Calculations for shortening the bunch length in storage rings using a harmonic cavity *

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Abstract: Using the Hefei Light Source phase Ⅱ project (HLS-Ⅱ) as an example, a theoretical analysis of shortening the bunch lengths using a higher harmonic cavity (HHC) is given. The threshold voltage of an active HHC and the threshold tuning angle of a passive HHC are first analysed. The optimum tuning angle for the constant detuning scenario and the optimum harmonic voltage for the constant voltage scenario are presented. The calculated results show that the reduced bunch length is about half that of the nominal bunch. The bunch lengths vary from 11 mm at 0.1 A to 7 mm at 0.4 A for the constant detuning scenario, while the bunch lengths are around 7 mm over the beam current range for the constant voltage scenario. In addition, the synchrotron frequency spread is increased. It indicates that HHC may be used to reduce the bunch length and increase the Landau damping of synchrotron oscillations in a storage ring.

Key words: shortening bunch, harmonic cavity, bunch length, operating scenario, synchrotron frequency spread

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1 Introduction

Higher harmoic cavities (HHC) can be employed in storage rings to control the bunch length [1]. To improve the beam lifetime, HHCs have been used to increase bunch length in many storage rings around the world [2–6]. HHCs are also an attractive option for shortening bunches. For example, they have been used in high luminosity storage rings to avoid luminosity reductions [1, 7]. In addition, storage ring Free Electron Laser (FEL) and time-resolved synchrotron radiation experiments also use HHCs to achieve the required bunch length [8].

The slope of the RF accelerating voltage can be steepened by the HHC, which increases the longitudinal restoring force, consequently shortening the bunch. Compared to the methods for decreasing the momentum acceptance or increasing the main RF voltage, using an HHC can avoid changing the storage ring lattice and requiring a higher power source, while the synchrotron frequency spread is increased and thus the coupled bunch instabilities may be suppressed [7].

Based on the Hefei Light Source Ⅱ(HLS-Ⅱ) parameters, for which the HHC [9] was originally designed to lengthen bunches with the purpose of improving the beam lifetime, here the theoretical calculations for using active and passive HHCs to shorten the bunches are given in detail. Firstly, the double RF system function, the potential function and the normalized charge density are given and reviewed in the presence of an active HHC. The phase diagram is depicted to determine the threshold of the relative harmonic voltage of the active cavity. Then the detailed study of how to tune a passive HHC at HLS-Ⅱ is given. The thresholds of the tuning angle will be determined at typical beam current to avoid affecting the RF acceptance too much. Furthermore, the optimum tuning angle for the constant detuning scenario and the optimum harmonic voltage for the constant voltage scenario are presented. We then investigate the effects of adjustments in the HHC constant voltage and constant detuning scenarios on the bunch length. Finally, the synchrotron frequency spreads are calculated for a double RF system with passive HHC or active HHC, before drawing some conclusions.

2 Harmonic cavity

2.1 Active harmonic cavity

The combined voltage of the main cavity and active harmonic cavity is written as [10]

$$V(\tau)=V_{\rm rf}[\sin(\omega_{\rm rf}\tau+\phi_{\rm s})+K\sin(n\omega_{\rm rf}\tau+n\phi_{\rm h})], \quad (1)$$

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where K is the relative harmonic voltage to the main RF voltage, ϕ_s is the nominal synchronous phase ($0<\phi_s<\pi$), and ϕ_h is the relative harmonic phase. The slope of the combined voltage seen by the synchronous particle is given by

$$V'(\tau)|_{\tau=0}=V_{rf}\omega_{rf}[\cos(\phi_s)+Kn\cos(n\phi_h)]. \tag{2}$$

To improve the beam lifetime, the HHC is added to flatten the voltage slope at the bunch center to decrease the peak charge density. It is therefore supposed that the steeper the voltage slope, the shorter the bunches that can be obtained. For a given K, the shortest bunch can be obtained when

$$n\phi_h=\begin{cases}\pi & \text{if } \dfrac{\pi}{2}<\phi_s<\pi\\ 0 & \text{if } 0<\phi_s<\dfrac{\pi}{2}\end{cases}. \tag{3}$$

This is called the neutral-phase operation [7]. The advantage of the neutral-phase operation is that the RF voltage and nominal synchronous phase will not change when the HHC is added.

