Anomalous spin of the Chern-Simons-Georgi-Glashow model*

HUO Qiu-Hong(霍秋红)[1;1)] JIANG Yun-Guo(姜云国)[2,3;2)] WANG Ru-Zhi(王如志)[1] YAN Hui(严辉)[1]

[1] Laboratory of Thin Film Materials, College of Materials Science and Engineering, Beijing University of Technology, Beijing 100024, China
[2] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[3] Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China

Abstract: With the Coulomb gauge, the Chern-Simons-Georgi-Glashow (CSGG) model is quantized in the Dirac formalism for the constrained system. Combining the Gauss law and Coulomb gauge consistency condition, the difference between the Schwinger angular momentum and canonical angular momentum of the system is found to be an anomalous spin. The reason for this result lies in the fact that the Schwinger energy momentum tensor and the canonical one have different symmetry properties in the presence of the Chern-Simons term.

Key words: Dirac quantization, Chern-Simons, anomalous spin

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1 Introduction

The Chern-Simons (C-S) field has applications in many branches of physics. It is a topological term which does not contribute to the dynamics of the gauge field, but it gives a topological mass to the gauge field [1, 2]. It only exists in the odd dimension of the space time, and can lead to fractional spin and fractional charge [3, 4]. In the Maxwell-Chern-Simons-Higgs (MCSH) theories, both Abrikosov-Nielsen-Olesen (ANO) vortices and C-S vortices can be constructed [5, 6]. C-S vortices have ring-shaped magnetic flux, different from that of the ANO vortices with a Gauss-like shape [7]. Many C-S models are used to explain the quantum Hall effects and high temperature superconductors [8, 9], since C-S vortices carry fractional spin and behave as anyons-like objects.

The fractional spin in many C-S theories has structural similarity, this is a remarkable feature. When the theory involves the Maxwell term, or the non-Abelian Yang-Mill piece, the calculation of the spin becomes subtle. A novel method to calculate the fractional spin was invented to overcome these difficulties [10–12], by calculating the difference between the Schwinger angular momentum and the canonical angular momentum. The difference is a compact form and can be interpreted as the spin of the vortex. When the asymptotic form of the gauge form in a vortex configuration is used, the difference becomes the common fractional spin. A question arises naturally: Why does this method succeed? In this letter, this method will be used to calculate the anomalous spin of the CSGG model. Since the scalar field in the Chern-Simons-Georgi-Glashow (CSGG) model is in the adjoint representation, no vortex configurations can be constructed. Combining the Gauss law and Coulomb gauge consistency condition, the solution of the gauge field is given. With this solution, the anomalous spin term is still obtained, and this method is valid in the presence of the C-S term. The reason lies in the fact that the Schwinger energy momentum tensor has different symmetry properties from the canonical one. In Section 3，we are going to explain this with more detail.

The advantage of the Coulomb gauge is that there is no time derivative term in the gauge fixed action. The infrared divergence in the Maxwell-Chern-Simons (MCS) theory occurs when the Coulomb gauge is applied [1]. There are also some ambiguities in the Yang-Mills Feynman integrals due to the absence of the time derivatives in the action [13]. The consistency of the non-Abelian C-S theory in the Coulomb gauge at any perturbation order was investigated by Ferrai and Lazzizzera [14]. With the pure non-Abelian C-S term, the Hamilton is zero because the C-S term contributes nothing to the dynamics. However, taking account of the Gauss law and the Coulomb gauge, the communication relation between the

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1) E-mail: hqh@emails.bjut.edu.cn

2) E-mail: jiangyg@ihep.ac.cn

gauge fields vanish identically at any perturbative order [14]. In Section 2, we exploit the quantization of the non-Abelian C-S theory with matter field, i.e. the CSGG model, by the Dirac quantization formalism. We will show that even with the matter field, the combination of Gauss law and the Coulomb gauge also leads to the disappearance of the communication relation between the gauge fields. This is helpful for studying the perturbation at any order, for example the quantum scattering amplitudes.

