A corresponding-state approach to quark-cluster matter *

GUO Yan-Jun(郭彦君)[1;1)] LAI Xiao-Yu(来小禹)[2;2)] XU Ren-Xin(徐仁新)[1,3;3)]
[1] School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[2] School of Physics, Xinjiang University, Urumqi 830046, China
[3] Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China

Abstract: The state of super-dense matter is essential for us to understand the nature of pulsars; however, non-perturbative quantum chromodynamics makes it very difficult to make direct calculations of the state of cold matter at realistic baryon number densities inside compact stars. Nevertheless, from an observational point of view, it is conjectured that pulsars could be made up of quark clusters since the strong coupling between quarks might render the quarks to be grouped in clusters. In this paper, we attempt to find an equation of state of condensed quark-cluster matter in a phenomenological way. Supposing that the quark-clusters could be analogized to inert gases, we apply here the corresponding-state approach to derive the equation of state of quark-cluster matter, as was similarly demonstrated for nuclear and neutron-star matter in the 1970s. According to the calculations that we have presented, the quark-cluster stars, which are composed of quark-cluster matter, could have a high maximum mass that is consistent with observations and, in turn, further observations of pulsar mass could also place a constraint on the properties of quark-cluster matter. We will also briefly discuss the melting heat during the solid-liquid phase conversion and its related astrophysical consequences.

Key words: pulsars, neutron stars, elementary particles

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1 Introduction

The state of matter above the nuclear matter density, ρ_0, is still far from certain, whereas it is essential for us to explore the nature of compact stars. At an average density higher than $\sim 2\rho_0$, the quark degree of freedom inside would not be negligible, and historically such compact stars are called quark stars [1–4]. Although cold quark matter is difficult to create in laboratories or to study by direct non-perturbative quantum chromodynamic (QCD) calculations, some efforts have been made to model quark matter and quark stars, from the MIT bag model to the color super-conductivity model [5]. In most of these models, quark matter is usually characterized by a soft equation of state because the asymptotic freedom of QCD tells us that as the energy scale goes higher the interaction between quarks becomes weaker. Nonetheless, the astrophysical phenomenology of compact stars can still not rule out this striking physical possibility, making pulsars superb astrophysical laboratories [see e.g. Refs. [6, 7] and references therein].

However, at realistic baryon densities of compact stars, $\rho \sim (2\text{–}10)\rho_0$, the energy scale is usually below 0.8 GeV, which is much lower than the scale where the asymptotic freedom could apply. In contrast, the non-perturbative effect should be significant, making quarks couple strongly with each other. Quark-clustering is, therefore, conjectured to occur in cold dense matter inside compact stars by the condensation of quarks in position space due to the strong coupling between quarks [8]. Consequently, a realistic quark star could actually be a "quark-cluster star", and solidification could be a natural result if the kinetic energy of quark-clusters is much lower than the residual interaction energy between the clusters. The idea of clustering quark matter could enable us to understand the different manifestations of pulsar-like compact stars [9, 10].

How then can one model the equation of sate of quark-cluster matter? Due to the lack of both theoretical and experimental evidence, the hypothetical quark-clusters in cold dense matter have not been confirmed. It is also difficult for us to derive the properties of quark-cluster matter from the first principles of QCD calculations. Nevertheless, in the 1970s an empirical method

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1) E-mail: guoyj10@pku.edu.cn
2) E-mail: xylai4861@gmail.com
3) E-mail: r.x.xu@pku.edu.cn

was employed and seriously discussed in a corresponding-state approach in order to deduce the properties of nuclear matter [e.g. 11, 12]. To establish a model that could be tested by observations, we have adopted an empirical method by analogizing quark-clusters to inert gases and applying the corresponding-state approach. A quark-cluster is usually assumed to be colorless, just like an inert atom is electrically neutral. The interaction between inert gas atoms is the result of residual electromagnetic force and similarly the interaction of quark-clusters could be regarded as the result of residual strong force, both of which should be characterized by the short-distance repulsion and long-distance attraction. In this paper, we assume that the interaction between quark-clusters could be described approximately in the same form as that between inert gas atoms (i.e. the Lennard-Jones potential), only with different parameters indicating stronger interaction and larger densities.

In fact, quark matter in the Lennard-Jones model has previously been studied, where the equation of state is derived via summing the interaction energy of all quark-clusters [13]. Previously, a polytropic model [14] and a two-Gaussian component soft-core model [15] for quark-cluster stars have been applied. However, the so-called corresponding-state approach that we demonstrate in this paper is an empirical approach to derive the properties of quark-cluster matter just by comparison to the experimental data of inert gases, based on the law of corresponding states. The law of corresponding states was first proposed by de Boer [16], who found that the properties of inert gases, such as pressure and density, could be written in a reduced form. After reducing to dimensionless terms, the experimental data of various inter gases can be fitted in smooth curves with a single quantum parameter. If the quark-cluster matter is assumed to be similar to inert gases, the corresponding-state approach can also be applicable to study the state of quark-cluster matter, even without knowing its exact structure.

With a similar form of interaction, we may derive the equation of state of quark-cluster matter from the empirical data of inert gases through the corresponding-state approach. The masses and radii of quark-cluster stars can then be derived and compared with observations. We find that the maximum mass of quark-cluster stars can be well above $2M_\odot$. Although, in principle, one may obtain a maximum mass high enough to explain observations in any kinds of unphysical models, we call attention to the point that the quark-cluster star model has meaningful implications for one to understand different manifestations, such as of the surface [17]. Additionally, the melting heat is also discussed, and it is shown that the solidification of newly born quark-cluster stars might explain the plateau of γ-ray bursts.