According to Eq. (1), above the transition energy the phase stability will be located in the beta quadrant while below the transition energy the phase stability will be located in the first quadrant. The energy of HLS-II is higher than the transition energy. The RF potential formed by the total voltages is given by

$$\begin{aligned}\Phi(\tau) &= \frac{\alpha}{E_0T_0}\int_0^{\tau}[eV(\tau)-U_0]\mathrm{d}\tau\\ &= \frac{\alpha V_{rf}}{E_0T_0\omega_{rf}}\Big[-\cos(\omega_{rf}\tau+\phi_s)+\frac{K}{n}\cos(n\omega_{rf}\tau)\\ &\quad -\omega_{rf}\sin(\phi_s)\tau+\cos(\phi_s)-\frac{K}{n}\Big],\end{aligned} \tag{4}$$

where α is the slip phase factor, E_0 is the nominal energy, T_0 is the revolution period and U_0 is the energy loss per turn. The neutral-phase operation supplies the most linear voltage over the range of bunch length. The restoring force of the combined voltage is approximately linear and for a Gaussian bunch, the resulting longitudinal distribution is also Gaussian. The bunch length is confined by the RF potential. The longitudinal density distribution of the bunch is given by [11]

$$\rho(z)=\rho_0\mathrm{e}^{-\frac{\Phi(z)}{\alpha^2\sigma_\varepsilon^2}}, \tag{5}$$

where σ_ε is the energy spread and ρ_0 is the normalization constant. The reduced bunch length is give by [11]:

$$\sigma_z=\sqrt{\frac{\int\rho_0\exp\left(-\dfrac{\Phi}{\alpha^2\sigma_\varepsilon^2}\right)z^2\mathrm{d}z}{\int\rho_0\exp\left(-\dfrac{\Phi}{\alpha^2\sigma_\varepsilon^2}\right)\mathrm{d}z}}. \tag{6}$$

Since the active harmonic cavity is fed by an external RF source, the relative harmonic voltage can be considered to be a variable. Fig. 1 presents the potential wells and the bunch density distributions of a single RF system (K=0) and double RF system. A higher relative harmonic voltage K will result in a narrower bottom to the potential well, consequently leading to a more concentrated distribution of the charge density and thus making the bunch shorter. As far as the RF acceptance is concerned, however, the relative voltage K cannot be increased indefinitely.

Figure 2 shows the separatrices of the longitudinal phase space for different relative harmonic voltage K. The threshold of the relative harmonic voltage is found to be 0.86 by scanning K from 0 to 1.0. When the relative harmonic voltage is greater than 0.86, the RF acceptance of the double RF system is sharply reduced. Fig. 3 clearly shows that the bunch length is reduced by means of increasing the relative harmonic voltage. So, 0.86 can be considered as the optimum relative harmonic voltage at the typical beam current of 0.3 A.

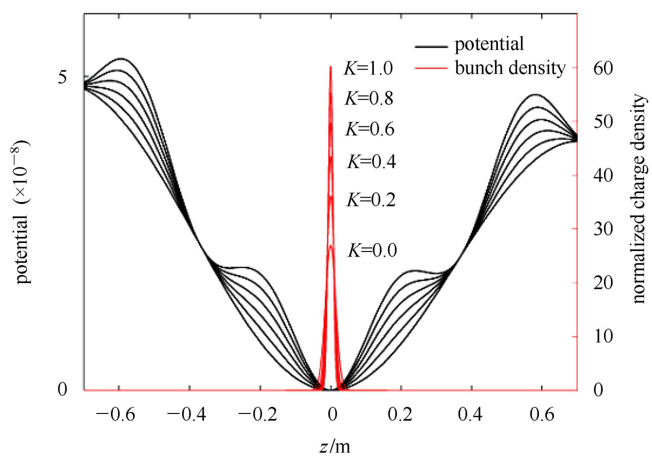

Fig. 1. Potentials and density distributions for several values of K.

