

Rearrangement term with relativistic density-dependent hyperon potentials*

JIANG Wei-Zhou(蒋维洲)¹⁾

Department of Physics, Southeast University, Nanjing 211189, China

Abstract: An appropriate density dependence of hyperon potentials is important for the stiffening of the equation of state and massive neutron stars. To persist in covariance and thermodynamic consistency, the rearrangement term is indispensable. In this work, we derive the rearrangement term for hyperon potentials with arbitrary density-dependence. The importance of the rearrangement term is also exhibited in numerical instances.

Key words: rearrangement term, hyperon potentials, relativistic mean-field models

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1 Introduction

Baryon-baryon interactions are fundamentally important to study the nuclear equation of state and accordingly the nuclear phenomenology. The nucleon-nucleon interactions have been extensively investigated in free space by the phase shift experiments, and a variety of theoretical potential models reproduced the low-momentum physics consistently [1]. In nuclear medium, the nucleon-nucleon interactions can be studied successfully by using many-body approaches such as the Brueckner theory [2]. However, knowledge about in-medium hyperon-baryon interactions is still rather limited due to the fact that direct hyperon-baryon scattering experiments are not readily available. This turns out to be evident in astrophysical applications. For instance, it was found that hyperonization may reduce the maximum mass of neutron stars by as much as $3/4M_\odot$ [3, 4]. A recent study of the in-medium hyperon potentials in the Brueckner approach indicates that the resulting hyperon equation of state (EOS) can just produce the maximum mass of neutron stars below $1.4M_\odot$, which is inconsistent with the observation of the massive neutron stars with $2M_\odot$ [5–7]. In particular, the $2M_\odot$ pulsar J1614-2230, measured rather accurately with the Shapiro delay [8], sets a stringent constraint on the theoretical models. Provided the hyperons are included, the conventional consideration that previously assumes similar in-medium hyperon potentials to those of nucleons is thus mendable so as to obtain a stiff EOS for more massive neutron stars.

The starting point of this work is the relativistic

density-dependent model where the density-dependence is induced by the Brown-Rho (BR) scaling [9–11]. For the strange sector, we assume various density dependencies of the hyperon potentials with the aim of stiffening the hyperon EOS. To persist in the covariance and thermodynamic consistency, we first derive the rearrangement term for arbitrary density-dependencies of hyperon potentials. This derivation also provides a complementary note to the rearrangement term used in Ref. [12]. Then, the numerical instances are carried out to show the importance of the rearrangement term in the pressure of isospin-asymmetric matter and the mass-radius relation of neutron stars.

2 Rearrangement term with inclusion of hyperons

The derivation of the rearrangement term is first performed in the model without hyperons. With this primer, the formula is extended to include the hyperons with arbitrary density-dependent coupling with non-strange mesons.

2.1 A simple model without hyperons

The model Lagrangian with the density-dependent parameters is written as [9]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_N [i\gamma_\mu \partial^\mu - M_N^* + g_\sigma^* \sigma - g_\omega^* \gamma_\mu \omega^\mu \\ & - g_{\rho N}^* \gamma_\mu \tau_3 b_0^\mu] \psi_N + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^{*2} \sigma^2) \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^{*2} \omega_\mu \omega^\mu - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_\rho^{*2} b_{0\mu} b_0^\mu, \quad (1) \end{aligned}$$

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1) Also at Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China

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where ψ_B , σ , ω , and b_0 are the fields of the baryons, scalar, vector, and isovector-vector mesons, with their masses M_B^* , m_σ^* , m_ω^* , and m_ρ^* , respectively. $F_{\mu\nu}$ and $B_{\mu\nu}$ are the strength tensors of the ω and ρ mesons, respectively. The meson coupling constants and masses with asterisks denote the density dependence, given by the BR scaling [9, 13, 14]. The energy density and pressure read, respectively,

$$\mathcal{E} = \frac{1}{2}m_\omega^{*2}\omega_0^2 + \frac{1}{2}m_\rho^{*2}b_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\sigma^{*2}\sigma^2 + \frac{1}{2}m_\sigma^{*2}\sigma^{*2} + \sum_i \frac{2}{(2\pi)^3} \int_0^{k_{Fi}} d^3k E_i^*, \quad (2)$$

$$p = \frac{1}{2}m_\omega^{*2}\omega_0^2 + \frac{1}{2}m_\rho^{*2}b_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 - \frac{1}{2}m_\sigma^{*2}\sigma^2 - \frac{1}{2}m_\sigma^{*2}\sigma^{*2} - \Sigma_0^R \rho + \frac{1}{3} \sum_i \frac{2}{(2\pi)^3} \int_0^{k_{Fi}} d^3k \frac{k^2}{E_i^*} \quad (3)$$