This paper is organized as follows. We summarize the properties and observational implications of quark-cluster stars in Section 2, and a brief introduction of the law of corresponding states is given in Section 3. The equation of state of quark-cluster matter and the mass-radius curve of quark-cluster stars are derived in Section 4 using the corresponding-state approach. The melting heat of solid quark-cluster stars and the related astrophysical consequences are discussed in Section 5. Section 6 includes our conclusions and discussions.

2 Quark-cluster matter

A lot of basic intuition questions are frequently asked about quark-cluster matter, even though such a state was proposed ten years ago [8]. For example, would quark-cluster matter be more energetically favored than nuclear matter or strange quark matter? Can quark-clusters be analogous to inert gases? Is a quark-cluster star model really necessary in the astrophysics of compact stars? Certainly, we cannot present clear and final answers to these questions because of both micro- and astro-problems, as well as their entanglement. In order for readers to have a thorough and comprehensive view of the quark-cluster star concept, we here make some rough estimations of the quark-cluster's existence and demonstrate that the answers could be positive in some regions of the parameter space in QCD phase diagram. This section will also summarize some observational hints.

2.1 The stability of quark-cluster matter

Understanding stability from first principles is an interesting but difficult problem. A special kind of quark-cluster, the so-called H-cluster, has been studied extensively. By comparing energy per baryon at fixed density, it has been found that H-cluster matter might be more stable than both neutron matter and nuclear matter when the density is larger than $2\rho_0$, where the in-medium effect plays a crucial role in stabilizing H-cluster matter [see Ref. [18] Section 2]. In addition, an order of magnitude estimation could help compare the stability of these three states.

Nuclear matter vs. Quark-cluster matter. In the low energy region of the QCD phase diagram, quarks are confined in nucleons. However, at the density of realistic compact stars, the confined state may not be simply that of hadrons because a light-flavor symmetry is likely to be restored. In an ordinary case, electrons are outside the nucleus and their energy E_e is far lower than 1 MeV, so the atoms can be stable with 2-flavor symmetry. Nevertheless, things are different in the case of a pulsar because here the electrons are inside the gigantic nucleus and the Fermi energy of electrons would be $E_e\sim 10^2$ MeV, which is even larger than the mass difference between s quark and u/d quark. Such a high energy might intensify the interaction e+p→n+ν_e, thus E_e decreases but the nuclear symmetry energy increases. Therefore, the s quark is likely to be excited in a gigantic nucleus, the number of which may be slightly ($\sim 10^{-5}$) less than the u/d quark because the s quark is heavier. If 3-flavor symmetry is restored, the number of electrons in a pulsar would be much lower, which makes $E_e \sim 10$ MeV, and the gigantic nucleus would be stable. Furthermore, three flavors of quarks could be grouped together to form a new hadron-like confined (quark-cluster) state in a gigantic nucleus if the coupling between quarks is still strong.

Strange quark matter vs. Quark-cluster matter. At a high density, low temperature regime, cold dense quark matter could be a Fermi gas or liquid if the interaction between quarks is negligible. However, can the density in realistic compact stars be so high that we can neglect the interaction? The average density of a pulsar-like star with typical mass of 1.4 $M_\odot$ and radius of 10 km is only $\sim 2.4\rho_0$. For 3-flavor quark matter with density of $3\rho_0$, we have number densities for each flavor of quark, u, d, and s, of $n_u \approx n_d \approx n_s \sim (3\times 0.16 = 0.48)$ fm^{-3}. A further calculation of Fermi energy gives, $E_F^{NR} \approx \frac{\hbar^2}{2m_q}(3\pi^2)^{2/3} \cdot n^{2/3} = 380$ MeV if quarks are considered moving non-relativistically, or $E_F^{ER} \approx \hbar c(3\pi^2)^{1/3} \cdot n^{1/3} = 480$ MeV if quarks are considered moving extremely relativistically.

However, the interaction between quarks may play an important role in determining the real state. For a quark with length scale l, from Heisenberg's uncertainty relation, the kinetic energy would be of $\sim p^2/m_q \sim \hbar^2/(m_q l^2)$, which has to be comparable to the color interaction energy of $E \sim \alpha_s \hbar c/l$ in order to have a bound state, where α_s is the coupling constant of strong interaction. One then finds if quarks are dressed, with a mass of m_q=300 MeV,

$$l \sim \frac{1}{\alpha_s}\frac{\hbar c}{m_q c^2} \approx \frac{1}{\alpha_s}\ \text{fm},\ E \sim \alpha_s^2 m_q c^2 \approx 300\alpha_s^2\ \text{MeV}. \tag{1}$$

This is dangerous for the Fermi state of matter since E is approaching and even greater than the Fermi energy of $\sim$0.4 GeV if the running coupling constant $\alpha_s > 1$, and a Dyson-Schwinger equation approach to non-perturbative QCD shows that the color coupling should be very strong rather than weak, with $\alpha_s \gtrsim 2$ at a few nuclear densities in compact stars [9]. Such strong interactions could group the quarks into clusters, rather than condensing in momentum space to form a color super-conductivity state.

2.2 Properties of quark-cluster matter

From the arguments above, we can see that a symmetry of light flavor quarks is restored in the quark-clustering phase, which is different from the usual hadron phase. The color interaction is still strong here. On the other hand, the quark-clustering phase is also different from the conventional quark matter phase, which is composed of relativistic and weakly interacting quarks. The quark-clustering phase could thus be considered as an intermediate state between hadron phase and free-quark phase.