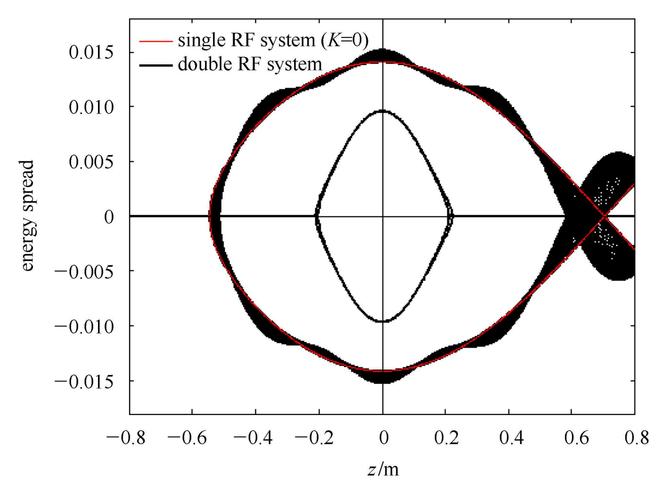

Fig. 2. RF buckets for several values of K.

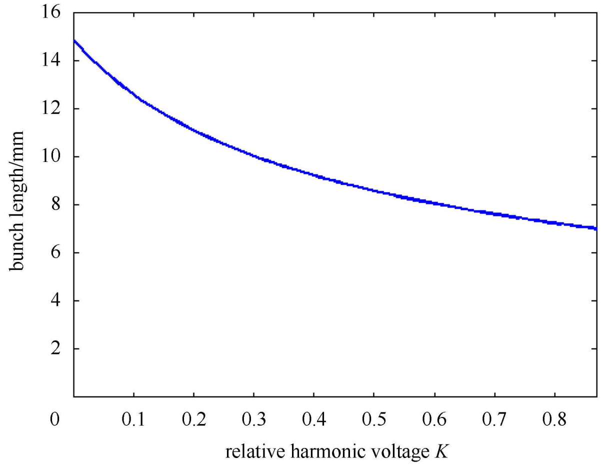

Fig. 3. Bunch lengths for several values of K smaller than the optimum value of 0.86.

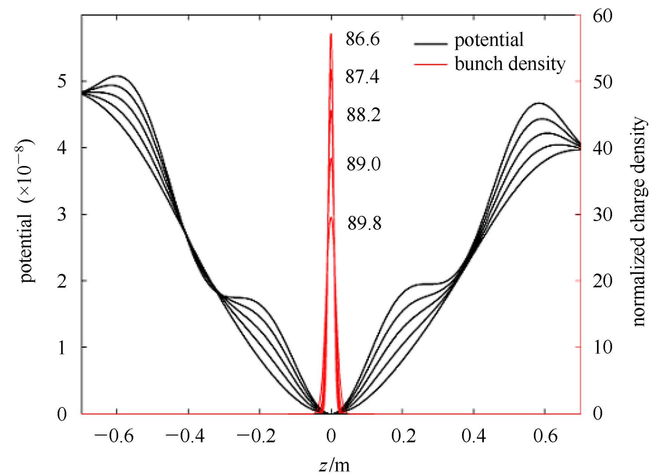

Fig. 4. Potential and density distributions for several values of the tuning angle.

