Astrophysical *S*-factor of the ^{12}C(α, γ) ^{16}O reaction at solar energies

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Abstract: The astrophysical *S*-factor of the ^{4}He+^{12}C radiative capture is calculated in the potential model at the energy range 0.1–2.0 MeV. Radiative capture ^{12}C(α, γ) ^{16}O is extremely relevant for the fate of massive stars and determines if the remnant of a supernova explosion becomes a black hole or a neutron star. Because this reaction occurs at low energies, the experimental measurements are very difficult and perhaps impossible. In this paper, radiative capture of the ^{12}C(α, γ) ^{16}O reaction at very low energies is taken as a case study. In comparison with other theoretical methods and available experimental data, good agreement is achieved for the astrophysical *S*-factor of this process.

Key words: radiative capture, the astrophysical *S*-factor, potential model

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1 Introduction

During a star's hydrogen-burning phase transition, the star is in the helium-burning phase. The thermal energy at 1.5×10^8 K is sufficient for the fusion of two helium nuclei into the unstable ^{8}Be nucleus. If the conditions are suitable, ^{8}Be nuclei are converted to ^{12}C nuclei by the capture of α-particle radiation. The process of forming the ^{12}C nucleus is called the triple-α process [1, 2].

$$^4\text{He}+^4\text{He}+^4\text{He}\leftrightarrow^8\text{Be}+^4\text{He}\rightarrow^{12}\text{C}+\gamma. \quad (1)$$

Therefore, conditions at 1.5×10^8 K are sufficient for α-particle capture by ^{12}C nuclei and ^{16}O nuclei are produced:

$$^{12}\text{C}+^4\text{He}\rightarrow^{16}\text{O}+\gamma. \quad (2)$$

These reactions are important because carbon and oxygen are the most abundant elements in the universe produced from the burning of helium, and heavier elements are often formed from these two elements. Our understanding of the reaction ^{12}C(α, γ) ^{16}O is therefore helpful to better understand the evolution of condensed stars, such as neutron stars and black holes. For example, a large cross section for this reaction leads to the production of heavier elements, while a small cross section can lead to the reverse situation and production of lighter elements. Thus, our main purpose is to calculate the cross section of this process.

The ^{12}C(α, γ) ^{16}O radiative capture process plays a major role in stars' fuel when they collapse. There is no accurate and complete information about these reactions, because the cross section of this reaction is low and impossible to produce in the laboratory directly at low energies [3–7].

In the past few decades, the yield of capture rays has been studied for E_α up to 42 MeV [8]. The cross section of the ^{12}C(α, γ) ^{16}O capture process has been obtained by fitting the measured cross sections and extrapolating to low energies using standard *R*-matrix, Hybrid *R*-matrix and *K*-matrix procedures. The influence of vacuum polarization effects on sub barrier fusion is also evaluated in Ref. [9], and the relevance of Coulomb dissociation of ^{16}O into ^{12}C+α is studied in Ref. [10–12]. Calculations to test the sensitivity of stellar nucleosynthesis to the level of ^{12}C at 7.74 MeV are described in Ref. [13].

At higher energies the E_2 cross section shows resonances at E_x=13.2, 15.9, 16.5, 18.3, 20.0, and 26.5 MeV. Some E_2 strength is also observed for E_x=14 to 15.5 and 20.5 to 23 MeV. In the range E_α=7 to 27.5 MeV with T=0, E_2 strength is 17 times the sum-rule value. It appears from this and other experiments that the E_2 centroid is at $E_x\sim$15 MeV, with a 15 MeV spread. Structures are observed in the yield of γ-rays from the decay to ^{16}O*(14.8± 0.1) for E_x=34–39 MeV. It is suggested that these correspond to a giant quadrupole excitation with $J^\pi=8^+$ built on the 6^+_1 state at E_x=14.815 MeV [8].