where i stands for the species of protons and neutrons. The rearrangement term $\Sigma^{R,0}$, caused by the density-dependent parameters, is first induced in the Dirac equation by assuming the field amplitude dependence of the parameters. In the RMF, this dependence turns out to be the density dependence [15]. In the evaluation of the derivatives over the density, the rearrangement term thus come up inevitably, for instance, in the expression of the pressure. Next, we calculate the chemical potential, which is a derivative of the energy density over the density. For the chemical potential of protons, it reads

$$\mu_p = \frac{\partial \mathcal{E}}{\partial \rho_p} = E_{Fp}^* + C_{\omega N}^2 \rho - C_{\rho N}^2 \rho_3 - \Sigma_{0p}^R, \quad (4)$$

and

$$\Sigma_{0p}^R = -\rho^2 C_{\omega N} \frac{\partial C_{\omega N}}{\partial \rho_p} - \rho_3^2 C_\rho \frac{\partial C_{\rho N}}{\partial \rho_p} - \tilde{C}_{\sigma N} \frac{\partial \tilde{C}_{\sigma N}}{\partial \rho_p} (g_\sigma^* \sigma)^2 - \tilde{C}_{\sigma N}^2 g_\sigma^* \sigma \frac{\partial (g_\sigma^* \sigma)}{\partial \rho_p} - \frac{\partial m_N^*}{\partial \rho_p} \rho_s, \quad (5)$$

where $\rho_3 = \rho_n - \rho_p$, $C_{\omega N} = g_\omega^*/m_\omega^*$, $C_{\rho N} = g_\rho^*/m_\rho^*$, and $\tilde{C}_{\sigma N} = m_\sigma^*/g_\sigma^*$ with $m_N^* = M_N^* - g_\sigma^* \sigma$ being the nucleon effective mass. The last two terms in Eq. (5) merge into the term $-\rho_s \partial M_N^*/\partial \rho_p$ according to the equation of motion of the σ meson, and it disappears in models without the BR scaling. For neutrons, the chemical potential can be given similarly as,

$$\mu_n = \frac{\partial \mathcal{E}}{\partial \rho_n} = E_{Fn}^* + C_{\omega N}^2 \rho + C_{\rho N}^2 \rho_3 - \Sigma_{0n}^R, \quad (6)$$

and

$$\Sigma_{0n}^R = -\rho^2 C_{\omega N} \frac{\partial C_{\omega N}}{\partial \rho_n} - \rho_3^2 C_\rho \frac{\partial C_{\rho N}}{\partial \rho_n} - \tilde{C}_{\sigma N} \frac{\partial \tilde{C}_{\sigma N}}{\partial \rho_n} (g_\sigma^* \sigma)^2 - \rho_s \frac{\partial M_N^*}{\partial \rho_n}. \quad (7)$$

Because the proton and neutron densities can be expressed as a function of the total density and the isospin asymmetry parameter, namely

$$\rho_n = \rho_n(\rho, \delta) = \rho(1+\delta)/2, \quad (8)$$

$$\rho_p = \rho_p(\rho, \delta) = \rho(1-\delta)/2,$$

the derivative of the energy density over the total density is given as

$$\frac{\partial \mathcal{E}}{\partial \rho} = \frac{\partial \mathcal{E}}{\partial \rho_p} \frac{\partial \rho_p}{\partial \rho} + \frac{\partial \mathcal{E}}{\partial \rho_n} \frac{\partial \rho_n}{\partial \rho}. \quad (9)$$

With this relation, we thus give the rearrangement term as

$$\Sigma_0^R = \Sigma_{0n}^R \frac{\partial \rho_n}{\partial \rho} + \Sigma_{0p}^R \frac{\partial \rho_p}{\partial \rho} = -\rho^2 C_{\omega N} \frac{\partial C_{\omega N}}{\partial \rho} - \rho_3^2 C_\rho \frac{\partial C_{\rho N}}{\partial \rho} - \tilde{C}_{\sigma N} \frac{\partial \tilde{C}_{\sigma N}}{\partial \rho} (g_\sigma^* \sigma)^2 - \rho_s \frac{\partial M_N^*}{\partial \rho}. \quad (10)$$

The rearrangement term is usually regarded to be independent of isospin asymmetry, and Eq. (10) gives the rearrangement term in nucleonic matter.