2.2 Passive harmonic cavity

The harmonic voltage of the passive harmonic cavity is induced by the beam itself. The combined voltage of a double RF system can be written as

$$V(\tau)=V_{\rm rf}\sin(\omega_{\rm rf}\tau+\phi_{\rm s})-2I_{\rm DC}FR_{\rm s}\cos(\psi_{\rm h})\cos(n\omega_{\rm rf}\tau-\psi_{\rm h}), \tag{7}$$

where F, defined as $F={\rm e}^{-(n\omega_{\rm rf}\sigma_z)^2}$, is the bunch form factor, $R_{\rm s}$ is the shunt impedance of the passive HHC and $\psi_{\rm h}$, defined by $\psi_{\rm h}=2Q(\omega_{\rm r}-n\omega_{\rm rf})/\omega_{\rm r}$, is the HHC tuning angle. The slope of the combined voltage seen by the bunch center is given by

$$V'(0)=V_{\rm rf}\omega_{\rm rf}\cos(\phi_{\rm s})-I_{\rm DC}FR_{\rm s}n\omega_{\rm rf}\sin(2\psi_{\rm h}). \tag{8}$$

From Eq. (8), below the transition energy ($0<\phi_{\rm s}<\pi/2$) the tuning angle is in the delta quadrant ($-\pi/2<\psi_{\rm h}<0$) while above the transition energy the tuning angle is in the first quadrant ($0<\psi_{\rm h}<\pi/2$). The synchronous phase will be changed to compensate for the energy loss caused by the passive HHC. The new synchronous phase is given by

$$\phi_{\rm s}=\arcsin\left(\frac{U_0+2I_{\rm dc}FR_{\rm s}\cos^2(\psi_{\rm h})}{eV_{\rm rf}}\right). \tag{9}$$

Figure 4 shows the potential and density distributions for several values of the tuning angle at the typical beam current (0.3 A) of HLS-Ⅱ. The smaller tuning angle leads to a narrower potential well and thus the bunch will get shorter. As for the active HHC, the threshold of the tuning angle can be obtained according to the phase diagram. Fig. 5 shows the separatrices of the longitudinal phase space for several tuning angles. The threshold of the tuning angle is found to be 86.7° by scanning tuning angles from 86.6° to 88.2°. When tuning angle is adjusted to less than 86.7°, the RF acceptance of the double RF system is greatly reduced.

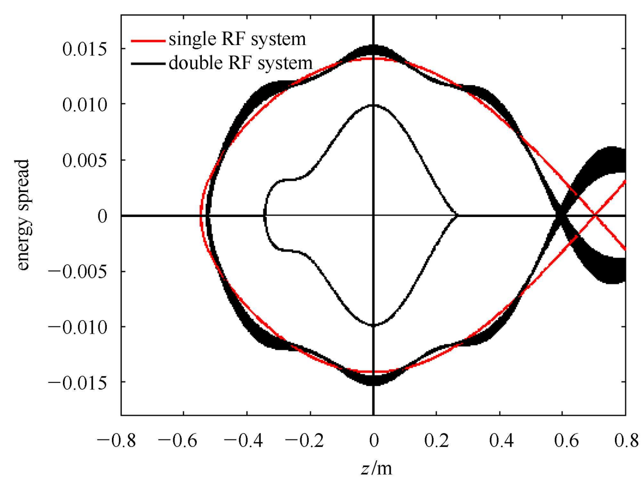

Fig. 5. RF buckets for several values of tuning angle.

Figure 6 indicates that the bunch length is reduced by passive HHCs. The smaller the tuning angle, the shorter the bunches that can be obtained, which applies to any beam current. Considering the reduction in the RF acceptance shown in Fig. 5, the tuning angle cannot be decreased indefinitely. The angle of 86.7° may be considered the optimum tuning angle at the typical beam current. Applying the optimum tuning angle to Eq. (9), the synchronous phase can be calculated in this optimum condition. Above the optimum tuning angle, the RF acceptance is large enough and varies slightly.