Recently, Dubovichenko et al. have calculated the astrophysical *S*-factor of ^{4}He+^{12}C radiative capture using the cluster model at the energy range 0.1–4.0 MeV. They show that the approach used, which takes into account E_2 transitions only, gives a good description of the new

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experimental data for adjusted parameters of potentials and leads to the value $S(300)$=16.0 keV·b [14, 15]. More recently, Bertulani presented a computer program aiming at the calculation of bound and continuum state observables for a nuclear system, such as the reduced transition probabilities, phase-shifts, photo-disintegration cross sections, radiative capture cross sections, and astrophysical S-factors [16–18]. The code is based on a potential model type and can be used to calculate nuclear reaction rates in numerous astrophysical reactions. In order to calculate the direct capture cross sections, one needs to solve the many-body problem for the bound and continuum states of relevance for the capture process. A model based on potential can be applied to obtain single-particle energies and wave functions. In numerous situations, this solution is good enough to obtain cross section results which fit the experimental data.

This paper is organized as follows: a brief review of multipole matrix elements and reduced transition probabilities is given in Section 2, along with definitions of the relevant formalism and parameters, electric and magnetic multipole matrix elements and reduced transition probabilities. In Section 3, the findings of the model with asymptotic wave functions are corroborated in more realistic calculations using wave function generated from the Woods-Saxon potentials and experimental data, in Section 3. A summary and conclusions follow in Section 4.

2 Brief review of theoretical framework

The computer code RADCAP calculates various quantities of interesting radiative capture reactions. The bound state wave functions of final nuclei are given by $\Psi_{JM}(\boldsymbol{r})$ and the ground-state wave function is normalized so that $\int \mathrm{d}^3r\ |\Psi_{JM}(\boldsymbol{r})|^2=1$.

The wave functions are calculated using the central ($V_0(r)$), spin-orbit ($V_\mathrm{S}(r)$) and the Coulomb potential ($V_\mathrm{C}(r)$) potentials. The potentials $V_0(r)$ and $V_\mathrm{S}(r)$ are given by

$$V_0(r) = V_0\ f_0(r),\ V_\mathrm{S}(r)=-\ V_{\mathrm{S}0}\left(\frac{\hbar}{m_\pi c}\right)^2\ \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}f_\mathrm{S}(r),$$
$$f_\mathrm{i}(r) = \left[1+\exp\left(\frac{r-R_\mathrm{i}}{a_\mathrm{i}}\right)\right]^{-1}, \quad (3)$$

where V_0, $V_{\mathrm{S}0}$, R_0, a_0, $R_{\mathrm{S}0}$, and $a_{\mathrm{S}0}$ are adjusted so that the ground state energy E_B or the energy of an excited state, is reproduced.

The radial Schrödinger equation for calculating the bound state is given by solving

$$-\frac{\hbar^2}{2m_{\mathrm{ab}}}\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}-\frac{l(l+1)}{r^2}\right]u^J_{lj}(r)+[V_0(r)+V_\mathrm{C}(r) +\langle \boldsymbol{s}\cdot\boldsymbol{l}\rangle\ V_{\mathrm{S}0}(r)]u^J_{lj}(r)=E_\mathrm{i}u^J_{lj}(r), \quad (4)$$

with $\langle \boldsymbol{s}\cdot\boldsymbol{l}\rangle=[j(j+1)-l(l+1)-s(s+1)]/2$.

The electric and magnetic dipole transitions are given by introducing the following operators [19]

$$\mathcal{O}_{E\lambda\mu}=e_\lambda r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}}),$$
$$\mathcal{O}_{M1\mu}=\sqrt{\frac{3}{4\pi}}\mu_\mathrm{N}\ \left[e_M l_\mu+\sum_{\mathrm{i=a,b}}g_\mathrm{i}(s_\mathrm{i})_\mu\right], \quad (5)$$

where

$$e_\lambda=Z_\mathrm{b}e\left(-\frac{m_\mathrm{a}}{m_\mathrm{c}}\right)^\lambda+Z_\mathrm{a}e\left(\frac{m_\mathrm{b}}{m_\mathrm{c}}\right)^\lambda$$