Compact stars composed of pure quark-clusters are electrically neutral, but in reality there could be some flavor symmetry breaking that leads to non-equality among u, d and s, usually with less s than u and d. The positively charged quark matter is necessary because it allows the existence of electrons that might be crucial for us to understand the radiation behaviors of compact stars.

What could be a realistic quark-cluster? We know that Λ particles (with structure uds) possess light-flavor symmetry, and one may think that a kind of quark-cluster would be Λ-like. However, the interaction between Λ is attractive, so H-cluster with structure uuddss could emerge, which has previously been predicted to be a stable state or resonance [19], and recently Lattice QCD simulations have shown possible evidence for its existence [20, 21]. Besides the H-cluster, an 18-quark cluster, that is quark-α, being completely symmetric in spin, color and flavor space, has also been speculated to exist [22]. The number of quarks in one cluster is left as a free parameter in this paper, and we set $N_q = 6$ and $N_q = 18$ for the sake of simplicity in the calculations as follows, corresponding to H-cluster and quark-α, respectively.

An estimate for the length scale l_{qc} of a quark-cluster gives $l_{qc} \sim 1/\alpha_s$ fm $\lesssim 1$ fm, which would be less than the average distance between quark-clusters $d \approx (3\times 0.16/N_q)^{-1/3} \gtrsim 1$ fm. Although quark-clusters consist of more quarks, they might not be larger than nucleons, and the distance between quark-clusters would be larger. Consequently, it is not likely that quark-clusters would be in closer proximity than nucleons. The quantum effect would not be significant if the residual short-distant repulsing interaction works, and the quark-cluster can be considered as a classical particle rather than a quantum gas. What is more, a quark-cluster may move non-relativistically due to its large mass and it could be localized in a lattice at low temperatures.

When justifying the corresponding-state approach, one may doubt why the law of corresponding states should apply to quarks. Theoretically, the corresponding-state law reflects the statistical behavior of a system. What is required is just the same form of interaction potential, while certain values would not influence the conclusion if the parameters are re-scaled. It could also be asked if the interaction between quark-clusters could be described by the Lennard-Jones potential? Although it is hard to know the accurate form of the interaction between quark-clusters, it could have a similar shape to a Lennard-Jones potential, considering the property of short-distance repulsion and long-distance attraction. A quark-cluster could be analogized to nucleons, except for light-flavor symmetry. Since strong interaction is not sensitive to flavor, the interaction between quark-clusters should be similar to that of nucleons, which is found to be Lennard-Jones-like by both experiment and modeling [23].

The interaction between quark-clusters may not be perfectly described by Lennard-Jones potential, the long range part of which may be proportional to r^{-7} instead of r^{-6} [24]. As a zeroth approximation, the Lennard-Jones potential assumption may lead to a violation of the law of corresponding states, but the reduced properties of quark-cluster matter should at least be in the same range with, if not exactly falling in, the experimental lines of inert gases. So our approach is to some degree reasonable, especially when the exact approach under QCD calculations seems to be impossible due to the significant non-perturbative effect.

2.3 Observational hints for the nature of pulsar

In the absence of QCD calculations due to non-perturbative effect, estimations in Section 2.1 could only demonstrate the possibility of stable quark-cluster matter, while no further conclusion could be made and the stability of quark-cluster remains to be justified. Nevertheless, a quark-cluster was speculated to exist primarily for understanding astrophysical observations of pulsar-like compact stars. In addition to first principles, pulsar-like compact stars could also provide valuable information for properties of super-dense matter, different manifestations of which provide hints of the state of matter at supra-nuclear density. Various observational phenomena could be understood in terms of a quark-cluster star model, including those that are challenging in conventional neutron star models [25].

What if the pulsar is made of quark-cluster matter? There are at least three consequences of this that are relevant to observational phenomena:

A stiff equation of state. It is conventionally thought that the state of dense matter softens and thus cannot result in high maximum mass if pulsars are quark stars. The discovery of $2M_{\odot}$ pulsar PSR J1614-2230 [26] may make pulsars unlikely to be quark stars. However, a quark-cluster star would have a stiff equation of state because the quark-cluster should be a non-relativistic particle because of its large mass and also because there would be a strong short-distance repulsion between the quark-clusters. It may well be possible to obtain a maximum mass of $\geqslant 2M_{\odot}$. Certainly, finding a stiffer equation of state is not enough to claim that quark-cluster matter exist, but other observations may hint at a self-bound surface and global solid structure, which could also favor the existence of quark-cluster.

A self-bound surface. In contrast to traditional neutron stars, a quark-cluster star would be self-bound by residual color-interaction between clusters, which could be a crucial difference that enables observational manifestations to distinguish the two models.

The drifting subpulses phenomena in radio pulsars suggest the existence of Ruderman-Sutherland-like gap-sparking and, thus, strong binding of particles on pulsar polar caps to form vacuum gaps; however, the calculated binding energy in normal neutron star models could not be so high, unless the magnetic field is extremely strong. This problem could be naturally solved in quark-cluster star scenario due to the strong self bound nature on the surface [27, 28].

In addition, many theoretical calculations predict the existence of atomic features in the thermal X-ray emission of neutron star atmospheres, while none of the expected spectral features has been detected with certainty up to now, which hints that the speculated atmospheres in conventional models may not exist. Although modified neutron star atmospheres with very strong surface magnetic fields [29, 30] may reproduce a featureless spectrum, a natural suggestion to understand the general observation could be that pulsars are actually quark-cluster stars without atoms on the surface [31].