For an active harmonic cavity, the phase of the RF source may be adjusted to meet the neutral-phase. For the passive harmonic cavity, the harmonic voltage is generated by the bunch, thus only the tuning angle is tunable. The tuner plug should be moved in or out to reach the optimum tuning angle at each beam current.

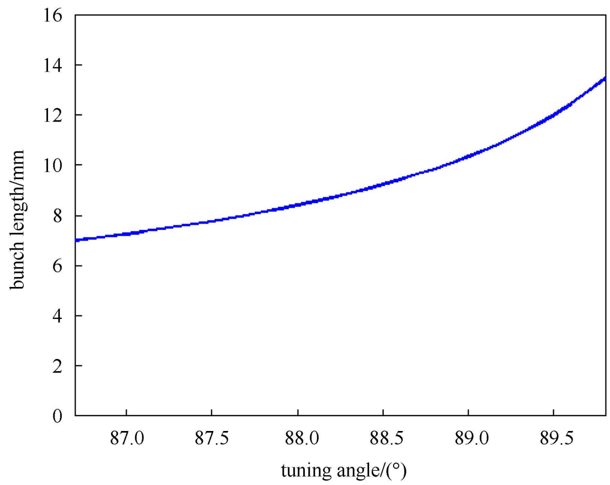

Fig. 6. Bunch lengths for several tuning angles greater than 86.7°.

3 Operating scenarios of the passive HHC

If the active HHC is used to shorten the bunches, the tuning angle must be adjusted with the beam current to give the optimum matching, which means that much more energy will flow into the cavity. As passive HHCs have the potential advantage of reducing costs, here we study in detail how to tune a passive HHC over the beam current with the aim of shortening the bunches in HLS-Ⅱ.

The optimum tuning angles for each beam current are shown in Fig. 7. The optimum tuning angle is proportional to the beam current. Based on the optimum tuning angles shown in Fig. 7, the bunch lengths are calculated at each beam current, as shown in Fig. 8.

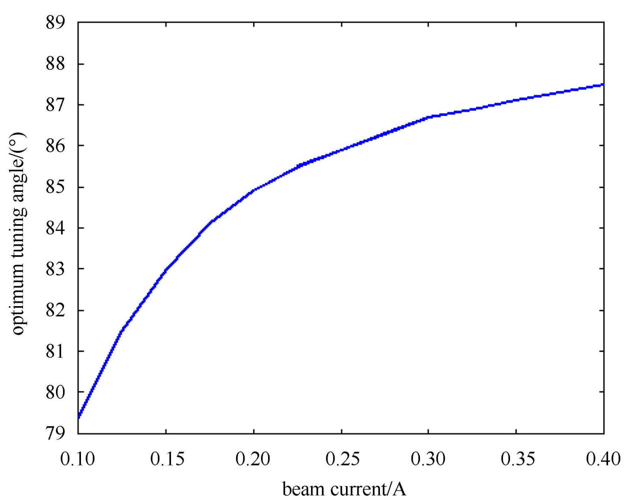

Fig. 7. Optimum tuning angle for each beam current.

In theory, the passive HHC should be adjusted to achieve the optimum tuning angle at each beam current, in theory. However, these optimum tuning angles can hardly be used as the reference quantities to move the tuner plug in or out. The passive HHC is therefore chosen to keep constant detuning and constant harmonic voltage over the range of the beam current. These are called the constant voltage scenario and the constant detuning scenario. The optimum values of the fixed voltage and the fixed tuning angle will be determined respectively.

From Fig. 7, the optimum value of the fixed tuning angle is 87.5°, which corresponds to the optimum tuning angle at 0.4 A, for the constant detuning scenario. When the tuning angle is less than this optimum value, the RF acceptance will suddenly decrease at some beam currents. The shortest possible bunch will not be obtained if the tuning angle is greater than this optimum value.