2 Canonical quantization

The 2+1 dimensional Lagrangian of the CSGG model in the component form is written as [15]

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a}+\frac{\kappa}{4}\epsilon^{\mu\nu\rho}\left[F^a_{\mu\nu}A^a_\rho-\frac{g}{3}f^{abc}A^a_\mu A^b_\nu A^c_\rho\right] +\frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a)-\frac{m^2}{2}\phi^a\phi^a-\frac{\lambda}{4}(\phi^a\phi^a)^2, \quad (1)$$

where the a, b, c denote the group indices, the Greek indices $\mu,\nu,\rho,\ldots=0,1,2$ denote the space-time. Under the gauge transformation, the variation of the Lagrangian has a variation term proportional to κ. Thus, $\kappa=g^2n/(4\pi)$ ($n\in\mathbb{Z}$) must be a quantized constant in order to leave the quantum amplitude gauge invariant. The ϕ^a carries the index of the gauge group, so it belongs to the adjoint representation. This gauge group is not fixed in the present discussion, it can be $SU(N)$, $SO(N)$, $USp(2N)$, etc. If we take it to be $SO(3)$, the well-known 't Hooft-Polyakov monopole solution exists in this model. f^{abc} is an antisymmetric tensor. g and λ are the coupling constants for A_μ and ϕ^a, respectively. The gauge field strength and the covariant derivative D_μ are written as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu, \quad (2)$$

$$D_\mu\phi^a = \partial_\mu\phi^a+gf^{abc}A^b_\mu\phi^c. \quad (3)$$

The Euler-Lagrangian equation can be calculated

$$D_\nu F^{\nu\mu,a}+\frac{\kappa}{2}\epsilon^{\mu\nu\rho}F^a_{\nu\rho}+gf^{abc}D^\mu\phi^b\phi^c = 0, \quad (4)$$

$$\left[D_\nu D^\nu-m^2+\lambda(\phi^b\phi^b)\right]\phi^a = 0. \quad (5)$$

The canonical momentum of the field is defined to be $\pi_X\equiv\partial\mathcal{L}/\partial\dot{X}$, where X denotes any field. With this definition, the canonical momenta of the gauge and the matter fields are

$$\pi^{\mu,a}=-F^{0\mu,a}+\frac{\kappa}{2}\epsilon^{0\mu\rho}A^a_\rho, \quad (6)$$

$$\pi^a_\phi=D^0\phi^a. \quad (7)$$

For $\mu=0$, $\pi^{0,a}=0$ gives a constraint. For $\mu=i$, one has $\pi^{i,a}=-F^{0i,a}+\frac{\kappa}{2}\epsilon^{0ij}A^a_j$, which contains a time derivative term. Therefore, the gauge field A^a_μ has dynamics in the CSGG model. The canonical Hamiltonian is written as

$$\begin{aligned}\mathcal{H}_\mathrm{C}=&-\frac{1}{2}\pi^a_i\pi^{i,a}-A^a_0\partial^i\pi^a_i-gf^{abc}\pi^a_iA^b_0A^{i,c}+\frac{\kappa}{2}\epsilon^{ij}\pi^a_iA^a_j\\&+\frac{1}{2}\pi^a_\phi\pi^a_\phi-gf^{abc}\pi^a_\phi A^b_0\phi^c-\frac{\kappa^2}{8}A^{i,a}A^a_i+\frac{1}{4}F^a_{ij}F^{ij,a}\\&+\frac{\kappa g}{12}\epsilon^{\mu\nu\rho}f^{abc}A^a_\mu A^b_\nu A^c_\rho-\frac{\kappa}{4}\epsilon^{ij}A^a_0F^a_{ij}\\&-\frac{1}{2}D_i\phi^aD^i\phi^a+\frac{m^2}{2}\phi^a\phi^a+\frac{\lambda}{4}(\phi^a\phi^a)^2. \quad (8)\end{aligned}$$