In addition, the bare and chromatically confined surface of a quark-cluster star could overcome the baryon contamination problem and create a clean fireball for a γ-ray burst and supernova. The strong surface binding would result in extremely energetic explosions because the photon/lepton luminosity of a quark-cluster surface is not limited by the Eddington limit, and supernova and γ-ray bursts could then be photon/lepton-driven [32–34]. It has recently been shown that the magnetic field observed for some compact stars could be generated by small amounts of differential rotation between the quark matter core and the electron sea [35].

A global solid structure. A quark-cluster star could be in a global solid state, like "cooked eggs", if the kinetic energy is less than the interaction energy between quark-clusters, while for normal neutron stars only the crust is solid, like "raw" eggs. A rigid body would precess naturally when spinning, either freely or by torque, and the observation of possible precession or even free precession of B1821-11 [36] and others could suggest a global solid structure of pulsars.

A star-quake is a peculiar action of solid compact stars, during which huge free energy (such as gravitational and elastic energy) is released. For a pulsar with mass $M\sim M_{\odot}$ and radius $R\sim 10$ km, the stored gravitational energy is $\approx GM^2/R\sim 10^{53}$ erg, so the energy released would be $\sim 10^{53}\Delta R/R$ when the radius changes from R to $R-\Delta R$. Compared with magnetars powered by magnetic energy, quake-induced energy in solid quark-cluster stars may also be enough to power the bursts, flares and even superflares of soft γ-ray repeaters and anomalous X-ray pulsars [37].

By combining surface and global properties, we think that the quark-cluster star model would be reasonable to describe pulsar-like stars, while this paper is mainly focused on the equation of state for quark-cluster stars via a phenomenological method.

3 The law of corresponding states

The law of corresponding states, advocated by de Boer [16], shows that the equation of state of substances with same form of interaction can be written in a reduced and universal form. Consider a group of substances with the following properties: (1) the total potential energy due to interaction can be written as a sum of identical expressions $\varphi(r_{\rm ik})$, each of which depends only on the distance $r_{\rm ik}$ between two particles i and k; (2) $\varphi(r)=\varepsilon f(r/\sigma)$, where f is a function same for all substances, and ε, σ are characteristic energies and lengths for different species. The macroscopic quantities, such as pressure P, volume V and temperature T, can be expressed in dimensionless terms:

$$P^* = P\sigma^3/\varepsilon, \tag{2}$$

$$V^* = V/(N\sigma^3), \tag{3}$$

$$T^* = kT/\varepsilon. \tag{4}$$

Another dimensionless parameter is

$$\Lambda^*=h/(\sigma\sqrt{m\varepsilon}), \tag{5}$$

corresponding to the de Broglie wavelength, which is constructed to measure the importance of quantum effects. It can be proven that the reduced equation of states expressed in dimensionless quantities is a universal relation

$$P^*=f(V^*,T^*,\Lambda^*), \tag{6}$$

which is the formulation of the law of corresponding states [16].

Although the so-called universal equation of states is just formally written as Eq. (6), a formula that is difficult to be derived theoretically for most cases, it can be used to obtain information on the equation of state of a substance which we are unfamiliar with. For determined V^* and T^*, P^* depends on the value of Λ^*, and the $P^*-\Lambda^*$ curve can be drawn using experimental data of laboratory substances. If the curve is smooth enough, the value of P^* for unfamiliar matter at such a state can be predicted, provided that its Λ^* is known.

For some substances described by Lennard-Jones 6–12 potential

$$\varphi(r)=\varepsilon\left\{\frac{4}{(r/\sigma)^{12}}-\frac{4}{(r/\sigma)^{6}}\right\}, \tag{7}$$

de Boer had determined ε and σ of noble gases and some permanent gases ($r=\sigma$ is the distance where $\varphi(r)=0$, and ε is the depth of potential well) [16]. Then, the experimental data of P^*, T^* or V^* for different substances turn out to be smooth functions of Λ^* as corresponding states. In Fig. 1, the experimental data of the volume V_0 at zero temperature and zero pressure, reduced to $V_0^*=V_0/(N\sigma^3)$, are plotted with Λ^* for some substances.

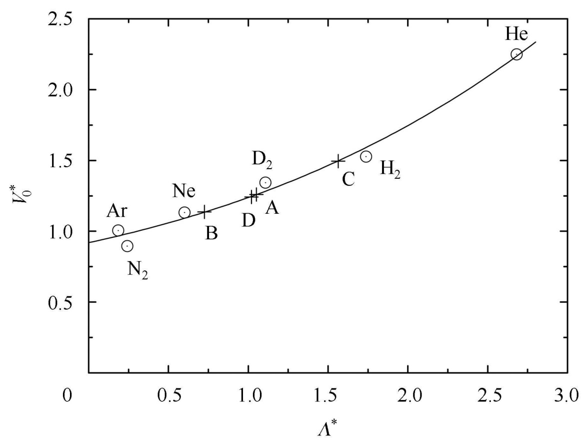

Fig. 1. The experimental data of reduced volume V_0^* and Λ^* at zero temperature and pressure for different inert gases are shown by dots [16], and the fitted curve of Eq. (8) is shown by solid line. Four cases A, B, C, and D are denoted by crosses, which will be explained in Sect. 4.1.