2.2 Inclusion of hyperons

The Lagrangian (1) can be extended to include hyperons. The coupling of mesons with hyperons can generally be given as the parameters $X_{\sigma Y}$, $X_{\omega Y}$, and $X_{\rho Y}$, which are ratios of the meson coupling with hyperons to that with nucleons. In this work, their ratio parameters are regarded to be density-dependent. In the following, we derive the rearrangement term with these additional parameters. For hyperons, we include additionally the \mathcal{L}_Y that is characterized by the strange meson exchange:

$$\mathcal{L}_Y = \bar{\psi}_Y [g_{\sigma^* Y} \sigma^* - g_{\phi Y} \gamma_\mu \phi^\mu] \psi_Y - \frac{1}{4} \Phi^{\mu\nu} \Phi_{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu + \frac{1}{2} (\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}), \quad (11)$$

where $\Phi_{\mu\nu}$ is the strength tensor of the ϕ field, and the parameters are assumed to be density independent. Here, σ^* (i.e., f_0 , 975 MeV) and ϕ (1020 MeV) are the scalar and vector strange mesons, respectively. The strange mesons σ^* and ϕ are essential to describe the strong $\Lambda\Lambda$ attraction [16, 17].

We start with the energy density provided by the ω meson, termed \mathcal{E}_ω :

$$\mathcal{E}_\omega = \frac{1}{2} m_\omega^* \omega_0^2 = \frac{1}{2} \left(\sum_i \frac{g_{\omega i}^*}{m_\omega^*} \rho_i \right)^2, \quad (12)$$

where the first equality accords to the equation of motion for the ω meson. We may define the parameter $C_{\omega i} = g_{\omega i}^*/m_\omega^*$ where the subscript i runs over all species of baryons and $g_{\omega i}^* = g_\omega^*$ for nucleons. The derivative with respect to the respective density contributes to the chemical potential of baryon, similar to the derivation of Eqs. (4), (6), whereas the rearrangement term just needs the derivative over the total density in the following

$$\frac{\partial \mathcal{E}_\omega}{\partial \rho} = \sum_i C_{\omega i} \rho_i \sum_j \left(C_{\omega j} \frac{\partial \rho_j}{\partial \rho} + \rho_j \frac{\partial C_{\omega j}}{\partial \rho} \right), \quad (13)$$

where the second term contributes to the rearrangement term:

$$\Sigma_{0\omega}^R = -m_\omega^* \omega_0 \sum_j \rho_j \frac{\partial C_{\omega j}}{\partial \rho}. \quad (14)$$

For the ρ meson, the expression is similar

$$\Sigma_{0\rho}^R = -m_\rho^* b_0 \sum_j \rho_j \tau_{3j} \frac{\partial C_{\rho j}}{\partial \rho}. \quad (15)$$

Now, we write down the terms concerning the σ and σ^* mesons,

$$\begin{aligned} \frac{\partial \mathcal{E}_\sigma}{\partial \rho} &= \tilde{C}_{\sigma N} \frac{\partial \tilde{C}_{\sigma N}}{\partial \rho} (g_\sigma^* \sigma)^2 + \sum_i \frac{g_{\sigma i}^*}{g_\sigma^*} \rho_{si} \frac{\partial g_\sigma^* \sigma}{\partial \rho} + \\ &\sum_Y \rho_{sY} \frac{\partial g_{\sigma^* Y} \sigma^*}{\partial \rho} + \sum_i \rho_{si} \frac{\partial m_{Bi}^*}{\partial \rho}, \end{aligned} \quad (16)$$

where the last term comes from the derivative of the kinetic term, and we take the mass and coupling constant of the strange meson to be density independent. Using the definition of the nucleon effective mass and hyperon effective mass ($m_Y^* = M_Y^* - g_{\sigma Y}^* \sigma - g_{\sigma^* Y} \sigma^*$), it arrives at

$$\frac{\partial \mathcal{E}_\sigma}{\partial \rho} = \tilde{C}_{\sigma N} \frac{\partial \tilde{C}_{\sigma N}}{\partial \rho} (g_\sigma^* \sigma)^2 + \sum_i \rho_{si} \frac{\partial M_i^*}{\partial \rho} + \Delta_\sigma \quad (17)$$

where $X_{\sigma Y} = g_{\sigma Y}^*/g_\sigma^*$, and

$$\begin{aligned} \Delta_\sigma &= \sum_Y \rho_{sY} \left(X_{\sigma Y} \frac{\partial (g_\sigma^* \sigma)}{\partial \rho} - \frac{\partial (g_{\sigma Y}^* \sigma)}{\partial \rho} \right) \\ &= - \sum_Y \rho_{sY} g_\sigma^* \sigma \frac{\partial X_{\sigma Y}}{\partial \rho}. \end{aligned} \quad (18)$$