and

$$e_M=\left(\frac{m_\mathrm{a}^2Z_\mathrm{a}}{m_\mathrm{c}^2}+\frac{m_\mathrm{b}^2Z_\mathrm{b}}{m_\mathrm{c}^2}\right)$$

are the effective electric and magnetic charges, respectively. l_μ and s_μ are the spherical components of order μ ($\mu=-1,0,1$) of the orbital and spin angular momentum ($\boldsymbol{l}=-\mathrm{i}\boldsymbol{r}\times\nabla$, and $\boldsymbol{s}=\sigma/2$) and g_i are the gyromagnetic factors of particles a and b. μ_N is the nuclear magneton.

The matrix element for the transition $J_0M_0\longrightarrow JM$ is given by [19, 20]

$$\langle JM|\mathcal{O}_{E\lambda\mu}|J_0M_0\rangle = \langle J_0M_0\lambda\mu|JM\rangle\frac{\langle J\|\mathcal{O}_{E\lambda}\|J_0\rangle}{\sqrt{2J+1}},$$
$$\langle J\|\mathcal{O}_{E\lambda}\|J_0\rangle = (-1)^{j+I_\mathrm{a}+J_0+\lambda}[(2J+1)(2J_0+1)]^{1/2} \times\left\{\begin{matrix} j & J & I_\mathrm{a}\\ J_0 & j_0 & \lambda\end{matrix}\right\}\langle lj\|\mathcal{O}_{E\lambda}\|l_0j_0\rangle_J, \quad (6)$$

where the subscript J is a reminder that the matrix element is spin dependent. For $l_0+l+\lambda$=odd, the reduced matrix element is null and for $l_0+l+\lambda$=even, is given by

$$\langle lj\|\mathcal{O}_{E\lambda}\|l_0j_0\rangle_J = \frac{e_\lambda}{\sqrt{4\pi}}(-1)^{l_0+l+j_0-j}\frac{\hat{\lambda}\hat{j}_0}{\hat{j}}\left\langle j_0\frac{1}{2}\lambda0|j\frac{1}{2}\right\rangle \times\int_0^\infty \mathrm{d}rr^\lambda\ u^J_{lj}(r)u^{J_0}_{l_0j_0}(r). \quad (7)$$

At very low energies, the transitions will be much smaller than the electric transitions. The M_1 contribution has to be considered in the cross sections for neutron photo-dissociation or radiative capture. The M_1 transitions, in the case of sharp resonances, play a role in the $J=1^+$ state in ^{8}B at $E_\mathrm{R}=630$ keV above the proton separation threshold [21, 22].

For the reduced matrix elements of the M_1 transition, the magnetic dipole matrix element is zero for $l\neq l_0$, and for $l=l_0$, it is given by [23]

$$\langle lj\|\mathcal{O}_{M1}\|l_0j_0\rangle_J = (-1)^{j+I_a+J_0+1}\sqrt{\frac{3}{4\pi}}\,\widehat{J}\widehat{J}_0\begin{Bmatrix} j & J & I_a \\ J_0 & j_0 & 1\end{Bmatrix}\mu_N$$
$$\times\left\{\frac{1}{\widetilde{l}_0}e_M\left[\frac{2\widetilde{j}_0}{\widetilde{l}_0}(l_0\delta_{j_0,\ l_0+1/2}+(l_0+1)\delta_{j_0,\ l_0-1/2})+(-1)^{l_0+1/2-j}\frac{\widehat{j}_0}{\sqrt{2}}\delta_{j_0,\ l_0\pm1/2}\delta_{j,\ l_0\mp1/2}\right]\right.$$
$$+g_N\frac{1}{\widehat{l}_0^2}\left[(-1)^{l_0+1/2-j_0}\widetilde{j}_0\delta_{j,\ j_0}-(-1)^{l_0+1/2-j}\frac{\widehat{j}_0}{\sqrt{2}}\delta_{j_0,l_0\pm1/2}\delta_{j,\ l_0\mp1/2}\right]$$
$$\left.+g_a(-1)^{I_a+j_0+J+1}\widehat{J}_0\widehat{J}\widehat{I}_a\widetilde{I}_a\begin{Bmatrix} I_a & J & j_0 \\ J_0 & I_a & 1\end{Bmatrix}\right\}\int_0^\infty \mathrm{d}r\ u^J_{lj}(r)\ u^{J_0}_{l_0j_0}(r). \quad (8)$$