The definition of π^a_μ gives that

$$\Lambda^a_1=\pi^a_0\approx0, \quad (9)$$

which is called the primary constraint by Dirac [16]. The total Hamiltonian is given by

$$\mathcal{H}_\mathrm{T}=\mathcal{H}_\mathrm{C}+\eta^a\Lambda^a_1, \quad (10)$$

where η^a is the Lagrangian multiplier. The consistency condition requires that

$$\begin{aligned}\dot{\Lambda}^a_1=&\left\{\pi^{0,a},\int\mathrm{d}^2x\mathcal{H}_\mathrm{T}\right\}\\=&\partial^i\pi^a_i+\frac{\kappa}{2}\epsilon^{ij}\partial_iA^a_j+gf^{abc}A^{i,b}\pi^c_i+gf^{abc}\pi^b_\phi\phi^c\\\equiv&\Lambda^a_2\approx0. \quad (11)\end{aligned}$$

This new constraint Λ^a_2 is the secondary constraint, which is the Gauss law. No further constraint is produced when $\dot{\Lambda}^a_2$ is considered. One can verify that $\{\Lambda^a_1,\Lambda^a_2\}\approx0$, this means both Λ^a_1 and Λ^a_2 are first class constraints. Dirac conjectured that all the first class constraints can be added to the Hamiltonian and the dynamics of the system do not change [16]. Costa et al. proved that Dirac conjecture is valid when all the constraints are first class [17]. π^a_0 is not the physical degree of freedom, since a photon has only two physical degrees. Thus, Λ^a_1 can be eliminated without changing the dynamics, i.e., the Hamiltonian equations. The constraint Λ^a_2 is the generator of the gauge transformation [18].

In order to quantize the system, two gauge fixing conditions should be introduced in order to fix the gauge. First we consider the Coulomb gauge

$$\Omega^b_1\equiv\partial_iA^{i,b}\approx0. \quad (12)$$

This gauge has advantages when we consider the static configuration of the soliton system, since it has no time derivative. In Dirac's procedure, the extended Hamiltonian can be obtained by adding the secondary first class constraint, which is written as

$$\mathcal{H}_\mathrm{E}=\mathcal{H}_\mathrm{C}+\eta^a_1\Lambda^a_1+\eta^a_2\Lambda^a_2\cong H_\mathrm{T}. \quad (13)$$

The sign "$\cong$" means that the Lagrangian multiplier η^a_2 can be absorbed to A^a_0 by redefining $A^{a\prime}_0=A^a_0+\eta^a_2$. This is evident support for the Dirac conjecture. Another condition Ω_2^b should be consistent with Ω_1^b, Requiring $\{\Omega_1^b, H_{\rm T}\}\approx 0$, one obtains

$$\Omega_2^b=\{\Omega_1^b,H_{\rm T}\}=-\partial_i\pi^{i,a}+\frac{\kappa}{2}\epsilon^{ij}\partial_i A_j^a+\partial^i D_i A_0^a. \tag{14}$$

The non-trivial communication relations are listed as

$$\{\Lambda_1^a(x),\Omega_2^b(y)\}=-\partial_y^i D_i^{ab}(y)\delta(x-y), \tag{15}$$

$$\{\Lambda_2^a(x),\Omega_1^b(y)\}=-\partial_y^i D_i^{ab}(x)\delta(x-y), \tag{16}$$

where $D_i^{ab}\equiv\partial_i\delta^{ab}+gf^{abc}A_i^c$. Now Λ_i^a, Ω_j^b can be considered to be the secondary class constraints, which form a non-singular matrix $C^{ab}(x,y)$

$$C^{ab}(x,y)=\begin{pmatrix} 0 & -\partial_y^i D_i^{ab}(y) \\ -\partial_y^i D_i^{ab}(x) & 0 \end{pmatrix}\delta(x-y). \tag{17}$$

The inverse matrix of the constraint $C^{ab}(x,y)$ can be solved by introducing a Green function $G^{cb}(x,y)$ [14, 19],

$$D_i^{ac}(x)\partial_x^i G^{cb}(x,y)=\delta^{ab}\delta(x-y). \tag{18}$$

Supposing $G^{ab}(x,y)$ has a good behavior at infinity, one obtains

$$(C^{-1})^{ab}(x,y)=\begin{pmatrix} 0 & G^{ab}(x,y) \\ -G^{ab}(x,y) & 0 \end{pmatrix}. \tag{19}$$