With results in Eqs. (14), (15), (17) and (18), we give the final expression of the rearrangement term including the contributions from hyperons

$$\Sigma_0^R = -m_\omega^* \omega_0 \sum_i \rho_i \frac{\partial C_{\omega i}}{\partial \rho} - m_\rho^* b_0 \sum_i \rho_i \tau_{3i} \frac{\partial C_{\rho i}}{\partial \rho}$$

$$- \tilde{C}_\sigma \frac{\partial \tilde{C}_\sigma}{\partial \rho} (m_N^* - M_N^*)^2 - \sum_i \rho_{si} \frac{\partial M_i^*}{\partial \rho} - \Delta_\sigma. \quad (19)$$

The parameters of the model are assumed to be only dependent on the density rather than the isospin asymmetry. This leads to the equality of the total and all individual rearrangement terms, and the chemical equilibrium is not affected by the rearrangement term. With this rearrangement term and the summation of kinetic terms over all species in Eqs. (2) and (3), one obtains the energy density and pressure of isospin-asymmetric hyperonized matter.

2.3 Numerics

Here, we manifest numerically the role of the term Δ_σ in Eq. (19) based on the RMF models SLC and SLCd [18]. The unique difference between the SLC and SLCd is the isospin-dependent interactions provided by the ρ meson, which produce different density dependencies of nuclear symmetry energy. Quantitatively, the slope parameter L at saturation density is $L = 92.3$ MeV and 61.5 MeV for the SLC and SLCd, respectively. After the RMF models SLC and SLCd are extended to include the hyperonizations, we need first of all to determine the parameters for hyperons. Apart from the usual case that the ratios of the meson-hyperon couplings to the meson-nucleon ones are taken to be constant, we consider here a scheme that the meson-hyperon coupling constant is separated to be density-dependent and density-independent parts. Nevertheless, the hyperon potentials [19–21]

$$U_\Lambda^{(N)} = -30 \text{ MeV}, \quad U_\Xi^{(N)} = -18 \text{ MeV}, \quad (20)$$

in nuclear matter at saturation density are used to preserve the relation between the vector and scalar meson coupling constants. For the strange mesons, we adopt the density-independent coupling constants for simplicity and follow the determination of parameters in Ref. [17] by fitting the potentials for the Λ and Ξ hyperons in Ξ matter $U_\Lambda^{(\Xi)} = U_\Xi^{(\Xi)} = -40$ MeV.

The density dependence of parameters is described by the scaling functions that are the ratios of the in-medium parameters to those in the free space. For the nucleonic sector, we take the scaling functions $\Phi_{iN}(\rho)$ with i denoting the meson species [18]. For the hyperonic sector, we consider the following scaling functions for the meson-hyperon coupling constants that consist of two terms:

$$\begin{aligned} \Phi_{\omega\Lambda}(\rho) &= \frac{1}{3} \Phi_{\omega N}(\rho_0) + \frac{2}{3} \Phi_{\omega N}(\rho), \\ \Phi_{\omega\Xi}(\rho) &= \frac{2}{3} \Phi_{\omega N}(\rho_0) + \frac{1}{3} \Phi_{\omega N}(\rho), \\ \Phi_{\sigma\Lambda}(\rho) &= (1 - f_{\sigma\Lambda}) \Phi_{\sigma N}(\rho_0) + f_{\sigma\Lambda} \Phi_{\sigma N}(\rho), \\ \Phi_{\sigma\Xi}(\rho) &= (1 - f_{\sigma\Xi}) \Phi_{\sigma N}(\rho_0) + f_{\sigma\Xi} \Phi_{\sigma N}(\rho), \end{aligned} \quad (21)$$

Table 1. Meson-hyperon coupling constants in various cases with models SLC and SLCd. The coupling constants in the medium are obtained as $g_{iY}^* = g_{iY}^0 \Phi_{iY}(\rho)$. The parameters listed here are free of the parameter $f_{\sigma Y}$. For the vector meson-hyperon couplings, we take in the calculation the relations: $g_{\omega\Sigma}^0 = g_{\omega\Lambda}^0$, $g_{\rho\Sigma}^0 = 2g_{\rho\Xi}^0$ and $g_{\rho\Lambda}^0 = 0$.