We use the notation $\hat{k}=\sqrt{2k+1}$, and $\widetilde{k}=\sqrt{k(k+1)}$ and g_N=5.586(−3.826) for the proton (neutron). $\mu_a=g_a\mu_N$ is also the magnetic moment of the core nucleus.

The reduced transition probability $\mathrm{d}B((E,B)\lambda)/\mathrm{d}E$ of the nucleus, i into j+k, contains the information on the structure in the initial ground state and the interaction in the final continuum state. The reduced transition probability for a specific electromagnetic transition $(E,B)\lambda$ to a final state with momentum $\hbar k$ in the continuum is given by [16]

$$\frac{\mathrm{d}B}{\mathrm{d}E}((E,B)\lambda,J_is\to kJ_fs)$$
$$=\frac{2J_f+1}{2J_i+1}\sum_{j_fl_f}\left|\sum_{j_il_ij_c}\langle kJ_fj_fl_fsj_c||\mathcal{M}((E,B)\lambda)||J_ij_il_isj_c\rangle\right|^2$$
$$\times\frac{\mu k}{(2\pi)^3\hbar^2}. \quad (9)$$

The electric excitations (E) with multipole operator are given by

$$\mathcal{M}(E\lambda\mu)=Z_{\text{eff}}^{(\lambda)}er^\lambda Y_{\lambda\mu}(\hat{r}), \quad (10)$$

where

$$Z_{\text{eff}}^{(\lambda)}=Z_b\left(\frac{m_c}{m_b+m_c}\right)^\lambda+Z_c\left(-\frac{m_b}{m_b+m_c}\right)^\lambda$$

is the effective charge number.

For proton radiative capture the effective charge numbers for E_1 and E_2 have to consider all contributions to the cross sections from Coulomb breakup, photo dissociation and radiative capture. In the case of neutron radiative capture, the E_1 transition dominates the low-lying electromagnetic strength and the E_2 contribution can be neglected.

The initial and final state are given by the following wave functions [16]

$$\Phi_i(\boldsymbol{r}) = \langle\boldsymbol{r}|J_ij_il_isj_c\rangle=\frac{1}{r}\sum_{m_im_c}(j_im_ij_cm_c|J_iM_i)f^{j_c}_{J_ij_il_i}(r)$$
$$\times\mathcal{Y}^{l_is}_{j_im_i}(\hat{\boldsymbol{r}})\phi_{j_cm_c}, \quad (11)$$
$$\Phi_f(\boldsymbol{r}) = \langle\boldsymbol{r}|\boldsymbol{k}J_fj_fl_fsj_c\rangle$$
$$=\frac{4\pi}{kr}\sum_{m_fm_c}(j_fm_fj_cm_c|J_fM_f)g^{j_c}_{J_fj_fl_f}(r)i^{l_f}Y^*_{l_fm_f}(\hat{k})$$
$$\times\mathcal{Y}^{l_fs}_{j_fm_f}(\hat{r})\phi_{j_cm_c},$$

where $f^{j_c}_{J_ij_il_i}(r)$ and $g^{j_c}_{J_fj_fl_f}(r)$ are the radial wave functions and $\phi_{j_cm_c}$ is the wave function of the core. The spinor spherical harmonics are denoted by $\mathcal{Y}^{ls}_{jm}=\sum_{m_lm_s}(lm_lsm_s|jm)Y_{lm}(\hat{r})\chi_{sm_s}$.