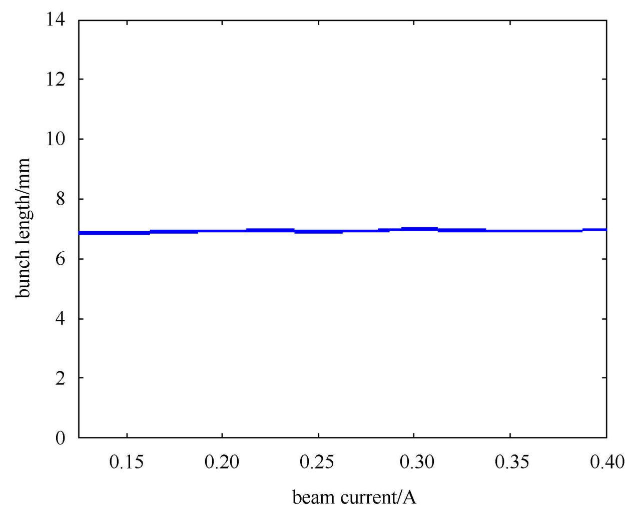

Fig. 8. Minimum bunch length at each beam current.

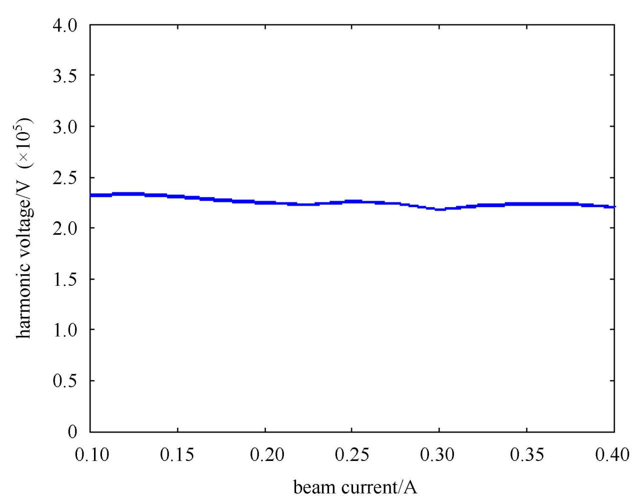

Fig. 9. Optimum harmonic voltage at each beam current.

In order to determine the optimum value of the fixed voltage, the optimum harmonic voltages are calculated with the beam current varying from 0.1 A to 0.4 A, as shown in Fig. 9. The optimum harmonic voltage varies slightly with the beam current. To avoid great reduction in RF acceptance for all beam currents, the minimum value (218 kV) of the optimum harmonic voltages is defined as the optimum value of the fixed voltage. The tuner should be moved to change the detuning of the harmonic cavity to keep a constant voltage of 218 kV over the beam current. Fig. 10 shows the linear relationship between the detuning and beam current for the constant voltage scenario.

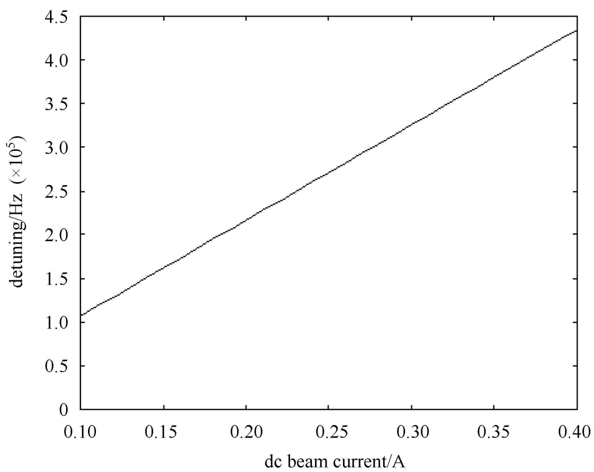

Fig. 10. Relationship of detuning to beam current for the constant voltage scenario.