A smooth curve can be drawn by fitting all the points, which forms the bases of our prediction via corresponding states law. The formula for the fitted curve is

$$V_0^*=0.57+9.45\times10^{-5}(\Lambda^*+6.35)^{4.44}. \tag{8}$$

Considering the property of short-distance repulsion and long-distance attraction shown by the Lennard-Jones potential, we assume that the interaction between quark-clusters can also be described by this form. The distinctions between quark-cluster matter and ordinary substances should be a much deeper potential well (larger ε) and higher density (smaller σ). With the same form of interaction as that of inert gas, we could apply the law of corresponding states to derive the properties of quark-cluster matter. If we find the quantum parameter Λ^* corresponding to quark-cluster matter, then V_0^* and other properties that vary smoothly with Λ^* can be determined by simply looking at the experimental curves of that property vs Λ^* for inert gases.

4 The state of quark cluster matter

4.1 Parameters

To apply the law of corresponding states to quark-cluster matter, we must first determine ε, σ and the mass m of each quark-cluster. m depends on the number of quarks N_q and the mass of each quark m_0 in one cluster. We give each quark a constituent mass and assume m_0 is one-third of the nuclear mass. N_q is left as a free parameter in this paper, and we set N_q=6 and N_q=18 for our calculations, corresponding to H-cluster and quark-α respectively.

Since no experimental attempt has been made to get the values of ε and σ, we try to constrain their values by the surface density ρ_s of quark-cluster stars. The temperature of quark stars can be approximated to be zero, and the pressure also reaches zero at the surface of stars. Given the value of V_0^*, we can calculate the surface density ρ_s (rest-mass density). It is obvious that ρ_s can be written as

$$\rho_s = N \cdot N_q m_0 / V_0, \tag{9}$$

and comparing Eq. (3) with Eq. (9) we can get

$$\rho_s = N_q m_0 / (V_0^* \sigma^3). \tag{10}$$

For certain values of N_q, ε and σ, we can calculate Λ^* of quark cluster matter by Eq. (5), and V_0^* can be found according to the fitted relation Eq. (8) of V_0^*-Λ^* curve, then we may determine ρ_s using Eq. (10). In Fig. 2, pairs of ε and σ that correspond to the same surface density ρ_s are plotted, respectively, for N_q=6 and N_q=18, where values of ρ_s are chosen to be once, twice and three times of nuclear matter density ρ_0. The lines of ε and σ, giving the same Λ^* with values 1, 2 and 3, are also drawn here for a further limit.

The surface density of quark stars is assumed to be in the range $1<\rho_s/\rho_0<3$. Quark-clusters could condensate to form solid state like classical particles, so the quantum effects may not be large enough for quark-cluster matter, then Λ^* should satisfy $\Lambda^*<2$. We select four points numbered A, B, C and D, representatively, to deduce the equation of state for quark-cluster matter and then the mass-radius relation of quark-cluster star. The values of ε, σ and the resulting ρ_s, Λ^* at points A to D are given in Table 1. They are also plotted in Fig. 1, corresponding to four different cases for quark-cluster matter, in which the equation of state will be calculated, respectively.

Table 1. Four groups of parameters selected for further calculation of quark-cluster, including N_q, ε, σ and the resulting Λ^*, ρ_s.

	N_q	ε/MeV	σ/fm	Λ^*	ρ_s/ρ_0
A	18	40	2.5	1.05	1.87
B	18	100	2.3	0.72	2.72
C	6	150	1.5	1.56	2.47
D	6	200	2.0	1.02	1.23

Our choice of parameters ε and σ comes from the following consideration. The depth of potential well for nuclear matter is about 100 MeV, so it may be reasonable that $\varepsilon = \mathcal{O}$ (100 MeV). The average inter-cluster distance d at the surface of quark-cluster stars is given by

$$d = \left[\frac{3\times 0.16(\rho_s/\rho_0)}{N_q}\right]^{-\frac{1}{3}}. \tag{11}$$

With $\rho_s/\rho_0 = 2$, we get $d = 1.84$ fm for $N_q = 6$ and $d = 2.66$ fm for $N_q = 18$ and then $\sigma = \mathcal{O}$ (1 fm) because it should have the same order of magnitude as d. It can be seen that the selected parameters are consistent with the above estimation.

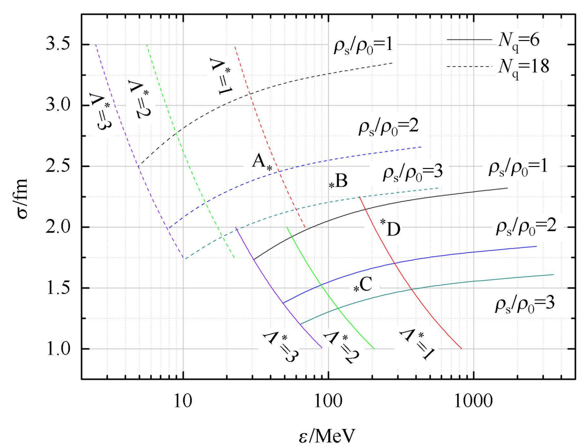

Fig. 2. Contour lines of surface density ρ_s and Λ^*, with solid lines representing N_q=6 and the dashed lines representing N_q=18, including ρ_s/ρ_0=1,2,3 and Λ^* = 1,2,3. Four cases A, B, C and D are denoted by stars.

4.2 The equation of state

Given ε and σ, we can deduce the state of quark-cluster matter by a corresponding-state approach, in the zero temperature case. If we know the experimental P^*-Λ^* curve at a certain V^* and zero temperature, we can find the value of P^* corresponding to Λ^* of quark cluster. According to Eq. (2) and the number density of quark-clusters $n=1/(V^*\sigma^3)$, the reduced quantities P^* and V^* can be converted to P and n, and then we can get the pressure at a certain number density of quark-clusters. Combining this with the relation between mass density ρ (rest-mass density plus interaction energy density) and number density n of quark-cluster matter, the equation of state can be derived.