The reduced matrix element in (9) can be expressed as [16]

$$\langle kJ_fj_fl_fsj_c||\mathcal{M}(E\lambda)||J_ij_il_isj_c\rangle$$
$$=\frac{4\pi Z^{(\lambda)}_{\text{eff}}e}{k}D^{J_fj_fl_f}_{J_ij_il_i}(\lambda sj_c)(-i)^{l_f}I^{J_fj_fl_f}_{J_ij_il_i}(\lambda j_c), \quad (12)$$

where the angular momentum coupling coefficient $D^{J_fj_fl_f}_{J_ij_il_i}(\lambda sj_c)$ and the radial integral $I^{J_fj_fl_f}_{J_ij_il_i}(\lambda j_c)$ are given by

$$D^{J_fj_fl_f}_{J_ij_il_i}(\lambda sj_c) = (-1)^{s+j_i+l_f+\lambda}(-1)^{j_c+J_i+j_f+\lambda}(l_i0\lambda0|l_f0)$$
$$\times\sqrt{2j_i+1}\sqrt{2l_i+1}\sqrt{2J_i+1}\sqrt{2j_f+1}$$
$$\times\sqrt{\frac{2\lambda+1}{4\pi}}\begin{Bmatrix} l_i & s & j_i \\ j_f & \lambda & l_f\end{Bmatrix}\begin{Bmatrix} j_i & j_c & J_i \\ J_f & \lambda & j_f\end{Bmatrix},$$
$$I^{J_fj_fl_f}_{J_ij_il_i}(\lambda j_c) = \int_0^\infty \mathrm{d}rg^{j_c*}_{J_fj_fl_f}(r)r^\lambda f^{j_c}_{J_ij_il_i}(r), \quad (13)$$

with the asymptotic radial wave functions for the bound state

$$f^{j_c}_{J_ij_il_i}(r)\to C^{j_c}_{J_ij_il_i}W_{-\eta_i,l_i+1/2}(2qr), \quad (14)$$

Table 1. The set of Woods-Saxon potential parameters, applied for calculation.

V_0/MeV	R_0/fm	a_0/fm	V_{S0}/MeV	R_{S0}/fm	a_{S0}/fm	R_C/fm
−51.8	2.41	0.644	39.54	2.291	0.644	2.41

and the asymptotic form of the continuum state for the scattering state

$$g^{j_c}_{J_f j_f l_f}(r)\rightarrow\exp\left[\mathrm{i}(\sigma_{l_f}+\delta^{j_c}_{J_f j_f l_f})\right]\times\left[\cos(\delta^{j_c}_{J_f j_f l_f})F_{l_f}(\eta_f;kr)\right.$$
$$\left.+\sin(\delta^{j_c}_{J_f j_f l_f})G_{l_f}(\eta_f;kr)\right], \tag{15}$$

where $C^{j_c}_{J_i j_i l_i}$, $W_{-\eta_i,l_i+1/2}$, F_{l_f}, G_{l_f} and $\eta_f=\eta_i/x$ are the asymptotic normalization coefficient, Whittaker function, regular Coulomb wave functions, irregular Coulomb wave functions and the Sommerfeld parameter, respectively [24].

The cross-section for different particles without spin is defined as follows:

$$\sigma^{\mathrm{cap}}_{(E,B)l}(E)=\frac{\pi\hbar^2}{2\mu\varepsilon}(2l+1)T_{(E,B)l}, \tag{16}$$

where $T_{(E,B)l}$ is the transition probability. Finally, the total cross section for an arbitrary transition is:

$$\sigma^{\mathrm{cap}}(E)=\sum_l(\sigma^{\mathrm{cap}}_{El}(E)+\sigma^{\mathrm{cap}}_{Ml}(E)). \tag{17}$$

The total cross section for an arbitrary transition can also be written:

$$\sigma^{\mathrm{cap}}(E)=S(E)\frac{1}{E}\mathrm{e}^{-2\pi\eta}. \tag{18}$$

In this equation the $S(E)$ astrophysical factor and

$$\eta=\frac{Z_c Z_\alpha e^2}{\hbar}\sqrt{\frac{\mu}{2E}}$$

are Sommerfeld parameters. We use the astrophysical S-factor because it is a well-defined function which changes little and is easy to analyze.