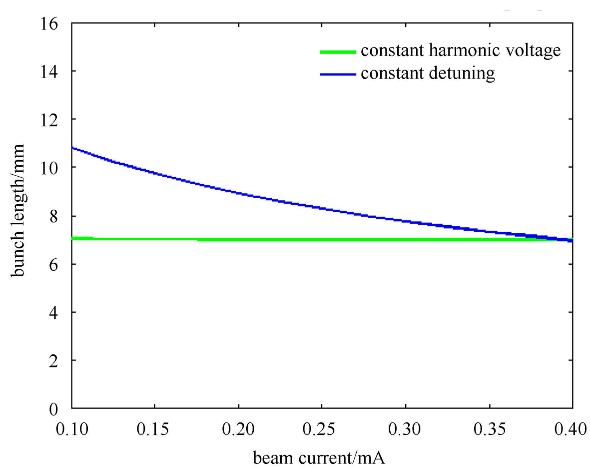

Fig. 11. Reduced bunch length vs. beam current for the constant detuning (87.5°) and the constant voltage (218 kV) scenarios.

The blue line shown in Fig. 11 is a plot of the reduced bunch length as a function of the beam current for a fixed tuning angle of 87.5°. The green line in Fig. 7 shows the bunch length with different beam currents for a fixed voltage of 218 kV. The reduced bunch lengths vary from 11 mm at 0.1 A and to 7 mm at 0.4 A for the constant detuning scenario, and the bunch lengths are around 7 mm over the beam current range for the constant voltage scenario. It is clear that the bunch length under constant voltage is compressed shorter than that under constant detuning. However, one attractive advantage for the constant detuning scenario is that it reduces the mechanical limitations of tuners [10].

4 Frequency spread of double RF system

The passive HHC may be used to increase the beam lifetime in storage rings and increase the Landau damping of synchrotron oscillations to suppress the coupled bunch instabilities [12]. The Landau damping rate is proportional to the synchrotron frequency spread. The synchrotron frequency spread of the double RF system with the HHC to reduce the bunch will be calculated in this section.

From the Eqs. (4) and (7), the Taylor expansion of the effective synchrotron potential is given by

$$\Phi(\tau)\approx a\tau^2+b\tau^3+c\tau^4, \tag{10}$$

where the coefficients a, b and c are defined as

$$\begin{aligned} a &= \frac{\alpha\omega_{\rm rf}}{2E_0T_0}[V_{\rm rf}\cos(\phi_{\rm s})-2I_{\rm dc}FR_{\rm s}n\cos(\psi_{\rm h})\sin(\psi_{\rm h})], \\ b &= \frac{\alpha V_{\rm rf}\omega_{\rm rf}^2}{6E_0T_0}[-V_{\rm rf}\sin(\phi_{\rm s})+2I_{\rm dc}FR_{\rm s}n^2\cos^2(\psi_{\rm h})], \\ c &= \frac{\alpha\omega_{\rm rf}^3}{24E_0T_0}[-V_{\rm rf}\cos(\phi_{\rm s})+2I_{\rm dc}FR_{\rm s}n^3\cos(\psi_{\rm h})\sin(\psi_{\rm h})]. \end{aligned} \tag{11}$$

For small oscillations, the synchrotron frequency is given by [13]

$$\omega_{\rm s}=\sqrt{2a}. \tag{12}$$

The synchrotron frequency spread is defined as

$$\sigma_{\omega_{\rm s}}=\frac{\alpha^2(\sigma_{\rm E}/E)^2}{\omega_{\rm s}}\left|\frac{3c}{\omega_{\rm s}^2}-\left(\frac{3b}{\omega_{\rm s}^2}\right)^2\right|. \tag{13}$$

For the constant detuning scenario, the tuning angle $\psi_{\rm h}$ is adjusted to keep the value of 87.5° over the beam current. For the constant voltage scenario, the tuner plug is moved in or out to keep the harmonic voltage of 218 kV. The relationship between detuning and beam current is shown in Fig. 10. The synchrotron frequency spread for the two scenarios are shown in Fig. 12. The black line shows the frequency spread of the single RF system, while the blue line and the green line show the frequency spread of the double RF system for the constant detuning and the constant voltage scenarios respectively. It indicates that the passive HHC dramatically increases the synchrotron frequency spread, which may increase the Landau damping rate.