The quantized communication relation (QCR) should be realized by the Dirac bracket, which is defined as

$$\begin{aligned}\{F^a(x),D^b(y)\}^*=&\{F^a(x),D^b(y)\}-\int{\rm d}z{\rm d}w\\ &\times\{F^a(x),\Lambda_i^c(z)\}\cdot(C^{-1})_{ij}^{cd}(z,w)\\ &\cdot\{\Omega_j^d(w),D^b(y)\},\end{aligned} \tag{20}$$

where $C_{ij}^{ab}(x,y)\equiv\{\Lambda_i^a(x),\Omega_j^b(y)\}$. To quantize the theory, we need to replace the Dirac brackets with the commutators. The QCR of the gauge field A_i^a and π_j^b is calculated to be

$$\left[A_i^a(x),\pi_j^b(y)\right]=\delta_{ij}\delta^{ab}\delta(x-y). \tag{21}$$

One can also verify that $[A_i^a(x),A_j^b(y)]=0$. This gives the correct quantization result for the CSGG model. In a pure non-Abelian C-S theory, the C-S field theories are shown to be finite and free in the Coulomb gauge [14]. It was questioned whether such commutation relationship (CR) vanishes at all perturbative orders in the presence of the matter field. Our analysis above shows that the CR of the gauge fields vanishes identically by taking into account the Gauss law and the Coulomb gauge fixing in the CSGG model.

3 Anomalous spin

Banerjee and Mukherjee proposed a novel method to calculate the fractional spin term in the C-S system [11]. This spin is interpreted as the difference between the angular momentum obtained by modifying Schwinger's energy-momentum tensor with Gauss constraint, and the canonical (Noether) angular momentum. The fractional spin term can also be calculated by the canonical angular momentum in the quantized system [20, 21]. Here, we make use of this method to calculate the spin of CSGG model, and obtain an anomalous spin term. The success of this method is that the symmetric properties of the modified Schwinger's energy momentum tensor are different from that of the canonical energy momentum.

The Schwinger's energy-momentum tensor is given by [19]

$$\begin{aligned}\Theta_{\mu\nu}=&\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}=F_{\mu\rho}^a F_\nu^{\rho,a}\\ &+\frac{1}{2}(D_\mu\phi^a D_\nu\phi^a+D_\nu\phi^a D_\mu\phi^a)-g_{\mu\nu}\mathcal{L}.\end{aligned} \tag{22}$$

It is evident that this tensor is symmetric, i.e., $T^{\mu\nu}=T^{\nu\mu}$. The is because the metric of the system is symmetric, $g^{\mu\nu}=g^{\nu\mu}$. However, the C-S term does not contain the metric. Thus, the C-S term has no contribution to the Schwinger energy momentum tensor. In the presence of constraints, a more general expression for $\Theta_{\mu\nu}$ can be given by

$$\Theta_{\mu\nu}^{\rm T}=\Theta_{\mu\nu}+\Gamma_{\mu\nu}^a G^a, \tag{23}$$

where $\Gamma_{\mu\nu}^a$ is the Lagrangian multiplier. Since G^a is the generator of gauge transformations, $\Theta_{\mu\nu}^{\rm T}$ is the gauge invariant on the constraint surface [11], i.e.,

$$\{\Theta_{\mu\nu}^{\rm T},G^a\}\approx 0. \tag{24}$$

In order to keep the correct spatial translation, a suitable choice for Γ_{0i}^a is $\Gamma_{0i}^a=-A_i^a$ [11]. Thus, the Schwinger energy momentum conserves both the gauge and Lorentz symmetry.

The angular momentum operator is defined as

$$L=\int{\rm d}^2x\epsilon^{0ij}x_i\Theta_{0j}^{\rm T}. \tag{25}$$

Considering Eq. (22) and Eq. (23), $\Theta_{0j}^{\rm T}$ is calculated to be

$$\Theta_{0j}^{\rm T}\approx\pi_k^a F_j^{k,a}-\frac{\kappa}{2}\epsilon_{0kl}A^{l,a}F_j^{k,a}+D_j\phi^a\pi_\phi^a. \tag{26}$$

Here, we use the Dirac weak equality condition. Substituting Eq. (26) into Eq. (25), one obtains

$$\begin{aligned}L_{\rm S}=&\int{\rm d}^2x\epsilon^{ij}x_i\Theta_{0j}^{\rm T}\\ =&\int{\rm d}^2x\epsilon^{ij}\left(x_i\pi_k F_j^{k,a}+x_i D_j\phi^a\pi_\phi^a\right)\\ &+\frac{\kappa}{2}\int{\rm d}^2x\,x_i A^{j,a}F_j^{i,a}.\end{aligned} \tag{27}$$

Notice that only the canonical variables are used.