To draw the P^*-Λ^* curve at different V^*, we need to know the relationship between P^* and V^* of some substances at zero temperature. For the lack of new data, we just use the data provided by de Boer in his subsequent article [38], where values of P^* and Λ^* were given corresponding to different values of V^* for various inert gases. Taking $V^*=0.88$ for instance, the P^*-Λ^* curves are shown in Fig. 3. The data points are almost in linear relation, which makes our interpolation reliable. The value of Λ^* at point A is about 1.05, and then we find $P^*\approx 25$ from the P^*-Λ^* curve. The corresponding P and n to the reduced quantities P^* and V^* are $P=1.0\times10^{35}$ dyn/cm^2, $n/n_0=2.7$ (n_0 is the number density of nucleons in nuclear matter). Thus, we get $P(n=2.7n_0)=1.0\times10^{35}$ dyn/cm^2 for quark-cluster matter in case A. By taking different values of V^*, the pressure P at different densities can be determined in case A. The same procedure is also applicable to the other three cases.

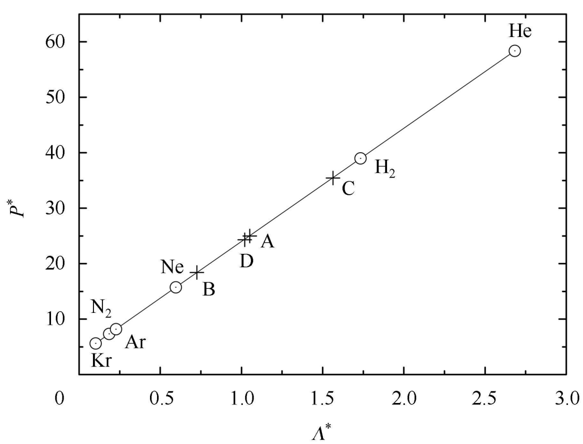

Fig. 3. Experimental data of P^* and Λ^* at zero temperature when $V^*=0.88$ are shown by dots [38], and the fitted curve is almost a straight line (solid line). The cases A, B, C, and D are denoted by crosses.

For each set of parameters, what we get is just a set of points in P-n diagram and not an analytic equation, and then we perform the curve fitting to get an approximate formula. The P-n relations derived from curve fitting are

$$P = (2.99\times10^{41}n^{5.63}-1.60\times10^{34})\ \text{dyn/cm}^2, \quad (12)$$

$$P = (1.99\times10^{41}n^{5.64}-7.63\times10^{34})\ \text{dyn/cm}^2, \quad (13)$$

$$P = (8.10\times10^{38}n^{5.24}-1.69\times10^{35})\ \text{dyn/cm}^2, \quad (14)$$

$$P = (6.69\times10^{40}n^{5.63}-1.63\times10^{35})\ \text{dyn/cm}^2 \quad (15)$$

for A, B, C and D, respectively, where n is in units of clusters/fm^3. Certainly, it is better to deduce the equation of state from a border range of densities, making the extrapolation more accurate. Nevertheless, lacking in experimental data of laboratory substances, we can only make such an approximation at this stage. It is worth mentioning that the approximation will not have much influence on the following calculations of the mass-radius curves.

According to $P=n^2\dfrac{\mathrm{d}E}{\mathrm{d}n}$, where E is the internal energy per cluster, we can get

$$E(n)-E(n_\mathrm{s})=\int_{n_\mathrm{s}}^{n}\frac{P(n)}{n^2}\mathrm{d}n, \quad (16)$$

where n_s is the number density of quark-clusters on the surface of stars. We may determine the value of $E(n_\mathrm{s})$ from a corresponding-state point of view, and then the relation between E and n can be derived by the above integral. Similar to V_0^*, $U_0^*=U_0/(N\varepsilon)$ can be approximated as a smooth function of Λ^*, where U_0 is the internal energy at zero temperature and zero pressure. From the data of laboratory substances [16], we derive a fitted formula for U_0^*,

$$U_0^*=-8.72+4.91\Lambda^*-0.71\Lambda^{*2}, \quad (17)$$

and $E(n_\mathrm{s})$ is thus

$$E(n_\mathrm{s})=U_0/N=U_0^*\varepsilon. \quad (18)$$

As both $P(n)$ and $E(n_\mathrm{s})$ are known, it is able to calculate $E(n)$ from Eq. (16). The results are plotted in Fig. 4, for four groups of parameters A to D, and we can see that the internal energy can be comparable to the rest-mass energy at some densities.

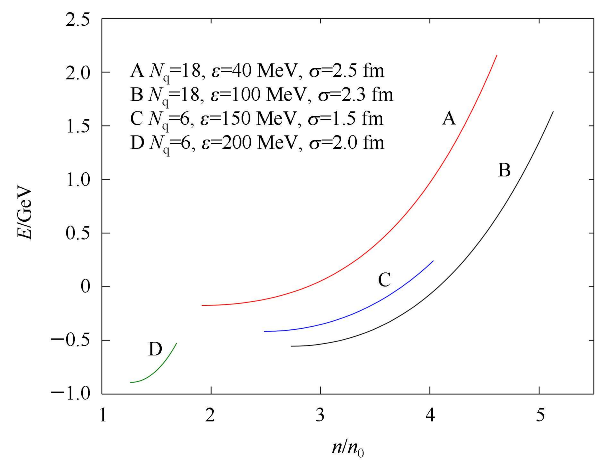

Fig. 4. The internal energy E per cluster for four groups of parameters A (red line), B (black line), C (blue line) and D (cyan line). The range of density n is from surface density n_s to the highest central density where the quark-cluster stars reaches the maximum mass.