3 Results and conclusions

The potential model and the RADCAP computer code are proper theoretical frameworks to describe the ground state properties of ^{16}O for the reaction ^{12}C(α, γ)^{16}O. To evaluate the radiation capture reaction ^{12}C(α, γ)^{16}O, the Schrödinger equation using the Woods-Saxon potential and with solved specific parameters and bound continuum states of the reaction is obtained with very good accuracy. Using the formulation from Section 2, the astrophysical S-factor is then calculated for transition E_2.

The set of Woods-Saxon potential parameters, applied for calculation are given in Table 1. The results for the astrophysical S-factor of ^{4}He+^{12}C radiative capture process is presented in Fig. 1, along with the experimental data [25–28], at solar energies 0.1–2 MeV.

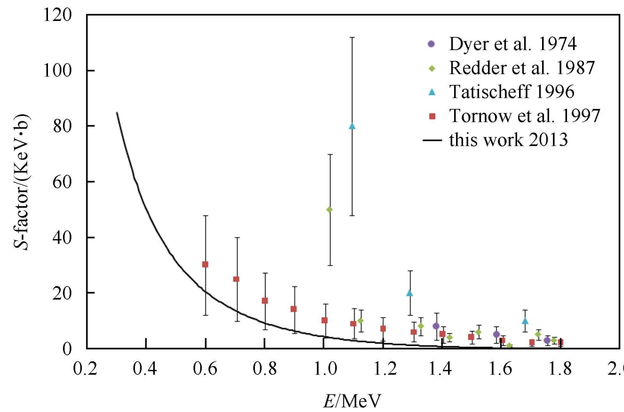

Fig. 1. The astrophysical S-factor for the ^{12}C(α, γ)^{16}O reaction.

The S-factor (E_{cm}=300 keV), at 300 keV transition energy, is found to be 84.97 keV·b, which is in reasonable agreement with some evaluated values for experimental data, shown in Table 2. Here, no significant difference has been seen between the results obtained with the present model and with some evaluated values for experimental data in other papers which use the potential model. In the other theoretical approach using the cluster model, the astrophysical S-factor of ^{4}He+^{12}C has been calculated to be S(300)=16.0 keV·b [15].

Table 2. Some evaluated astrophysical S-factor (E_{cm}=300 keV) values, in keV·b for experimental data for the ^{12}C(α, γ)^{16}O reaction.

reference	year	$S(E_2)$
Schürmann et al. [29]	2012	73.4
Oulebsir et al. [30]	2012	50±19
Hammer et al. [31]	2005	81±22
Kunz et al. [32]	2001	85±30
Redder et al. [26]	1987	80±25
this work	2013	84.97

4 Summary and conclusions

The radioactive capture process ^{12}C(α, γ)^{16}O is one of the most important reactions in nuclear astrophysics. The reaction cross section determines the relative abundance of most elements in red giant stars, neutron stars and black holes. In general, the electric dipole radiation E_1 is much stronger than the electric quadrupole radiation E_2. Electric dipole transitions between states with the same isospin are forbidden in the first order.

State 1^+ and 0^+ ground state ^{16}O nuclei have isospin T=0, the electric dipole radiations between the two levels are not at the first order; the electric dipole radiation is second order, and the electric dipole radiation is the same order as the electric quadrupole radiation. Therefore, we must consider the effects of both radiations. In comparison with other theoretical methods and available experimental data, good agreement is achieved for the astrophysical S-factor for this process.

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