990×10^{-6}	1.25×10^{-6}	0.21
	0	0	0	0.008	0.008	5.989×10^{-6}		
	0.048	0.20	0.006	0	0	5.990×10^{-6}		
	0.048	0.20	0.006	0.008	0.008	5.989×10^{-6}		
^{250}Fm	0	0	0	0	0	8.403×10^{3}	2.65×10^{3}	0.32
	0	0	0	0.235	0.057	8.284×10^{3}		
	0.235	0.033	−0.040	0	0	8.403×10^{3}		
	0.235	0.033	−0.040	0.235	0.057	8.284×10^{3}		
^{252}Fm	0	0	0	0	0	3.722×10^{5}	1.09×10^{5}	0.29
	0	0	0	0.235	0.040	3.674×10^{5}		
	0.245	0.018	−0.045	0	0	3.722×10^{5}		
	0.245	0.018	−0.045	0.235	0.040	3.674×10^{5}		
^{254}Fm	0	0	0	0	0	7.520×10^{4}	1.37×10^{4}	0.18
	0	0	0	0.245	0.026	7.418×10^{4}		
	0.237	0.008	−0.039	0	0	7.520×10^{4}		
	−0.039	0.237	0.008	0.245	0.026	7.418×10^{4}		
^{256}Rf	0	0	0	0	0	3.862×10^{0}	2.38×10^{0}	0.61
	0	0	0	0.236	0.024	3.809×10^{0}		
	0.247	−0.007	−0.043	0	0	3.862×10^{0}		
	0.247	−0.007	−0.043	0.236	0.024	3.809×10^{0}		

3 Numerical results

The alpha emission obeys the spin-parity selection rule [33]:

$$|I_{\mathrm{i}}-I_{\mathrm{f}}|\leqslant l\leqslant I_{\mathrm{i}}+I_{\mathrm{f}} \text{ and } \frac{\pi_{\mathrm{i}}}{\pi_{\mathrm{f}}}=(-1)^l, \tag{20}$$

where I_{i}, π_{i} and I_{f}, π_{f} are the spin and parity of the parent and daughter, respectively. We have calculated the half-lives and the Hf for the transitions to the different states. The Hf parameter reported in Table 1 and plotted in Fig. 5, is defined as

$$\mathrm{Hf}=\frac{T_{1/2}^{\mathrm{exp}}}{T_{1/2}^{\mathrm{cal}}}. \tag{21}$$

In Table 1, the first, fourth and fifth columns respectively represent different modes of transition and the deformation parameters of parent and daughter nuclei. In the seventh column we report our calculated half-lives. In Table 2 we have considered some deformed even-even nuclei for the ground state to ground state transition. Fig. 2 depicts $\lg T_{1/2}$ vs. neutron number of parent nucleus which shows increasing $\lg T_{1/2}$ for decreasing N_{p}. In Fig. 3 we have plotted $\lg T_{1/2}$vs. decay energy. We see that $\lg T_{1/2}$ decreases when the decay energy increases. The decreasing behavior of decay energy vs. neutron number of parent nucleus is shown in Fig. 4. We have compared Hfs of the present work with those of Ref. [33] in Fig. 5.

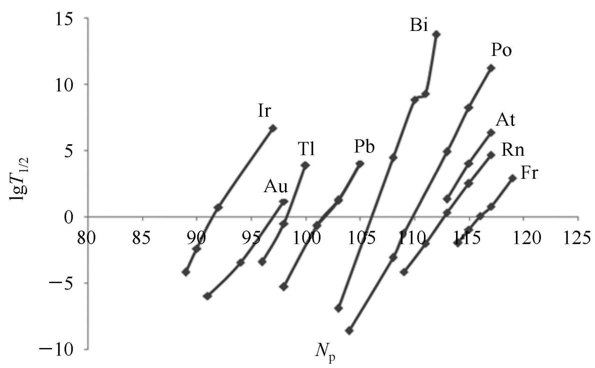

Fig. 2. $\lg T_{1/2}$ vs. neutron number of parent nucleus.

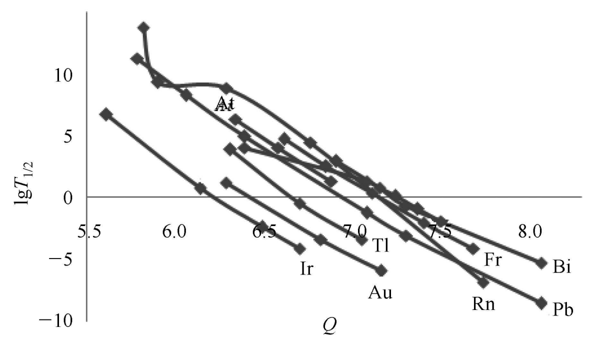

Fig. 3. $\lg T_{1/2}$ vs. decay energy.

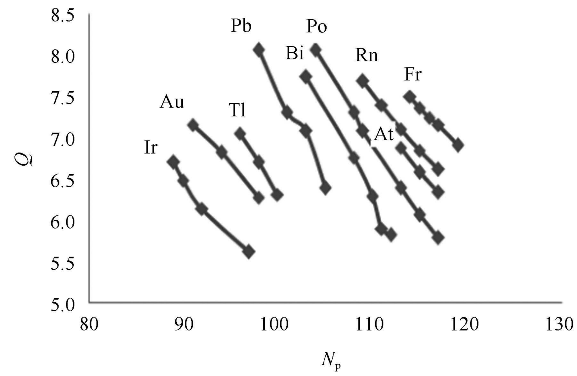

Fig. 4. Decay energy vs. neutron number of parent nucleus.

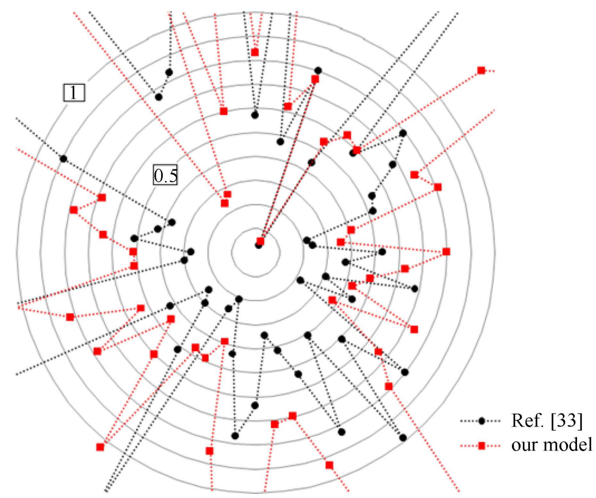

Fig. 5. Hfs in our work and Ref. [33].

4 Conclusion

We have calculated the alpha-decay half-lives under the Yukawa proximity potential model within the framework of multipole approximation for the daughter and parent nuclei. Our study reveals that taking into account the angular term, the theoretical results are improved. We considered the multiple approximation for all possible combinations, i.e. solely for the daughter or the parent and both of them. The results are in closer agreement with the experimental data in the first case.

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