model	$X_{\omega\Lambda}^0$	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Sigma}$	$g_{\sigma^*\Xi}$	$g_{\sigma\Lambda}^0$	$g_{\sigma\Sigma}^0$	$g_{\sigma\Xi}^0$	$g_{\omega\Lambda}^0$	$g_{\omega\Xi}^0$	$g_{\rho\Xi}^0$	ρ_{Λ}^0/ρ_0
SLC	1/3	5.998	7.545	9.713	3.475	1.416	3.132	3.442	3.442	3.802	2.3
	2/3	6.146	7.651	9.764	5.920	3.861	3.063	6.884	3.442	3.802	2.7
	0.8	6.548	9.595	10.320	6.689	4.840	6.555	8.261	8.261	3.802	3.0
SLCd	1/3	5.998	7.545	9.713	3.475	1.416	3.132	3.442	3.442	5.776	2.4
	2/3	6.146	7.651	9.764	5.920	3.861	3.063	6.884	3.442	5.776	2.9
	0.8	6.548	9.595	10.320	6.689	4.840	6.555	8.261	8.261	5.776	3.0

where ρ_0 is the saturation density, $f_{\sigma Y}$ is an adjustable constant, and the scaling function $\Phi_{\rho\Xi}$ for the ρ meson is taken as the same as that of the ω meson. The factors 1/3 and 2/3 are taken in the vector meson scaling functions according to constituent quark compositions and assuming the density-independent strange sector in hyperons. The above relation leads to the density-dependent coupling ratios

$$X_{iY} = X_{iY}^0 \Phi_{iY}(\rho) / \Phi_{iN}(\rho), \quad i = \sigma, \omega, \rho, \quad (22)$$

with the X_{iY}^0 being given at zero density. Eq. (21) produces the relation $\Phi_{iY} \equiv \Phi_{iN}$ at saturation density. Thus, we do not need to readjust the parameter $g_{\sigma Y}(\rho_0)$ as $f_{\sigma Y}$ changes. Though other parameters are free of choices of $f_{\sigma Y}$, large values of $f_{\sigma Y}$ are preferred in the calculation

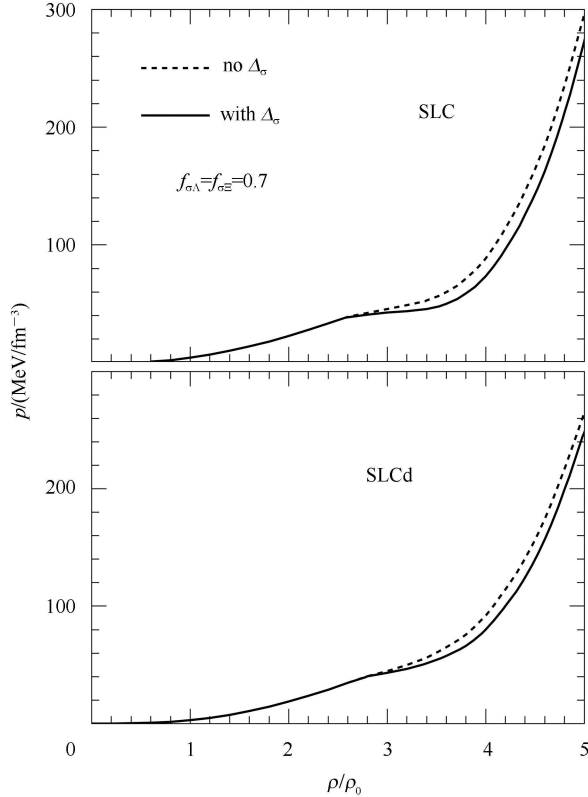


Fig. 1. The pressure of hyperonized matter as a function of density with and without the term Δ_σ .

to keep the monotonously rising trend of the pressure with the increase of the density. The ratio $X_{\sigma Y}$ changes with the parameter $f_{\sigma Y}$, giving rise to the nonzero contribution to the rearrangement term, see Eq. (19). The parameters are tabulated in Table 1.

In Fig. 1, the pressure of asymmetric matter with hyperonization is shown as a function of density. The term Δ_σ starts to contribute to the pressure with the appearance of hyperons at the density about $2.5\rho_0$. The magnitude of the Δ_σ term goes up with the descending $f_{\sigma Y}$. With the value 0.7 shown in Fig. 1, the Δ_σ term has a contribution as large as 20% to the total pressure. This indicates that a correct derivation of the rearrangement term in hyperonized matter is rather important.