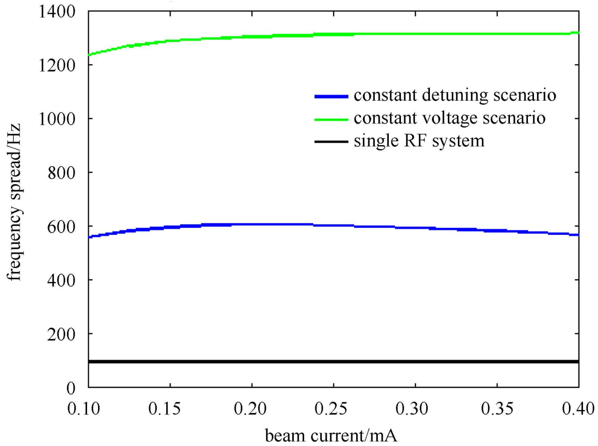

Fig. 12. Frequency spread for the constant detuning and the constant voltage scenarios.

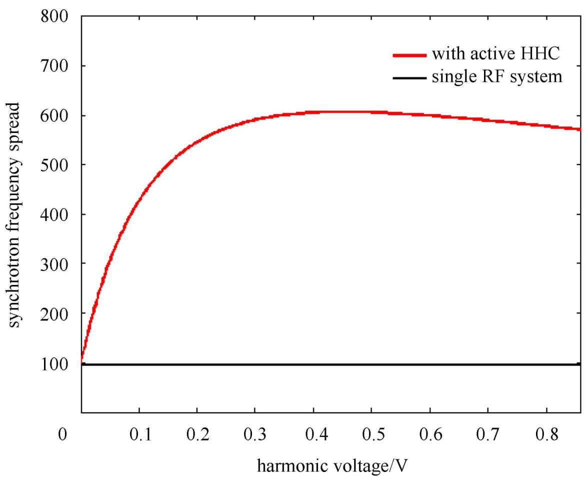

Fig. 13. Frequency spread variation with the voltage of the active HHC.

For the active HHC, the synchrotron frequency spreads are calculated with the harmonic voltage varying from 0 to the optimum harmonic voltage of 218 kV. Fig. 13 shows that the frequency spread increases sharply and then falls slowly. The calculated results show that both passive and active HHC may be used to increase the Landau damping rate.

5 Conclusions

In this paper, a HHC is considered as an effective measure to shorten the bunch length in storage rings. The HHC can steepen the voltage slope seen by the bunch in order to narrow the potential well and consequently reduce the bunch length. Based on the parameters of HLS-Ⅱ, both active and passive HHC are discussed.

The optimum relative harmonic voltage at the typical beam current of 0.3 A for an active HHC is 0.86, above which the RF acceptance will decrease suddenly. The optimum tuning angle at the typical bean current for passive HHC is 86.7°, below which the RF acceptance will decrease suddenly. Operating scenarios for the passive HHC are discussed in detail. The optimum value of the fixed tuning angle is 87.5° for the constant detuning scenario and the optimum value of the fixed voltage is 218 kV for the constant voltage scenario. Based on the determined optimum tuning angle and the optimum harmonic voltage, the paper compares the effects of the HHC operated in constant voltage and constant detuning on the bunch length. The reduced bunch lengths vary from 11 mm at 0.1 A to 7 mm at 0.4 A for the constant detuning scenario, while the bunch lengths are around 7 mm over the beam current range for the constant voltage scenario. The advantage of the constant detuning scenario is that it can reduce the mechanical limitations of tuners. The synchrotron frequency spreads of the double RF system are calculated and the results show that both passive and active HHCs may be used to reduce the bunch length and increase the Landau damping in a storage ring.

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