The canonical energy-momentum tensor is defined as [11]

$$T_{\mu\nu}=\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^a)}\partial_\nu\phi^a+\frac{\partial\mathcal{L}}{\partial(\partial^\mu A^a_\rho)}\partial_\nu A^a_\rho-g_{\mu\nu}\mathcal{L}, \tag{28}$$

and T_{0i} is calculated to be

$$T_{0i}\approx\pi^a_\phi\partial_i\phi^a+\pi^a_k\partial_i A^a_k, \tag{29}$$

where the term proportional to primary constraint Λ^a_0 is ignored. Notice that, $T_{0i}\neq T_{i0}$, since T_{i0} has a time derivative term $\pi^a_i\partial_0 A^a_i$, which should be converted into the canonical variables. The asymmetry of the canonical tensor originates from the C-S term. The canonical angular momentum tensor is defined as

$$M_{\mu\nu}=\int\mathrm{d}^2x\left[x_\mu T_{0\nu}-x_\nu T_{0\mu}+\pi^a_\rho\Sigma^{\rho\lambda}_{\mu\nu}A^a_\lambda\right]. \tag{30}$$

For the scalar field, $\Sigma^{\rho\lambda}_{\mu\nu}=0$, while for the vector field, $\Sigma^{\rho\lambda}_{\mu\nu}=\delta^\rho_\mu\delta^\lambda_\nu-\delta^\rho_\nu\delta^\lambda_\mu$. Thus, the angular momentum L can be written as

$$\begin{aligned}L_{\mathrm{C}}&=\frac{1}{2}\epsilon^{ij}M_{ij}\\&=\epsilon^{ij}\int\mathrm{d}^2x\left[x_i\pi^a_\phi\partial_j\phi^a+x_i\pi^a_k\partial_j A^a_k+\pi^a_i A^a_j\right].\end{aligned} \tag{31}$$

Comparing the Schwinger angular momentum L_{S} in Eq. (27) and the canonical angular momentum L_{C} in Eq. (31), it seems that L_{S} contains the C-S contribution while L_{C} does not. This is an illusion because the definition of π^a_i includes the C-S related term. In Eq. (27), the last term $\frac{\kappa}{2}\int\mathrm{d}^2x x_i A^{j,a}F^{i,a}_j$ diminishes with the C-S contribution in π^a_i. Thus, L_{S} is equivalent to the angular momentum in the Maxwell-Georgi-Glashow (MGG) theory.

The difference between the Schwinger angular momentum and the canonical angular momentum is written as

$$K=L_{\mathrm{S}}-L_{\mathrm{C}}\approx-\int\mathrm{d}^2x\partial^k\left[\pi^a_k\epsilon^{ij}x_i A^a_j\right], \tag{32}$$

which is a total boundary term. In the process of calculation, the constraint Λ^a_2 is considered. The matter component ϕ^a contributes the same gradient to L_{S} and L_{C}, and then disappears in K. For singular configurations, i.e., in the presence of C-S vortices, K does not vanish [11]. In the following, we will show that K is an anomalous spin term even without the existence of singular configurations.

Combining the secondary constraint Λ^a_2 (Gauss law) in Eq. (11) and the gauge fixing condition Ω^a_2 in Eq. (14), one obtains

$$\kappa\epsilon^{ij}\partial_i A^a_j=-\partial_i D^i A^a_0+gf^{abc}\left(\pi^b_\phi\phi^c+\pi^{i,b}A^c_i\right)\equiv J^a. \tag{33}$$