The mass density ρ consists of rest-mass density and energy density,

$$\rho=n(N_\mathrm{q}m_0+E/c^2), \quad (19)$$

then, the equation of state for quark-cluster matter can be derived by combining P-n relation and Eq. (19), and we show the results in Fig. 5, for the four groups of parameters A to D.

4.3 Mass-radius relation

Considering perfect fluid case and the general relativity, the hydrostatic equilibrium in spherically symmetry is described by Tolman-Oppenheimer-Volkoff equation,

$$\frac{\mathrm{d}P}{\mathrm{d}r}=-\frac{Gm(r)\rho}{r^2}\frac{\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{m(r)c^2}\right)}{1-\frac{2Gm(r)}{rc^2}}, \tag{20}$$

where $m(r)=\int_0^r \rho\cdot 4\pi r'^2 \mathrm{d}r'$. In the above discussions, we have established the equations of state, from which we can make a further calculation of the mass-radius and mass-central density (rest-mass density) relations for quark-cluster stars. The results are shown in Fig. 6 for the four groups of parameters A to D. We can see that the maximum masses are higher than three times the solar mass $M_\odot$, which are reached with central density less than $5\rho_0$, for all the selected groups of parameters. As a comparison, we also plot the mass-radius curves for homogeneous spheres with the same surface density corresponding to each of the four cases. This shows that the gravity cannot be negligible only when the star is near the maximum mass, which could be the result of the strong self-bounding of quark-cluster stars.

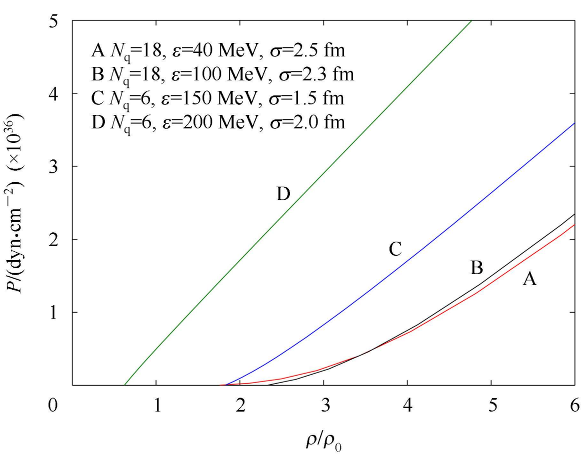

Fig. 5. Equations of states for the same four groups of parameters as in Fig. 4.

Conventional quark matter is characterized by the soft equation of state, and the emergence of quark matter inside compact stars is usually thought to be a reason for lowering their maximum mass. However, the quark-cluster matter could have a stiff equation of state due to the strong coupling. Although the corresponding-state approach is just a phenomenological and empirical method, we could still apply it to study the state of quark-cluster matter and then understand the observations of pulsar-like compact stars. The observed high-mass pulsar PSR J1614-2230 with mass $(1.97\pm0.04)M_\odot$ [26] has received a lot of attention, and we can see that the quark-cluster stars in our present model could be consistent with this observation. Moreover, our model of quark-cluster stars could not be ruled out even if the mass of the pulsar J1748-2021B ($2.74M_\odot$) in a galactic cluster is confirmed in the future.

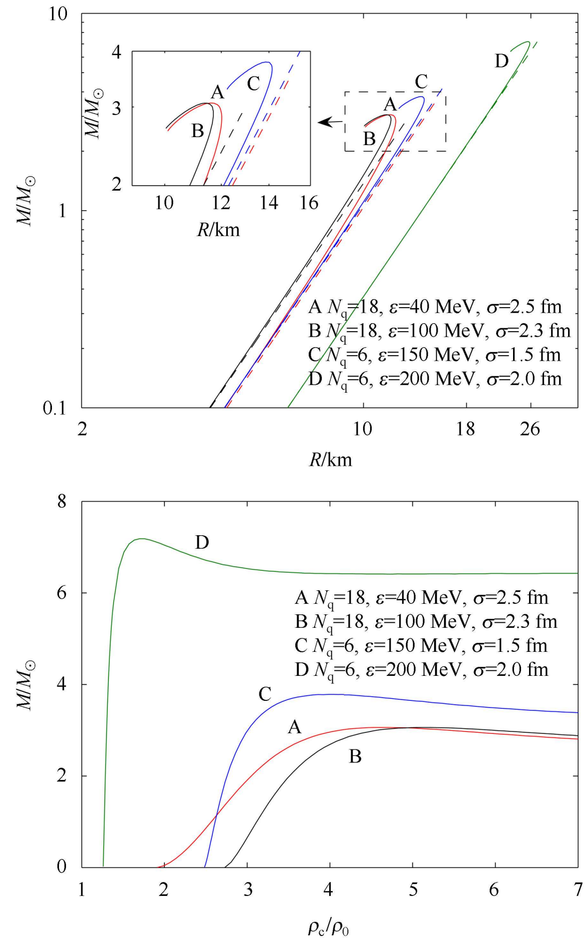

Fig. 6. The mass-radius and mass-central density (rest-mass density) curves, and different parameters are distinguished by their colors as the same in Fig. 4, and the corresponding dash lines represent $M=\rho_s\cdot 4\pi R^3/3$.