Shown in Fig. 2 is the mass-radius (M-R) relations of neutron stars obtained from solving the standard TOV equation. The Δ_σ term contributes significantly to the M-R relations, while it decreases with the increase of $f_{\sigma Y}$. On the other hand, the role of the Δ_σ is small in affecting the maximum mass of neutron stars. This is attributed to the modelling that suppresses the hyperon fractions at high densities.

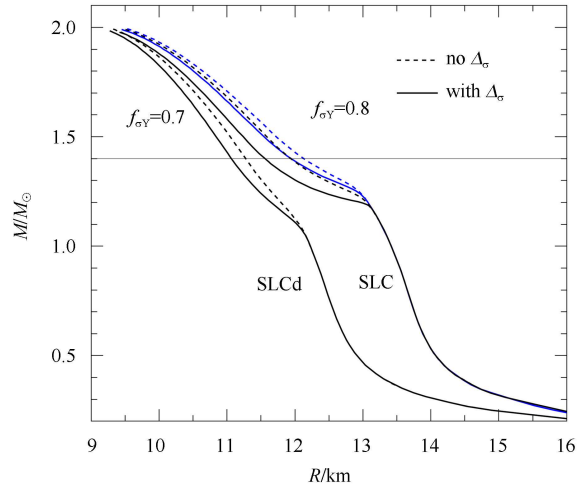


Fig. 2. (color online) The mass-radius relation of neutron stars. The curves are obtained with the parameter $f_{\sigma Y} = 0.7$ (black) and 0.8 (blue). The full (dashed) line stands for the result with (without) the term Δ_σ .

3 Summary

In this work, different density dependencies for the nucleon and hyperon potentials are considered to be consistent with the stiff EOS, which is suggested by the

recent observation of massive neutron stars. The rearrangement term is derived for hyperonized asymmetric matter with arbitrary density-dependence of the hyperon potential. The importance of the rearrangement term is exhibited numerically in the pressure of asymmetric matter and the mass-radius relation of neutron stars.

References

- 1 Bogner S K, Kuo T T S, Schwenk K. Phys. Rept., 2003, **386**: 1
- 2 Brockmann R, Machleidt R. Phys. Rev. C, 1990, **42**: 1965
- 3 Glendenning N K. Astrophys. J, 1985, **293**: 470
- 4 Glendenning N K, Moszkowski S A. Phys. Rev. Lett., 1991, **67**: 2424
- 5 Nice D J, Splaver E M, Stairs I H, Loehmer O et al. Astrophys. J, 2005, **634**: 1242
- 6 Nice D J, Stairs I H, Kasian. in AIP Conf. Ser., 2008, **983**: 453; 40 Years of Pulsars: Millisecond Pulsars, Magnetars and More, ed. Bassa C, Wang Z, Cumming A, Kaspi V M. AIP: New York
- 7 Özel F. Nature, 2006, **441**: 1115
- 8 Demorest P B, Pennucci T, Ransom S M, Roberts M S E, Hessels J W T. Nature, 2010, **467**: 1081
- 9 JIANG W Z, LI B A, CHEN L W. Phys. Lett. B, 2007, **653**: 184
- 10 Brown G E, Rho M. Phys. Rev. Lett., 1991, **66**: 2720
- 11 Brown G E, Holt J W, Lee C H, Rho M. Phys. Rep., 2007, **439**: 161
- 12 JIANG W Z, LI B A, CHEN L W. Astrophys. J, 2012, **756**: 56; arXiv:1207.1686 [astro-ph.SR]
- 13 Brown G E, Rho M. nucl-th/0509001; nucl-th/0509002
- 14 SONG C. Phys. Rep., 2001, **347**: 289
- 15 Fuchs C, Lenske H, Wolter H. Phys. Rev. C, 1995, **52**: 3043
- 16 Schaffner-Bielich J, Gal A. Phys. Rev. C, 2000, **62**: 034311
- 17 JIANG W Z. Phys. Lett. B, 2006, **642**: 28
- 18 JIANG W Z, LI B A, CHEN L W. Phys. Rev. C, 2007, **76**: 054314
- 19 Fukuda T et al. (E224 collaboration). Phys. Rev. C, 1998, **58**: 1306
- 20 Millener D J, Dover C B, Gal A. Phys. Rev. C, 1988, **38**: 2700
- 21 Hausmann R, Weise W. Nucl. Phys. A, 1989, **491**: 601