Integrating over the 2-dimensional spatial surface on both sides of Eq. (33), one obtains the non-Abelian flux of the system,

$$\Phi^a\equiv\int\mathrm{d}^2x\epsilon^{ij}\partial_i A^a_j=\frac{Q^a}{\kappa}, \tag{34}$$

where $Q^a=\int\mathrm{d}^2xJ^a$. Eq. (33) contains no time derivative term, one can construct a solution for A^a_i, which is written as

$$A^a_i=-\epsilon^{ij}\frac{x^j}{|x^2|}\frac{Q^a}{2\pi\kappa}. \tag{35}$$

Here the soliton configuration is not referred. Substituting Eq. (35) into K, one obtains

$$K=-\frac{Q^aQ^a}{4\pi\kappa}. \tag{36}$$

Up to now, the analysis is general in a sense that no soliton Ansatz is accounted, no specific gauge group is specified. The formula of K can be called the anomalous spin term.

In the CSGG model, no vortex solutions can be constructed, because ϕ^a is in the adjoint representation. Thus, whether K stands for a fractional spin term is in doubt. If there are three spatial dimensions, the 't Hooft-Polyakov monopole solution really exists in this model, namely the complex monopoles [15]. Since we work in only 2 spatial dimensions, the topological excitations forbid the existence of monopoles. In order to keep the gauge invariance, κ should be quantized as mentioned before, otherwise, the action changes under gauge transformation . For $\kappa=g^2n/(4\pi)$, one has $K=-Q^aQ^a/(g^2n)$. One can interpret such K as the fractional spin, it can be arbitrarily smaller if $n\to\infty$. Our analysis does not depend on the matter components, thus it also holds for the vortex system. For instance, if one replaces the adjoint scalar ϕ^a with a fundamental complex field φ, one can construct a non-Abelian C-S vortex [7]. In the vortex configuration, the asymptotic behavior of $D_i\varphi$ leads to $Q^aQ^a=m^2$. Thus, $K=-\frac{m^2}{g^2n}$, where m stands for the winding number of vortices, and n is an arbitrary integer.

4 Conclusion

In this letter, we investigate the 2+1 dimension CSGG theories with an arbitrary gauge group. By the Dirac formalism for the constrained system, we obtain two constraints, which are all first class. Taking the coulomb gauge and its consistency as gauge fixing conditions, the Dirac brackets of the gauge fields are deduced. Replacing the Dirac brackets with the commutators, the system is quantized in the canonical quantization formalism. The Schwinger angular momentum and Noether angular momentum of the system are calculated respectively. Their difference is found to be a compact form. Combining the Gauss law and Coulomb gauge consistency condition, the solution of the gauge field is given. With this solution, the difference of the two angular momenta is converted to be an anomalous spin term.

In our analysis, the reason for the difference between the Schwinger and the canonical angular momentum is given. The C-S term disappears when the action is varied with the metric, thus the Schwinger energy momentum tensor is symmetric about the indices. Meanwhile, the C-S term does contribute to the canonical energy momentum tensor, which is asymmetric when permuting the indices. Therefore, the difference originates from the C-S term in the Lagrangian. The matter fields and the Maxwell term contribute the same for both angular momentum, thus they do not appear in K. The anomalous spin term is also found in the canonical angular momentum of the C-S system at the quantum level[20, 21], which supports the analysis here.

Previously, the fractional spin of C-S systems is realized with the vortex configurations [1, 3–6, 11]. In this letter, no vortex configuration is used to deduce K. The Gauss law together with the Coulomb gauge are sufficient to guarantee the existence of the anomalous spin. Thus, the anomalous spin exists no matter whether there are soliton solutions or not. C-S vortices have broken axial symmetry, multi-fractional vortex-centers, ring-like flux structure, etc [7]. When the Ansatz of the vortex configuration is considered, the conserved charge Q^a is related to the winding number of the vortices. In this way, K can be explained as the fractional spin. In the 3+1 dimension, the C-S term violates both the Lorentz and parity invariance [22], but does not violate the gauge invariance up to a total derivative term. Therefore, the Lorentz symmetry holds for L_{S} but is broken in L_{C} in 3+1 dimension. In 3+1 dimension, also the 't Hooft-Polyakov monopole configuration exists in the CSGG model. Interesting future work will be to make use of the explicit Ansatz of monopoles to calculate K, which may be related to the winding number of monopoles.

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