5 Melting heat

If the kinetic energy of quark clusters is much lower than the inter-cluster potential energy, then they may form a solid state, which is meaningful for the thermal X-ray behaviors of compact stars [39]. We will estimate the latent heat of phase transition of quark-cluster stars from liquid to solid state by the corresponding-state approach.

We calculate the ratio of melting heat per particle H and ε for some ordinary substances [40], and find that there is also a good relation between $H^*=H/\varepsilon$ and Λ^*, as shown in Fig. 7. The fitted formula for H^* and Λ^* is

$$H^*=1.18\mathrm{e}^{-((\Lambda^*-0.12)/1.60)^2}. \tag{21}$$

For given N_q, ε and σ, we can determine Λ^* first and then get the value of H^* from Eq. (21), thus the melting heat $H=H^*\varepsilon$ can be derived.

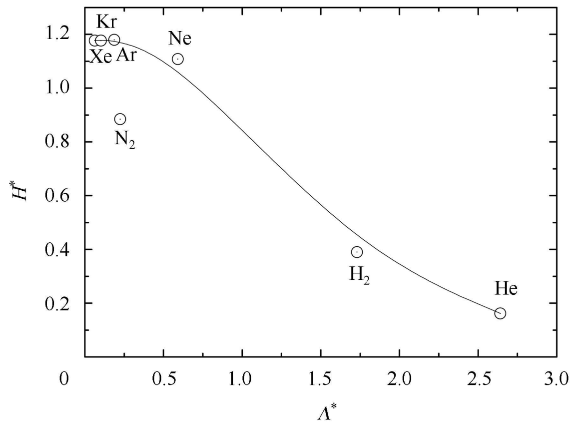

Fig. 7. Data points: experimental data of the reduced melting heat $H^*=H/\varepsilon$. Solid line: fitted curve of Eq. (21).

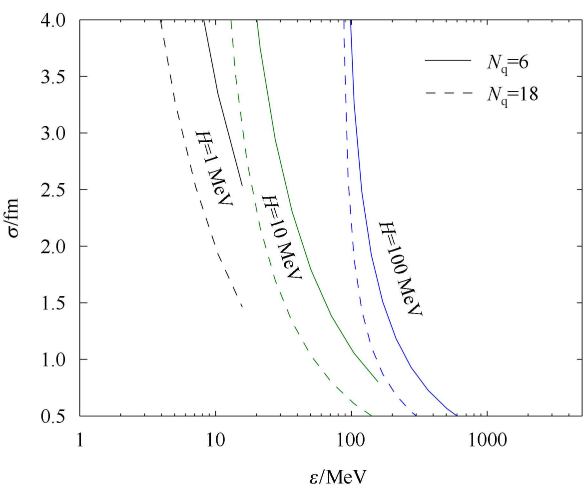

Fig. 8. ε and σ which determine the same melting heat of each cluster. H is chosen to be 1 (black line), 10 (cyan line) and 100 MeV (blue line), in two cases $N_\mathrm{q}=6$ (solid line) and $N_\mathrm{q}=18$ (dashed line).

In Fig. 8 pairs of ε and σ that determine the same melting heat are plotted, where values of H are chosen to be 1, 10 and 100 MeV, in two cases $N_\mathrm{q}=6$ and $N_\mathrm{q}=18$. The solidification of quark-cluster stars has been suggested to be relevant to the plateau of γ-ray burst [41], and it is found that if the energy released by each quark-cluster in the liquid to solid phase transition is larger than 1 MeV, the total released energy could produce the plateau. We can see that under a wide range of parameters in our model, the latent heat could be sufficient for this method of understanding the plateau of γ-ray burst.

6 Conclusions and discussions

In cold quark matter at realistic baryon densities of pulsar-like compact stars, the interaction between quarks would be so strong that they could condensate in position space, forming quark-clusters, and the stars are then called quark-cluster stars if the dominant component inside is quark-clusters. We propose that the interaction between quark-clusters could be analogous to that between inert gas atoms described by the Lennard-Jones potential, and apply the corresponding-state approach to derive the equation of state. As a phenomenological and empirical method, the corresponding-state approach can avoid detailed assumptions of quark-cluster matter as well as computation of the many-body effects, and we only need to be concerned about the differences between substances. Along with these advantages, there are large uncertainties in our results, which come from the Lennard-Jones approximation and lack of experimental data source. Even so, the corresponding-state approach could give us qualitative information about the properties of quark-cluster matter, while the exact approach under QCD calculations seems to be very difficult and even impossible now due to significant non-perturbative effects. Summarily, our two-parameter (ε and σ) empirical approach makes it possible to establish a model that could be tested by observations.

The equation of state that we have derived by the corresponding-state approach could be stiff enough to make a star stable, even if its mass is higher than $2M_\odot$, under reasonable parameters. This result is consistent with the recent observation of a high-mass pulsar, thus the emergence of such kind of exotic matter, "quark-cluster matter", could not be ruled out. The observations of pulsars with a higher mass (e.g. $>3M_\odot$) would support our quark-cluster star model and give further constraints to the parameters. Moreover, the latent heat released by the solidification of newly born quark-cluster stars could help us to understand the formation of the plateau of γ-ray burst.

Certainly, whether quark-cluster matter could exist at supra-nuclear densities, and what quark-clusters are composed of, as well as how to describe their interaction are still open questions. On the other hand, the nature of pulsar-like compact stars is still essentially related to significant non-perturbative effects of QCD, and we hope that future astrophysical observations, complementary to the terrestrial experiments, could give us hints to the answers of all of these problems.

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